# Problem Set #2
# Macroeconomic Theory III

## 1 Marginal Propensity to Consume out of Current Income

Consider the CEQ-PIH consumption function which leads to the random walk representation for consumption.

Consider the following income processes: ($\varepsilon_t$ has $E_{t-1}\varepsilon_t = 0$)
(a) i.i.d. $y_t = \bar{y} + \varepsilon_t$
(b) autoregressive: $y_t = \bar{y} + \rho y_{t-1} + \varepsilon_t$
(c) random walk: $y_t = \bar{y} + y_{t-1} + \varepsilon_t$
(d) persistent changes in income: $\Delta y_t = \bar{y} + \rho \Delta y_{t-1} + \varepsilon_t$ (i.e. $(y_t - y_{t-1}) = \bar{y} + \rho\left(y_{t-1} - y_{t-2}\right) + \varepsilon_t$)
The change in consumption at time $t$ can be expressed as:

$$\Delta c_t = \mu \varepsilon_t$$

For each of the processes above find $\mu$. Interpret your results. What are the implications of these results for the relative volatility of consumption and income?

## 2 Durable Goods PIH

Suppose consumer's have the following preferences over durable goods (there are no non-durables here):

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(S_t\right)$$

where

$$S_t = (1-\delta)\, S_{t-1} + c_t$$

where $S_t$ represents the stock of durables and $c_t$ the purchase of new durables, at time $t$. Consumer's have access to a financial market with no borrowing constraints. Labor income is the only source of uncertainty, the interest rate is constant and equal to $r$.

$$A_{t+1} = (1+r)\,(A_t + y_t - c_t)$$

(a) Write out the maximization problem the agent faces. Show that the first order condition for optimality is:

$$\sum_{j=0}^{\infty} \beta^j (1-\delta)^j E_t u'(S_{t+j}) = \beta(1+r) \sum_{j=0}^{\infty} \beta^j (1-\delta)^j E_t u'(S_{t+1+j}) \qquad (1)$$

(b) Show that (1) implies:

$$u'(S_t) = \beta R E_t u'(S_{t+1}) \qquad (2)$$

(hint: take (1) for $t+1$, multiply it by $\beta(1-\delta)$ and take $E_t(\cdot)$ on both sides; subtract this from (1) for period $t$).

(c) Alternate route: Show that the budget constraint and the accumulation equation implies that

$$\tilde{A}_{t+1} = (1+r)\left(\tilde{A}_t + y_t - S_t\left[1 - \frac{(1-\delta)}{(1+r)}\right]\right)$$

where $\tilde{A}_t = A_t + S_{t-1}(1-\delta)$. You can interpret $1 - \frac{(1-\delta)}{(1+r)}$ as the (shadow) cost of renting a unit of a durable good and $\tilde{A}$ as total net wealth.

Rewrite the problem for the consumer in terms of $\tilde{A}$ instead of $A$ and derive the first order condition. You should arrive at (2) directly.

(d) Show that if $u$ is quadratic and $\beta(1+r) = 1$ then (2) implies that,

$$\Delta c_t = u_t - (1-\delta)\, u_{t-1}.$$

i.e. the innovations in consumption have a MA(1). Interpret.

# 3 Hand-to-Mouth Workers in the Ramsey Growth Model

Consider the following variantion of the simplest neoclassical growth model. Half of the population, the 'hand-to-mouth' consumers, simply consume any labor income they earn each period – they never own any assets whatsoever. The other half, the 'savers', have preferences and choices as in the standard neoclassical model. There is no population growth and we conveniently normalize the total population to be (a continuum) of size 2.

The preferences for the savers are standard,

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

for some $\beta < 1$, and $u$ twice continuously differentiable, increasing and strictly concave with the INADA condition $\lim_{c \to 0} u'(c) = \infty$.

Technology is given by the constant returns to scale Cobb-Douglas production function

$$Y_t = K_t^{1/3} L_t^{2/3}.$$

Labor, $L_t$, is supplied inelastically by both types of agents each period with total labor supply normalized to 1. The savers and the hand-to-mouth agents each supply 1/2.

The resource constraint is,

$$C_t + K_{t+1} = Y_t + (1-\delta) K_t$$

where $C_t \equiv c_t^1 + c_t^2$ is aggregate consumption and $c^1$ represents consumption of hand to mouth consumers and $c^2$ consumption of savers.

Notice that we do not describe the preferences of the hand-to-mouth agents, just their behavior.

(a) Setup the standard description of markets for labor and capital, stating the budget constraints faced by savers and hand to mouth consumers, the (static) problem of the firm. Define a competitive equilibrium.

(b) Show that in equilibrium the labor income and consumption of the hand-to-mouth agents is a constant fraction $\lambda$ of output $Y_t$. Determine $\lambda$.

(c) Argue that the competitive equilibrium is Pareto Optimal for the 'savers' in the following sense, it solves:

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to,

$$c_t + K_{t+1} = (1-\lambda) K_t^{1/3} L_t^{2/3} + (1-\delta) K_t$$

where $\lambda$ is a constant fraction of output that goes to the hand-to-mouth agents found in (b).

(d) Does the introduction of the hand-to-mouth consumers affect the steady state level of capital?

(e) Does the introduction of the hand-to-mouth consumers affect the equilibrium dynamics of consumption, output and capital relative to the case without hand-to-mouth consumers? Discuss: stability, uniqueness of the steady-state, monotonicity and the speed of convergence to the steady state (hint: for the speed of convergence take a linear approximation around the steady state with and without the hand-to-mouth consumers)

# 4 Precautionary Savings in General Equilibrium

**(this problem will not be graded)**

Let utility be given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $u(c) = -\exp\{-c\}$. Assume the standard intertemporal budget constraint

$$A_{t+1} = (1+r)(A_t + y_t - c_t).$$

Note: we do not necessarily impose $\beta(1+r) = 1$. Assume that $y_t$ is *i.i.d.* across time and agents. Let $y_t = \bar{y} + \varepsilon_t$ where $\varepsilon_t$ is iid and $E_{t-1}\varepsilon_t = 0$.

(a) Show that the consumption function,

$$c_t = \frac{r}{1+r}\left[A_t + y_t + \frac{1}{r}\bar{y}\right] - \pi$$

for some $\pi$ implies

$$\Delta c_t = \frac{r}{1+r}[y_t - \bar{y}] + r\pi$$

(b) Use the Euler equation and your results in (a) to show that the consumption function in (a) is optimal for some $\pi$ (hint: use the Euler equation to guess and verify the optimality of the above consumption function) which depends on $r$ and the distribution of $\varepsilon$.

(c) Show that $\pi > 0$ if $\beta(1+r) = 1$. Compare this to the CEQ-PIH case. How does $\pi$ depend on the uncertainty in $y_t$?

(d) Argue that in a steady state equilibrium where aggregate consumption and assets are constant we must have $\pi = 0$. This pins down the equilibrium interest rate, $r$.

Compute the equilibrium interest rate $r^e$ for $\beta = .96$, $\bar{y} = 1$ and with $\varepsilon_t$ distributed normal with mean zero and standard deviation equal to 0.2 (this distributional assumption allows you to find an expression for $E \exp(-\varepsilon)$). Compare this to the interest rate that prevails without uncertainty.

If we added capital, what could be said about the capital stock?