Markov Chain Hallway and Poisson Forest Environment Generating Distributions

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1   Markov Chain Hallway Distribution

Random 2D hallway-type maps can be sampled from a generating distribution based on a Markov chain, which allows the frequency of turns and other characteristics to be embedded in the state-transition probabilities. Sampling a history of states in the Markov chain corresponds to sampling a sequence of straight hallway segments and turns, which are then used to construct a map.

To generate hallways that are unable to cross over themselves, we require that the Markov chain alternate between pairs of left turns and pairs of right turns (i.e. `LEFT-LEFT-RIGHT-RIGHT-LEFT-LEFT`, etc). Therefore, the state representation must store which type of turn preceded each straight segment, and which type of straight segment preceded each turn, regardless of how many segments ago the previous turn or straight segment occurred. Let $s \in \{1, 2, 3, 4, 5, 6, 7, 8\}$ be the state of the Markov chain. The eight states are:

1. `STRAIGHT` segment oriented left, following a #5 `LEFT TURN`
2. `STRAIGHT` segment oriented up, following a #7 `RIGHT TURN`
3. `STRAIGHT` segment oriented up, following a #6 `LEFT TURN`
4. `STRAIGHT` segment oriented right, following a #8 `RIGHT TURN`
5. `LEFT TURN` segment, following a #3 `STRAIGHT` segment
6. `LEFT TURN` segment, following a #4 `STRAIGHT` segment
7. `RIGHT TURN` segment, following a #1 `STRAIGHT` segment
8. `RIGHT TURN` segment, following a #2 `STRAIGHT` segment

The transition probability matrix is:

$$
T = \begin{bmatrix}
p_{ss} & 0 & 0 & 0 & 0 & 0 & p_{st} & 0 \\
0 & p_{ss} & 0 & 0 & 0 & 0 & 0 & p_{st} \\
0 & 0 & p_{ss} & 0 & p_{st} & 0 & 0 & 0 \\
0 & 0 & 0 & p_{ss} & 0 & p_{st} & 0 & 0 \\
p_{ts} & 0 & 0 & 0 & 0 & 0 & p_{tt} & 0 \\
0 & 0 & p_{ts} & 0 & p_{tt} & 0 & 0 & 0 \\
0 & p_{ts} & 0 & 0 & 0 & 0 & 0 & p_{tt} \\
0 & 0 & 0 & p_{ts} & 0 & p_{tt} & 0 & 0
\end{bmatrix}
\tag{1}
$$

where p_{ss}, p_{ts}, p_{tt}, and p_{st} specify the probability of a straight segment following a straight segment, a straight segment following a turn, a turn following a
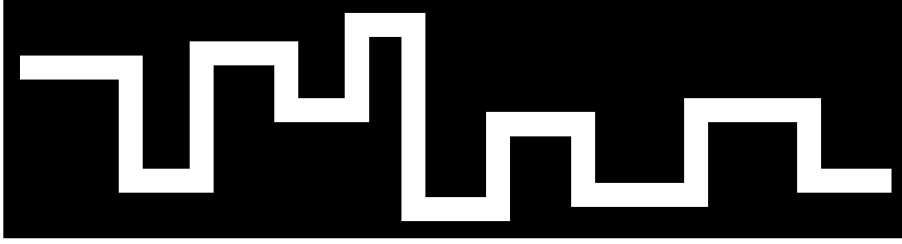
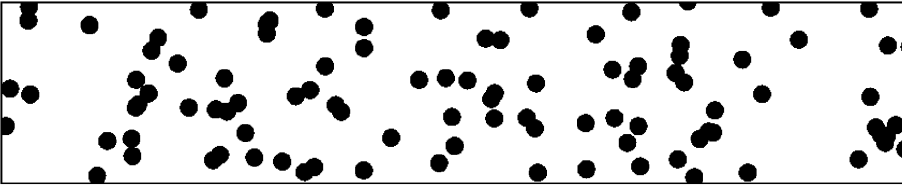Figure 1: Randomly generated Markov hallway map with $p_{st} = 0.4$ and $p_{tt} = 0$.

Figure 2: Randomly generated Poisson forest map with $\lambda = 0.05$ trees $\cdot$ m^{-2} and $r = 1$ m.

turn, and a turn following a straight segment, respectively. To ensure that the rows of T sum to one, we must have $p_{ss} + p_{st} = 1$ and $p_{ts} + p_{tt} = 1$. If desired, we can prevent U-turns (two turns following one another without a straight segment in between) by setting $p_{tt} = 0$, implying that $p_{ts} = 1$. We initialize the Markov chain to begin with one of the eight possible states, and for $k = 1, 2, \ldots, N-1$, we sample s_{k+1} from the discrete probability distribution defined by the $(s_k)^{th}$ row of T, where N is the number of segments (i.e., the length of the hallway). Figure 1 shows an example of a randomly sampled Markov hallway with $p_{st} = 0.4$ and $p_{tt} = 0$.

2 Poisson Forest Distribution

Random 2D forest-type environments can be generated using the Poisson distribution. Let A be the area of a rectangular forest. The number of trees in the forest, X, is distributed $X \sim Poisson(\lambda A)$, where λ is the mean number of trees per unit area. The sampled number of trees are then placed uniformly at random within the forest, with radius r. The Poisson process can be modified using rejection sampling during the tree-placement phase to enforce a minimum distance between tree centers if desired. Figure 2 shows an example of a randomly generated Poisson forest with $\lambda = 0.05$ trees $\cdot$ m^{-2} and $r = 1$ m.