Problem Set #1
Macroeconomic Theory III

Everyone should turn in their own problem set. You are encouraged to work with a group. If you do please indicate all the members of your group.

1 Intertemporal Elasticity of Substitution, Risk Aversion and the Cost of Fluctuations

The statement of the question is long precisely so your answer may be shorter – please do not display all the steps, just the main ones.

This problem seeks to familiarize you with the non-expected utility preference specification introduced by Kreps and Porteus (1978) and elaborated by Epstein and Zin (1991), Weil (1990). You are also asked to cost of fluctuations in consumption as Lucas (1985) with these preferences.

Let remaining lifetime utility *after the resolution of current consumption* at time t be given by v_t. Assume v_t satisfies:

$$v_t = \left[(1-\beta)\, c_t^{\rho} + \beta \left(E_t v_{t+1}^{\alpha} \right)^{\frac{\rho}{\alpha}} \right]^{\frac{1}{\rho}} \tag{1}$$

for $1 \geq \rho \neq 0$ and $1 \geq \alpha \neq 0$. Think of this equation as defining a stochastic process for $\{v_t\}$ given a stochastic process for $\{c_t\}$. In any period t *before the resolution of current consumption* we can define lifetime utility by u_t as:

$$u_t = \left(E_t v_t^{\alpha} \right)^{\frac{1}{\alpha}} \tag{2}$$

Questions:

(a) Show that if $\{v_t\}$ is implied by $\{c_t\}$ using (1) then $\{\lambda v_t\}$ is implied by $\{\lambda c_t\}$ using (1) for any $\lambda > 0$. Show that if $c_t = \psi$ for some constant ψ then v_t and u_t are also constant and equal to ψ. [1]

(b) For any stationary process $c = \{c_t\}$, with associated utility u_0, let $\bar{c}$ denote the deterministic and constant sequence of consumption $(Ec_t, Ec_t, \cdots)$. Define our measure of welfare loss from the variability of $\{c_t\}$ to be that constant percentage increase, η, in the process c that is required to make the individual indifferent between $(1+\eta)\,c$ and $\bar{c}$.

Use your results in (a) to show that:

$$1 + \eta = \frac{Ec_t}{u_0}$$

(c) Show that if $\alpha = \rho$ then:

$$v_0 = \left[(1-\beta) \left(c_0^\rho + E_0 \sum_{t=1}^{\infty} \beta^t c_t^\rho \right) \right]^{\frac{1}{\rho}} \tag{3}$$

which is just a monotonic transformation of the more familiar

$$\frac{c_0^\rho}{\rho} + E_0 \sum_{t=1}^{\infty} \beta^t \frac{c_t^\rho}{\rho}$$

utility specification.

(d) (i) Show that if $\{c_t\}$ is deterministic then (3) holds (without the need for the expectation operator of course).

Use this to compute the value of utility obtained if $c_{lh} = (c_l, c_h, c_l, c_h, ...)$, denote this lifetime utility by $v_{0,l}$. and $v_{0,h}$, $c_{hl} = (c_h, c_l, c_h, c_l, ...)$. How does your answer depend on the parameters α and ρ?

(ii) It is useful to think of the limit as $\beta \to 1$. Show that $v_{0,h}$ and $v_{0,l}$ converge to the same value. What is the intuition for this? Compute the welfare loss as a fraction of the constant consumption stream that equals the average $\bar{c} \equiv \frac{1}{2}c_h + \frac{1}{2}c_l$. How does your answer depend on the parameters α and ρ?

(iii) Compute the value of u_0 if the sequences c_{lh} and c_{hl} occur with probability 1/2 (which is revealed at the beginning of period 0) as $\beta \to 1$. This

[1] Given these results we could say that v_t (u_t) represents the “certainty equivalent constant consumption” from t on, after (before) the resolution of time t consumption.

should be very easy given your answer to (ii). How does your answer depend on the parameters α and ρ?

(e) (note: we no longer work with the limit $\beta \to 1$ here, we are back to β bounded away from 1) Find an expression for utility, u_0, when $\{c_t\}$ is such that $c = (c_h, c_h, ...)$ with probability 1/2 and $c = (c_l, c_l, ...)$ with probability 1/2. How does your answer depend on the parameters α, ρ?

(f) Let c_t be *i.i.d.* distributed over time with outcomes c_l and c_h each with probability 1/2. Combine (1) and (2) and the fact that with i.i.d. consumption $u_t = u_0$ to show that u_0 must solve of the form:

$$u_0 = \left[\frac{1}{2}\left[(1-\beta)\, c_l^{\rho} + \beta u_0^{\rho}\right]^{\frac{\alpha}{\rho}} + \frac{1}{2}\left[(1-\beta)\, c_h^{\rho} + \beta u_0^{\rho}\right]^{\frac{\alpha}{\rho}}\right]^{\frac{1}{\alpha}} \tag{4}$$

Think of this equation as implicitly defining u_0 (whether it does so uniquely is an interesting question I am not asking). Why does this equation involve both of the parameters α and ρ?

(g) You will now use equation (4) to compute u_0 and then the welfare loss, η, as defined in (b) of consuming the i.i.d. sequence described in (f).

Use the following parameters througout: $\beta = .96$ and $\log c_h = 1.02$ and $\log c_l = 0.98$ (which gives a std of log consumption of 2.7%, higher than that used by Lucas (1987)). Compute u_0 and η for all nine combinations of the parameters: $\alpha = 1,\ 1/2,\ -1$ and $\rho = 1,\ 1/2,\ -1$.Display your results for η only on a table in percentage terms (i.e. multiply by 100). Do your results seem to vary more with ρ or α?

Hint: to solve (2) you can iterate on this equation: plug in a good initial guess (such as $u_0 = 1$ which is a good guess based on the results in (a)) on the right hand side and compute the value of u_0 that results, then use this new number on the right hand side again, etc... Repeat this process until the numbers you get converge (that is, when the absolute value difference is less than 10^{-10}, say). You can work with any program for this: Matlab or Excel will certainly do. (you probably won't have to iterate more than 500 times (i.e. copy your formula down 500 rows if you are using excel).