

PFC/JA-89-30

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June 1989

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Submitted for publication in: Journal of Spacecraft and Rockets

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Abstract

The theory and experimental data for the efficacy of plasma contactors as current collectors is reviewed. Previous theoretical work on plasma contactors has fallen into two categories: collisionless double layer theory (describing space charge limited contactor clouds) and collisional quasineutral theory. Ground based experiments at low current are well explained by double layer theory, but this theory does not scale well to power generation by electrodynamic tethers in space, since very high anode potentials are needed to draw a substantial ambient electron current across the magnetic field in the absence of collisions (or effective collisions due to turbulence). Isotropic quasineutral models of contactor clouds, extending over a region where the effective collision frequency ν_e exceeds the electron cyclotron frequency ω_{ce} , have low anode potentials, but would collect very little ambient electron current, much less than the emitted ion current. We present a new model, for an anisotropic contactor cloud oriented along the magnetic field, with $\nu_e < \omega_{ce}$. The electron motion along the magnetic field is nearly collisionless, forming double layers in that direction, while across the magnetic field the electrons diffuse collisionally and the potential profile is quasilinear. Using a simplified expression for ν_e due to ion acoustic turbulence, an analytic solution has been found for this model, which should be applicable to current collection in space. The anode potential is low and the collected ambient electron current can be several times the emitted ion current.

1 Introduction

Plasma contactors are plasma clouds which allow the passage of charge between an electrode and an ambient plasma. They have been proposed for use in power generating devices such as electrodynamic tethers[1] because they may substantially reduce the impedance of the electron current collection from the ionosphere and make the emission of electrons much less energetically expensive than using an electron gun. In this paper we will concentrate on plasma contactors used at an anode to collect electrons in the ionosphere or some other ambient plasma. Such a contactor will emit ions, as well as collect electrons. Two figures of merit for such a contactor are its impedance ϕ_0/I , and the gain ξ , defined as

$$\xi = I/I_i(r_{anode})$$

where $I = I_i + I_e$ is the total current (emitted ions and collected electrons) at the anode (at $r = r_{anode}$), and ϕ_0 is the potential of the anode with respect to the ambient plasma. The impedance determines the maximum power that can be generated by a tether, since the total tether potential ϕ_{total} is fixed at $v_o B_0 L$, where v_o is the orbital velocity, B_0 is the ambient magnetic field, and L is the length of the tether. If R_{load} is the load resistance and R_t is the tether resistance (plus any other impedance in the circuit), then

$$\phi_{total} = R_{load}I + R_t I + \phi_0(I)$$

The maximum power $R_{load}I^2$ at fixed ϕ_{total} and R_t is obtained when $R_{load} = R_t + d\phi_0/dI \approx R_t + \phi_0/I$. (If the power generation is to be much more than 50% efficient, as it must be to compete with alternate means of power generation such as fuel cells, then the maximum power is somewhat lower than this.) The power is greatest when the contactor impedance is lowest. The gain is important because it determines the rate at which gas must be used (to produce ions), for a given total current. If the gain is high, less gas is used to collect a given current.

Both the impedance and the gain will depend on the current. In general there is a trade-off: at very low current, both high gain and low impedance are possible, but the power is low. While at high current, high gain can be obtained only at the cost of very high impedance (again resulting

in low power). Low impedance and high power are possible only with low gain. To illustrate these trends, we may consider the extreme limits. When the current is equal to the electron saturation current of the ambient plasma over the surface area of the physical anode, then the gain is infinite (since no ions need be emitted to draw this much electron current) and the contactor impedance is zero, but the power (for low earth orbit and practical tether and anode parameters) is at most tens of watts. Arbitrarily large current (and high power) may be obtained by emitting a large ion current, but unless the anode potential is high enough, it will not be possible to collect many electrons across the magnetic field, and the gain will approach 1. A basic goal of contactor research is to determine how large a gain is possible at a given power level. If it turns out that at the power levels of interest for tethers (typically tens of kW) the maximum gain is close to 1, then there is no point in using plasma contactors for current collection; in effect, the best plasma contactor is no better than an ion beam. If, on the other hand, gains at least a few times greater than 1 are possible at power levels of interest, then plasma contactors are useful as current collectors for tethers. We will present theoretical results suggesting that this is the case, although the gains are only moderate, in the range of 2 to 10. These theoretical results pertain to a regime (collisionless electron motion along the magnetic field, collisional diffusion across the magnetic field) which we expect to be valid in low earth orbit for high current contactors, but for which there have been no ground based experiments. Such experiments are very important for confirming the theory, or showing how it must be modified.

In previous work[2,3] it has been suggested that the plasma contactor cloud will consist of several different regions. First will be an inner core where the cloud will be isotropic because the two major directions of anisotropy, namely the earth's magnetic field and the direction of motion of the source will be shielded by the dense plasma from the contactor source. There will then be two outer regions where the two directions of anisotropy are manifested. Previously, it has generally been assumed that a substantial current of ambient electrons can be collected only from field lines that pass through the inner core region[2,4]. However, we will show in Sec. 5 that for conditions in low earth orbit it may also be possible to collect a significant electron current from the outer core region, where the anisotropy due to the magnetic field is important.

There has been much debate about the size of the core region over which electrons can be collected. One estimate is obtained by matching the cloud density to the ambient density[5]

$$n_{\text{cloud}}(r_{\text{core}}) \approx n_{\text{ea}}$$

and another by taking magnetic field effects into account[6]

$$\nu_e(r_{\text{core}}) \approx \omega_{ce}$$

where ν_e is the radially dependent electron collision frequency (including effective “collisions” due to turbulence) and ω_{ce} is the electron gyrofrequency. A third estimate is obtained by requiring regularity of the self-consistent potential[7]

$$\left. \frac{\partial \phi}{\partial r} \right|_{r_{\text{core}}} \approx 0$$

and finally a fourth estimate comes by requiring a consistent space charge limited flow inside the core[8]

$$m_i n_i u_i^2|_{r_{\text{core}}} \approx m_e n_e u_e^2|_{r_{\text{core}}}$$

where u_i is the outgoing ion flow velocity and u_e is the incoming electron flow velocity. These diverse theories give a wide range of current enhancement factors for the plasma cloud and suggest that determining the size of the core region is critical to the understanding of the current collection.

If we assume a spherical core cloud of radius r_{core} , then from continuity of current

$$I = I_i(r_{\text{anode}}) + I_e(r_{\text{anode}}) = I_i(r_{\text{core}}) + I_e(r_{\text{core}})$$

and the gain is

$$\xi = \frac{I_e(r_{\text{core}})}{I_i(r_{\text{anode}})} + \frac{I_i(r_{\text{core}}) - I_i(r_{\text{anode}})}{I_i(r_{\text{anode}})} + 1$$

Plasma contactor clouds enhance or produce electron current flow through two possible paths. First (the first term on the right hand side of the equation), they can serve as virtual anodes through which electrons from far away can be drawn and collected to the real anode at the center of the cloud. Secondly (the second term on the right hand side), the neutral gas associated with the cloud can become ionized, creating electron-ion pairs. The electrons will be collected to the anode,

and the ions will be repelled. For use in space with an electrodynamic tether, however, ionization of contactor neutrals is not an efficient use of neutral gas; if this is the only means by which the current is enhanced, then the same neutral gas can be used more efficiently by ionizing it internally in an ion source. Plasma contactors will be useful if they enable the ionosphere to supply electrons. The two sources of electrons in the ionosphere are the ionospheric plasma and the ionospheric neutrals. However the mean free path for ionization of the ionospheric neutral gas is so long (many kilometers) that ionization of this gas on the length scale of the plasma contactor cloud is highly unlikely. For this reason we shall assume that all ionization associated with contactors is ionization of contactor neutral gas. Therefore plasma contactors can be useful with electrodynamic tethers only if they enhance current by collecting ambient electrons from the ionosphere. The collected electron current $I_e(r_{core})$ will generally be the saturation current times the area of the core cloud $4\pi r_{core}^2$, or, if the contactor is only collecting electrons along magnetic field lines running into the core cloud, then $I_e(r_{core})$ will be the saturation current times $2\pi r_{core}^2$. (If, as we consider in Sec. 5, the core cloud is not spherical but is elongated in the direction of the magnetic field, then r_{core} is the minor radius, across the magnetic field.) For this reason the size of r_{core} is crucial to the effectiveness of plasma contactors as electron collectors in space.

In Sec. 2 we will review ground-based experiments on plasma contactors. In Sec. 3 a collisionless double layer theory will be derived, along the lines of Wei and Wilbur[8] and Katz[9], and it will be shown that this theory provides a good quantitative description of ground-based experiments at moderately low currents, but it will not be applicable to space-based contactors except at extremely low current and power. If the electrons are strictly collisionless, then the magnetic field prevents electrons from reaching the anode unless they originate on field lines that pass close to the anode (which limits the current that can be collected) or the anode potential is high enough to pull electrons across the magnetic field to the anode from some distance away. A necessary condition for this, which depends on the anode radius r_{anode} , was found by Parker and Murphy[10]. Another constraint on r_{anode} is that it must be less than the inner radius of the double layer. We will show that any spherically symmetric double layer with space-charge limited current greater than a very low limit (about 50 mA collected electron current, corresponding to 1 mA emitted ion current,

for dayside equatorial low earth orbit, and even lower current for nightside) which satisfies these constraints must have an anode radius that is close to r_{core} . Such a plasma contactor would serve no purpose, since it would hardly collect any more ambient electron current than the bare anode. This means that an unmagnetized collisionless space-charge limited double layer model, as analyzed by Wei and Wilbur[8], cannot apply in space, except at very low currents, no matter how great the potential is. If the anode emits a current greater than this, at zero initial velocity (i.e. space-charge limited), and if the electrons are assumed to be collisionless, then the double layer cannot be spherically symmetric, regardless of the potential. Electron collection will be inhibited across the magnetic field, and the collected electron current will be lower than predicted by the Wei-Wilbur theory[8] for that anode potential and emitted ion current. Although a theory valid in this regime is not available, we can still obtain an upper limit on the collisionless electron current that can be collected, and a lower limit on the anode potential, for a given ion current, by assuming that the Parker- Murphy condition is marginally satisfied for a double layer obeying the equations of Wei and Wilbur, and ignoring that constraint that the inner radius of such a double layer must occur at a greater radius than r_{anode} . We then obtain an upper limit to the power than can be generated by a plasma contactor collecting electrons to a 20 km long tether in space, in the absence of electron collisions. This maximum power is quite low, only a few hundred watts, less than an order of magnitude above the power that can be generated by a tether without a plasma contactor, using a bare anode to collect electrons.

At higher emitted ion current, there will be a region where the electrons cannot go straight to the anode, but where ambient electrons will be trapped, to keep the plasma quasineutral. These electrons will remain trapped for a time long compared to the time it would take for an unmagnetized electron to go straight to the anode. If there are effective collisions due to instabilities, some of these trapped electrons may be able to diffuse to the anode, and the collected electron current may be much greater than what would be found in the collisionless model.

In Sec. 4 we will describe a collisional quasineutral theory, related to the models of Dobrowolny and Iess[7] and Hastings and Blandino[4], which is more applicable to contactors emitting a large ion current (either in space or in ground based experiments) than the collisionless models. This model

assumes that ambient electrons can only be collected over the cross-section of the isotropic inner core region, where the effective collision frequency is greater than the electron gyrofrequency. With this restriction, the model still predicts that very little ambient electron current can be collected in space. In Sec. 5, we will describe preliminary work on a model of the outer core region, in which the motion along the magnetic field is collisionless, forming a double layer, but the motion across the magnetic field is collisional and quasineutral. This model, which is expected to be applicable to contactors in space, suggests that significant current may be collected from this outer core region, with low contactor impedance. Unfortunately there are, to our knowledge, no experiments in this regime, to which the theory can be compared. Conclusions will be presented in Sec. 6.

2 Brief Review of Experimental Work

In this section, three plasma contactor experimental setups and their resultant data sets are presented. This information will be used to analyze a plasma contactor's capability to enhance current collection. The work presented is by members of Istituto di Fisica dello Spazio Interplanetario (IFSI), Frascati, Italy and that done by P. Wilbur and colleagues at Colorado State University (CSU) as well as that of M.J. Patterson of NASA Lewis Research Center (LeRC).

2.1 Case 1: Frascati

Experiments have been carried out by Vannaroni, et al., [11] in both the Freiburg plasma chamber and the $0.5m^3$ Frascati vacuum chamber. The experiments pursued in the smaller Frascati chamber can be viewed as a characterization of the hollow cathode device later used in the Freiburg plasma facility. No plasma simulator was used at Frascati.

The dimensions of the Freiburg facility are 2.5 m in diameter and 5.5 m in length, with a Kaufman thruster used to simulate the ionospheric plasma. External Helmholtz coils were used to compensate for Earth's magnetic field as well as to generate field components within the chamber if desired. We present only the data set for which the terrestrial magnetic field compensation occurred. Preliminary analysis of the data indicates that parameters show a 10-20% variation[12] when the plasma is magnetized with the Helmholtz coils. The Kaufman thruster is operated with

argon. The Ar^+ was expelled from the thruster at an energy of 60 eV. The thruster plasma source and the hollow cathode assembly were separated by 370 cm.

For the hollow cathode device, a cathode to anode/keeper discharge current theoretically results in a high density region of weakly ionized, highly collisional plasma freely expanding into the surrounding vacuum. Upon expansion to large distances away from the contactor, the cloud is taken to be low density and collisionless. Experimentally, the Langmuir probe is used to obtain the plasma profile and it is assumed that the plasma potential in the plasma immediately surrounding the hollow cathode is equal to the keeper voltage, thereby normalizing the values of the plasma potential in order that plasma variation may be studied.

For purposes of comparison, the Freiburg operating condition of interest was that in which the hollow cathode and the plasma simulator functioned simultaneously. With both plasma sources operating, the chamber pressure was 3.2×10^{-4} Torr. For this operating condition, the plasma simulator was electrically connected to the chamber wall and the hollow cathode was then polarized with respect to the plasma simulator. This interaction study was hindered by the fact that the resolution of the Langmuir probe was not fine enough to fully examine the potential profile and that the profile did not span the entire distance along the chamber axis between the hollow cathode and the plasma simulator.

With the hollow cathode assembly at the same potential as the plasma simulator and the anode of the hollow cathode at +11V, an increase in the temperature of the electron population was detected along with an appreciable $d\phi/dr$ located between 15 cm and 30 cm from the hollow cathode plasma source.

2.2 Case 2: Colorado State University

The plasma contactor laboratory tests conducted under Wilbur's direction have been accomplished with an apparatus including two separate hollow cathode devices, one simulating the ambient space plasma and the other coupling to this "ambient" plasma as a spaceborne hollow cathode would. An anode design was chosen for the contactor hollow cathode such that the size of the contactor anode could be altered. Thus the effect of the anode size on the electron collection

Table 1: Experimental Cases

Experiment Location	Chamber Pressure (torr)	Gas	
		Hollow Cathode	Plasma Simulator
Frascati	2.4×10^{-3}	Xe	N/A
Freiburg	3.2×10^{-4}	Xe	Ar
CSU	4.3×10^{-6}	Xe	Xe
LeRC	2.3×10^{-6}	Xe	Xe
LeRC	2.3×10^{-6}	Xe	Xe

process could be examined. The capability existed to bias the contactor with respect to the “ambient” plasma, the simulator, and the chamber wall. When the contactor was operating in electron collection mode, the case of interest within the context of this paper, the hollow contactor was biased with respect to the simulator which was electrically connected to the chamber wall. The experimental parameters for the case chosen for study within this work are shown in Table 1. (See Section 3.2 for further discussion of CSU data and operating parameters.)

2.3 Case 3: NASA Lewis Research Center

Patterson [13] has conducted a series of plasma contactor studies at NASA LeRC. Chamber tests of the CSU contactor and simulator have also been performed in conjunction with Patterson at NASA LeRC [14]. While the electrical configurations of the hollow cathode and the plasma simulator in both the CSU and NASA LeRC test facilities remain the same, the dimensions and the pressure conditions of the two facilities do not, as Table 1 shows.

Double layer formation is seen in the Wilbur and Patterson data sets at low levels of electron current collection. According to Patterson, at high current levels (i.e. > 1.0 A), deviations from the spherical double-sheath theory [8] are seen in the data due to the development of sheath asymmetry and bulk ionization. Wilbur [15], however, has found a clearly demarcated double layer region at 1.2 A at a standoff distance from the hollow cathode between approximately 25 to 40 cm. (This was

for the 12 cm anode, with test conducted at LeRC.) Due to lack of resolution in the Frascati data set, the question remains as to whether hollow cathode/plasma simulator configurations besides that of CSU/LeRC yield the double layer result at low, or even high, electron current collection levels. Upcoming experiments in the new Frascati plasma facility will address such questions.

The 24 cm anode LeRC contactor vs. the 12 cm CSU contactor tested separately in the LeRC facility demonstrate an order of magnitude difference in current collected, favoring the larger anode size. This type of observation has also been noted at the CSU facility when the hollow cathode anode size was varied under the same plasma simulator operating conditions. Chamber wall effects and Langmuir probe saturation hindered the measurement of hollow cathode current collection in the Freiburg experiments; analysis of isolated cases of the hollow cathode biased with respect to the plasma source indicated that the current collected was an order of magnitude less than predicted by the Dobrowolny and Iess model[7].

The data indicates that there are five regions of plasma contactor operation occurring within the laboratory setting. The first region, with currents less than 100mA being collected, does not offer any particular structure in the plasma profile. In this case, apparently, the emitted ion density is less than the ambient density even at the anode, so any potential drop will occur in a sheath leaning against the anode, rather than in a double layer, and the collected current will just be the electron saturation current over the area of the outer surface of the sheath. A transition region then exists for current levels just above 100mA in which a spherical double layer[8] appears to be present but the contactor plume is unignited (i. e. there is no diffuse glow). The third region, traversing the current range up to 1A, has breakdown of the spherical double layer and multiple as well as cylindrical double layers appear; this region is also ignited flow (i. e. there is a diffuse glow). Just above 1A, the ignited flow causes increased ionization. Presumably, streaming instabilities will have set in in this fourth region. The spherical double layer model is completely invalid in this region. Plume domination then occurs in the fifth region, where currents are well in excess of 10A.

3 Double-Layer Theory and Implications

3.1 Collisionless Unmagnetized Model

Ground-based experiments in which double layers are seen are well described by a collisionless unmagnetized model as we will show. We assume two components of plasma, an ambient component and a contactor component. The ambient ions and electrons are maxwellian at positions r well beyond the double layer, with ion and electron temperatures T_{ia} and T_{ea} , and density n_{∞} . The contactor plasma has maxwellian electrons at temperature T_{ec} and cold ions streaming radially out from a plasma source localized near the anode, with ion current I_i . The potential drop ϕ_0 between the source, at $r = r_{source}$, and the ambient plasma at $r \rightarrow \infty$, is assumed to be much greater than any of the temperatures, and the radius at which the double layer forms is assumed to be much greater than a Debye length. With these assumptions, the plasma is quasineutral everywhere except inside the double layer, at $r_{inner} < r < r_{outer}$. (Here r_{outer} is the radius, called r_{core} in the Introduction, at which the ambient electron saturation current is collected.) Inside the contactor cloud, at $r < r_{inner}$, there are no ambient ions, and the density of ambient electrons, which have been accelerated in the double layer, is much less than the density of contactor electrons, so quasineutrality requires $n_{ec}(r) = n_{ic}(r)$. The densities of contactor electrons and ions are related to the potential ϕ (defined relative to $r \rightarrow \infty$) by

$$n_{ec} = n_{source} \exp[(\phi - \phi_0)/T_{ec}] \quad (1)$$

$$n_{ic} = n_{source} (r_{source}/r)^2 [1 + (\phi_0 - \phi)/T_{ec}]^{-1/2} \quad (2)$$

where we have assumed that ions are emerging from the source at the sound speed $(T_{ce}/m_i)^{1/2}$, due to acceleration in a Bohm presheath, and we have neglected any ionization or recombination occurring at $r > r_{source}$. Setting the right hand sides of Eqs.(1) and (2) equal to each other gives a transcendental equation for $\phi(r)$. It is evident that for $r \gg r_{source}$,

$$\phi(r) \approx \phi_0 - 2T_{ec} \ln(r/r_{source}) \quad (3)$$

so the potential only drops a few times T_{ec} inside the contactor cloud, much less than the total potential drop. The source density n_{source} is related to the ion current I_i by

$$I_i = 4\pi r_{source}^2 e n_{source} (T_{ec}/m_i)^{1/2} \quad (4)$$

Outside the double layer, at $r > r_{outer}$, the ambient electron density decreases from n_∞ as r decreases, because no electrons are emitted from the double layer. We assume that there are no sources of electrons, or collisions, which could fill in the resulting empty region of velocity space. From quasineutrality, the ambient ion density must also decrease as r decreases (even if the density of contactor ions, accelerated in the double layer, is small compared to the ambient ion density), so the potential must rise by an amount on the order of T_{ia} . If T_{ia} is much less than T_{ea} , then the ambient electron density is not affected by the potential, so it is reduced from n_∞ by a simple geometric factor

$$n_{ea}(r) = \frac{1}{2} n_\infty [1 + (1 - r_{outer}^2/r^2)^{1/2}] \quad (5)$$

and the potential is given by

$$\phi(r) = T_{ia} \ln(n_\infty/n_{ea}) \quad (6)$$

The potential drop from r_{outer} to ∞ is just $T_{ia} \ln 2$, much less than the total potential drop. Most of the potential drop must therefore occur in the double layer. Within the double layer, $r_{inner} < r < r_{outer}$, the plasma is not quasineutral, and Poisson's equation (for spherical symmetry)

$$\frac{1}{r^2} \frac{d}{dr} r^2 \frac{d\phi}{dr} = 4\pi(n_e - n_i) \quad (7)$$

must be satisfied subject to the boundary conditions that ϕ and $d\phi/dr$ be continuous at r_{inner} and r_{outer} . These four boundary conditions specify a solution to the second order differential equation, and the values of the free parameters r_{inner} and r_{outer} . Since most of the drop in potential occurs in the double layer, to good approximation the boundary conditions are

$$\phi(r_{inner}) = \phi_0 - 2T_{ec} \ln(r_{inner}/r_{source}) \quad (7a)$$

$$\phi(r_{outer}) = 0 \quad (7b)$$

$$d\phi/dr = 0 \text{ at } r_{inner} \text{ and } r_{outer} \quad (8)$$

If, as we have assumed, $T_{ia} \ll T_{ea}$, then the ambient ion density drops much more quickly than the ambient electron density as the potential starts to rise going inward from r_{outer} , and we can neglect the ambient ion density in Eq. (7). Similarly, since the energy of the contactor ions is greater than T_{ec} at r_{inner} , even if only by a logarithmic factor, the contactor electron density drops much more quickly than the contactor ion density in going outward from r_{inner} , and to rough approximation we can neglect the contactor electron density in the double layer. In the double layer, then, we must solve Poisson's equation, Eq. (7), with

$$n_e = \frac{n_\infty r_{outer}^2}{2 r^2} \exp(\phi/T_{ea}) [1 - \text{erf}(\sqrt{\phi/T_{ea}})] \quad (10)$$

$$n_i = n_{source} \frac{r_{source}^2}{r^2} \left(\frac{\phi_0 - \phi(r)}{T_{ec}} \right)^{-1/2} \quad (11)$$

An approximate analytic solution, which provides some physical insight, may be found when the double layer is thin, *i.e.* $r_{outer} - r_{inner} \ll r_{inner}$. Then, in the vicinity of r_{inner} , for $\lambda_D \ll r - r_{inner} \ll r_{outer} - r_{inner}$, the potential approximates a Child-Langmuir sheath, with negligible n_e

$$\phi(r_{inner}) - \phi(r) \approx 3^{4/3} T_{ec} \ln(r_{inner}/r_{source}) \left(\frac{r - r_{inner}}{\lambda_{D,inner}} \right)^{4/3} \quad (12)$$

where

$$\lambda_{D,inner}^2 = \frac{T_{ec} \ln(r_{inner}/r_{source})}{2\pi e^2 n_{source}} \left(\frac{r_{inner}}{r_{source}} \right)^2 \quad (13)$$

is the ion Debye length at r_{inner} . In the vicinity of r_{outer} , for $\lambda_D \ll r_{outer} - r \ll r_{outer} - r_{inner}$, the potential approximates an inverted Child-Langmuir sheath, with negligible n_i

$$\phi(r) \approx \frac{3^{4/3}}{2} T_{ea} \left(\frac{r_{outer} - r}{\lambda_{D,outer}} \right)^{4/3} \quad (14)$$

where

$$\lambda_{D,outer}^2 = \frac{T_{ea}}{2\pi e^2 n_\infty} \quad (15)$$

is the electron Debye length at r_{outer} . The transition from Eq. (12) to Eq. (14) occurs when $n_e \approx n_i$, which is to say at the point where the two expressions for $\phi(r)$, Eq. (12) and Eq. (14), have second derivatives that are equal in magnitude (but with opposite signs). At this point, the two expressions for $\phi(r)$ must have the same first derivative. This means that the transition from

Eq. (12) to Eq. (14) must occur half way between r_{inner} and r_{outer} , with $\phi(r)$ antisymmetric about this point, and the coefficients in front of the two expressions for $\phi(r)$ must be equal,

$$2T_{ec}\ln(r_{inner}/r_{source})\lambda_{D,inner}^{-4/3} = T_{ea}\lambda_{D,outer}^{-4/3} \quad (16)$$

Equation (16) leads immediately to the well known double layer requirement[17]

$$I_e/I_i = (m_i/m_e)^{1/2} \quad (17)$$

where $I_e = 2\pi r_{outer}^2 J_e^\infty$, and $J_e^\infty = en_\infty(2\pi T_{ea}/m_e)^{1/2}$ is the ambient electron saturation current. In other words, the contactor cloud will expand freely until the ion current density $I_i/4\pi r^2$ is equal to the ambient electron saturation current times $(m_e/m_i)^{1/2}$. If $T_{ea} \approx T_{ec}$, then this will occur when the density of the contactor plasma is comparable to the density of the ambient plasma.

From Eqs. (12), (14), and (16), the width of the double layer is related to the potential drop $\Delta\phi = \phi(r_{inner}) - \phi(r_{outer})$ by

$$r_{outer} - r_{inner} = \frac{2}{3}\lambda_{D,outer} \left(\frac{\Delta\phi}{T_{ea}} \right)^{3/4} \quad (18)$$

and these results are valid only if the width given by Eq. (18) is much less than r_{inner} . If this condition is not satisfied, then Poisson's equation must be solved numerically, as has been done by Wei and Wilbur[8] and by Williams[16], and in this case I_e/I_i will be smaller than $(m_i/m_e)^{1/2}$.

We note that this collisionless model is essentially identical to the collisional fluid model of Katz[9] in the limit that the resistivity η is sufficiently small, $e\eta J \ll \nabla P$, where P is the pressure and J is the current density. In this case, the potential gradient $e\nabla\phi = \nabla P + e\eta J$ is dominated by the barometric term ∇P , in Katz' terminology.

3.2 Comparison With Experiment

The model outlined above is in good agreement with the ground-based experiments of Wilbur[15] at CSU, in those conditions where double layers were seen. In these experiments, the anode had a radius $r_{anode} = 6$ cm, but the effective source radius, where most of the ionization occurred, was $r_{source} \approx 2$ cm. ϕ_0 could vary from 0 to 70V, and the collected electron current could vary from 0 to 1A. (At higher current, the effective collision frequency, due to streaming instabilities, was too

high for collisionless double layer theory to be valid.) Neutral gas, xenon, was introduced at the center of the anode at a rate that could vary from 1.8 to 13.7 sccm, which corresponded to a neutral density ranging from 3×10^{11} to 10^{12} cm^{-3} , concentrated within r_{source} of the origin. For ϕ_0 above some critical value, which depended on the neutral density, ambient electrons accelerated in the double layer had enough energy to ionize the gas, and the contactor cloud underwent a transition to an “ignited mode” in which this ionization was the major source of emitted ion current. The electron temperature and density and the plasma potential were measured as functions of position. The ambient ion temperature was much lower than the electron temperatures.

In a typical case, with $\phi_0 = 37\text{V}$, most of the potential drop, 25V, occurred in a double layer (more or less spherical) located between $r_{inner} = 8\text{cm}$ and $r_{outer} = 11\text{cm}$. The rest of the potential drop occurred between the anode and r_{inner} . The potential profile was virtually flat outside r_{outer} . The ambient electron temperature was 5.5eV, and the ambient electron density was $3 \times 10^7 \text{ cm}^{-3}$. These electrons have a Larmor radius of about 15 cm in the earth’s magnetic field, which is greater than $r_{outer} - r_{inner}$, and once they cross the double layer they have a Larmor radius of about 50 cm, which is greater than $r_{outer}^2/2r_{anode}$, so the electrons can easily reach the anode according to the Parker-Murphy criterion[10], and the assumption in our model of unmagnetized electrons was more or less valid. The assumption of collisionless electrons was also marginally satisfied if we use Parks’ and Katz’ estimate[5] of an effective collision frequency $\nu_e \approx 0.1\omega_{pe}$. At r_{outer} we find $\nu_e = 3 \times 10^7 \text{ s}^{-1}$, and the electron mean free path is about 3cm, comparable to the width of the double layer, while at r_{inner} we find $\nu_e = 2 \times 10^7 \text{ s}^{-1}$ and the mean free path of the accelerated ambient electrons is about 10cm, comparable to r_{inner} . Note that at densities a few times higher, the electron mean free path would be less than the double layer width, and double layers could not exist. This is in agreement with observations at currents above 1A, as discussed at the end of Sec. 2. There was also a 40eV ambient electron component (the “primary” electrons) of density $3 \times 10^6 \text{ cm}^{-3}$. Such a component of electrons was not included in our model, but their effect can be included by using an effective $T_{ea} \approx 9\text{eV}$ which would give the same electron saturation current as that obtained from the 5.5eV and 40eV components.

The collected electron current, 370mA, was in good agreement with this electron saturation

current integrated over the area of the double layer $2\pi r_{outer}^2$ (not $4\pi r_{outer}^2$, since it was a half sphere). The electrons in the contactor cloud had a temperature $T_{ec} = 2\text{eV}$, and a density which went from $8 \times 10^8 \text{cm}^{-3}$ at r_{source} down to $2 \times 10^7 \text{cm}^{-3}$ at r_{inner} . This ratio of $n_e(r_{source})/n_e(r_{inner})$ is close to $(r_{inner}/r_{source})^2 [(\phi_0 - \phi(r_{inner}))/T_{ce}]^{1/2}$, the value given by Eq.(2). The emitted ion current I_i would then be $2\pi r_{source}^2 e n_e(r_{source}) (T_{ec}/m_i)^{1/2} = 0.4\text{mA}$, fairly close to the ion current required by Eq.(17), $(m_e/m_i)^{1/2} I_e = 0.7\text{mA}$. The observed width of the double layer, $r_{outer} - r_{inner} \approx 3\text{cm}$, is a few times greater than the width of 0.6 cm predicted by Eq.(18), but it is likely that the measured width is smeared out by fluctuations in the position of the double layer.

3.3 Critical Potential for Transition to Ignited Mode

The emitted ion current in Wilbur's experiment consists of a small ion current I_0 produced by the hollow cathode source, independent of the incoming electron current, and a current of ions produced by ionization of neutral gas by the incoming ambient electrons, which have been accelerated by the double layer

$$I_i = I_0 + I_e \int dr n_0(r) \sigma \quad (19)$$

where $n_0(r)$ is the neutral density, and σ is the electron ionization cross-section for xenon at the energy of the incoming ambient electrons, $\phi_0 + T_{ea}$. Since, for a thin double layer, $I_i = (m_e/m_i)^{1/2} I_e$, it follows from Eq.(19) that

$$I_e = I_0 \left[(m_e/m_i)^{1/2} - \int dr n_0(r) \sigma \right]^{-1} \quad (20)$$

Equation (20) sets the radius of the double layer by the fact that I_e must be equal to the saturation current integrated over the surface of the double layer, $I_e = 2\pi r_{outer}^2 J_e^\infty$. This expression for I_e is self-consistent if it gives $r_{outer} \gg \lambda_{De} (\phi_0/T_{ea})^{3/4}$. Otherwise, the double layer will not be thin, and $(m_e/m_i)^{1/2}$ must be increased by the appropriate factor[8], which will further reduce r_{outer} and I_e . A consequence of Eq.(20) is that, as ϕ_0 (and hence σ) is increased from zero, I_e will gradually increase until ϕ_0 reaches a critical value, where

$$\int dr n_0(r) \sigma = (m_e/m_i)^{1/2} \quad (21)$$

Table 2: Transition to ignited mode

ϕ_0 (Volts)	σ (cm ²)	$\int dr n_0$ (cm ⁻²)	Gas flow (sccm)
11	2.3×10^{-16}	9×10^{12}	13.7
16	3.3×10^{-16}	6×10^{12}	9.6
19	3.6×10^{-16}	5.4×10^{12}	6.8
27	4.8×10^{-16}	4×10^{12}	4.1
36	5.5×10^{-16}	3.4×10^{12}	2.7

At this ϕ_0 , according to Eq. (20), I_e will blow up. In practice, I_e will not become infinite, but will be limited by several factors: 1) if r_{outer} is too much greater than r_{source} , the incoming electrons will not be able to converge completely on the source, and they will not all be available for ionization; 2) I_i cannot be greater than the flow rate of neutral gas; 3) r_{outer} cannot be greater than the size of the tank. However, we expect qualitatively that, at this critical ϕ_0 , there will be a sudden increase in I_e and in r_{outer} , and that the critical ϕ_0 will be a decreasing function of neutral line density $\int dr n_0$. Such a transition to an “ignited mode” at a critical ϕ_0 was seen in Wilbur’s experiments[15]. Table 2 gives σ (for xenon, at electron energy $\phi_0 + T_{ea}$, with $T_{ea} = 9eV$), the required neutral line density for this transition to occur at each of several values of ϕ_0 , and the gas flow rate at which the transition was observed, for each value of ϕ_0 .

Measurements of the spatial distribution of neutral gas were made, yielding neutral line densities, within $r_{source} \approx 2cm$ of the center of the anode, in good agreement with the theoretical values shown in Table 2. However, the neutral line density near the center of the anode may not be the relevant neutral line density. Measurements were made of the density and energy of the incoming hot electrons as a function of radius inside the contactor cloud; while the energies were close to the expected values of $\phi(r) + T_{ea}$, and the density at r_{inner} was close to the ambient density times $[T_{ea}/\phi(r_{inner})]^{1/2}$, at smaller r the density increased more slowly than r^{-2} , and was only 3 times greater at r_{source} than at r_{inner} . This indicates that the incoming electrons were not converging to within r_{source} of the center of the anode, but were spread out over much of the full anode radius of

6 cm. This failure of the electrons to fully converge may be due to their angular momentum, and perhaps to the effect of the ambient magnetic field, effects which were not included in our model. The neutral line density over most of this area was considerably lower than it was within r_{source} of the center of the anode, giving a neutral line density that, according to Eq. (21), is lower than that required for the transition to the ignited mode. A possible explanation for this discrepancy is that there may have been a substantial flux of secondary emission electrons in the vicinity of the anode, contributing to the ionization rate.

3.4 Requirement of Supersonic Ambient Electrons

The double layer theory developed above assumed that $T_{ea} \gg T_{ia}$, an approximation that allowed us to define a sharp boundary r_{outer} to the double layer, and to neglect the ambient ion density within the double layer. This assumption is well satisfied in ground based experiments, but not in the equatorial region of the ionosphere in low earth orbit, where $T_{ea} \approx T_{ia}$. The question arises as to whether a double layer equilibrium potential $\phi(r)$, can join smoothly onto a quasineutral potential at $r > r_{outer}$ in this case. For that matter, we have not really demonstrated that such an equilibrium is possible even when $T_{ea} \gg T_{ia}$, since we did not consider the very outer edge of the double layer, where the ambient ion density is comparable to the ambient electron density, and where the transition from the interior of the double layer (where the ambient ion density is negligible) to the quasineutral region occurs. It turns out that a double layer equilibrium exists for any T_{ea}/T_{ia} . We will show this explicitly for $T_{ea} \gg T_{ia}$ and for $T_{ea} \ll T_{ia}$; it has already been shown by Alpert, Gurevich and Pitaevskii[18] for the more difficult case of $T_{ea} = T_{ia}$.

In order to have a potential $\phi(r)$ which asymptotically approaches a quasineutral solution at large r , it is necessary to have

$$\frac{d}{d\phi}(n_e - n_i) > 0 \quad (22)$$

In other words, the electrons must be supersonic as they approach the double layer from the outer (low potential) side. A similar condition exists, with the opposite sign, for the existence of a Debye sheath at a wall, which joins smoothly onto a quasineutral plasma with a potential that is positive with respect to the wall. In that case, which also applies to the double layer on the

inner (high potential) side, the requirement is that the ions be supersonic as they approach the sheath. The question of why the ions are always supersonic going into the sheath was considered by Tonks and Langmuir[19], and later by Bohm[20], who showed that an electric field (the “Bohm presheath”) must exist wherever there is a plasma source in the quasineutral region, and that it always accelerates ions to supersonic velocities before they reach the sheath. On the outer side of the double layer there is no plasma source, but there is a quasineutral presheath, *viz.* the potential rise of order T_{ia} associated with the empty region of electron velocity space, due to the fact that electrons are not emitted from the double layer. This presheath plays the same role in accelerating electrons that the Bohm presheath plays in accelerating ions. When $T_{ea} \gg T_{ia}$, this potential is given by Eqs. (5) and (6). At r_{outer} , the potential is $T_{ia}\ln 2$. In the vicinity of r_{outer} , that is for $\phi(r) \ll T_{ea}$, the ambient electron density, from Eq. (10), is

$$n_e(r) \approx \frac{n_\infty}{2} \left(1 - \frac{2}{\sqrt{\pi}} \sqrt{\frac{\phi}{T_{ea}}} \right) \quad (23)$$

and $dn_e/d\phi$, evaluated at $\phi = T_{ia}\ln 2$, is $-(4\pi\ln 2)^{-1/2}n_\infty(T_{ia}T_{ea})^{-1/2}$. The ambient ion density is just

$$n_i(r) = n_\infty \exp(-\phi/T_{ia}) \quad (24)$$

so $dn_i/d\phi$, evaluated at $\phi = T_{ia}\ln 2$, is $-(n_\infty/2)T_{ia}^{-1}$. Then Eq.(22) is always satisfied if $T_{ea} \gg T_{ia}$. Note that this would not be true if there were no presheath outside the double layer, since in that case $dn_e/d\phi$ would blow up at $\phi = 0$.

When $T_{ia} \gg T_{ea}$, then the ambient electron density is greatly affected by the presheath, and to find $\phi(r_{outer})$ we must set $n_e(r)$ from Eq.(10) equal to $n_i(r)$ from Eq.(24), in the limit that $\phi \gg T_{ea}$. In this limit Eq.(10) becomes

$$n_e(r) \approx \frac{n_\infty}{2\sqrt{\pi}} \left(\frac{T_{ea}}{\phi} \right)^{1/2} \quad (25)$$

and we find $\phi(r_{outer}) \approx \frac{1}{2}T_{ia}\ln(T_{ia}/T_{ea})$ which is greater, by a logarithmic factor, than T_{ia} . It is obvious in this case that the electrons are supersonic, and Eq.(22) is satisfied. Since Eq.(22) is satisfied for either $T_{ia} \gg T_{ea}$ or $T_{ia} \ll T_{ea}$, and since we know from Alpert, Gurevich and Pitaevskii[18] that it is also satisfied for $T_{ia} = T_{ea}$, it appears very likely that it is satisfied for all T_{ea}/T_{ia} , although we have not found a simple proof of this. It is worth noting that Eq.(22), and the

analogous condition for the inner side of the double layer, are only satisfied because of processes going on some distance away from the double layer, and that misleading results could be obtained by computer simulations of double layers which do not include these distant regions, and do not properly treat the plasma coming into the double layer.

3.5 Limitations of Wei and Wilbur Model Due to Magnetized Electrons

In Wilbur's ground based experiments[15] the Larmor radius of the ambient electrons in the earth's 0.3G magnetic field is about 20cm, much greater than the 3 cm thickness of the double layer, so the magnetic field will not significantly deflect the electrons as they cross the double layer. Once they cross the double layer, they will have a Larmor radius of about 50 cm, and in the 8 cm they have to traverse to get to the anode, they will be deflected by about $\frac{1}{2}(8)^2/50 = 0.7$ cm, less than the 6 cm radius of the anode, consequently the magnetic field will not inhibit the electrons from getting to the anode[10]. Hence our model, which assumed unmagnetized electrons, ought to be valid. An additional requirement of our model, $r_{inner} > r_{anode}$, is also satisfied in Wilbur's experiments.

In space, on the other hand, the ambient electron temperature, at least in the equatorial region, is much less, only about 0.1eV, so the Larmor radius is about 2.5cm, and the density is much less than in the ground based experiments (about 10^5cm^{-3} rather than $3 \times 10^7\text{cm}^{-3}$). Therefore, to collect an electron current of several amps from the ambient plasma will require r_{outer} of tens of meters, much greater than the electron Larmor radius. The electrons can traverse such a distance only if they undergo collisions (or effective collisions due to some kind of instability), or if they can gain enough energy as they cross the double layer to remain, in effect, unmagnetized, until they reach the anode. We have considered the latter possibility, and have found that, even with rather optimistic assumptions, it requires a sheath impedance that is undesirably large, since it would result in most of the tether potential drop occurring in the sheath. We conclude that effective collisions of some kind are needed in a plasma contactor in space, in order to collect a large electron current from the ambient plasma, at a reasonable impedance.

Parker and Murphy[10] have shown that, in the absence of collisions, and for $e\phi_0 \gg T_{ea}$, a

necessary condition which must be satisfied for electrons at r_{outer} to reach the anode is

$$r_{outer}^2/r_{anode}^2 < 1 + (8e\phi_0/m_e\omega_{ce}^2 r_{anode}^2)^{1/2} \quad (26)$$

Equation (26) is also a sufficient condition if all of the potential drop occurs in a thin double layer at r_{outer} . If the double layer is thick, or if a significant part of the potential drop occurs in the quasineutral regions on either side of the double layer, then an even more stringent condition must be satisfied, in order for electrons to reach the anode. Another condition that must be satisfied is $r_{inner} \geq r_{anode}$. It turns out that for most parameters of interest this condition and Eq. (26) cannot both be satisfied, for a spherically symmetric space-charge limited collisionless double layer, as described by Wei and Wilbur[8]. This is true except at very low currents, or for anodes with r_{anode} almost equal to r_{outer} . If higher ion currents are emitted from an anode (with $r_{anode} \ll r_{outer}$) with zero initial velocity, and there are no collisions or turbulence allowing electrons to be transported across the magnetic field, then a spherically symmetric double layer cannot develop, no matter how great the potential is. Electron collection will necessarily be inhibited in the direction across the magnetic field; in this direction the potential profile will not follow the form found by Wei and Wilbur[8], because the collected ambient electron current will not be space-charge limited, but will be limited by magnetic field effects. A theory giving the electron current and potential in this anisotropic collisionless regime is not available. However, if we ignore the requirement that $r_{inner} > r_{anode}$ and assume that only Eq. (26) and the Wei-Wilbur equations must be satisfied, then we can obtain an upper limit for the electron current than can be collected, and a lower limit for the potential, for a given ion current and anode radius.

The electron current I_e is related to r_{outer} by

$$I_e = 2\pi r_{outer}^2 J_e^\infty \quad (27)$$

where $J_e^\infty = en_{ea}(T_{ea}/2\pi m_e)^{1/2}$ is the ambient electron saturation current. We have calculated what the impedance of the double layer will be assuming Eq. (26) is barely satisfied, for $r_{anode} = 10$ cm. If, as turns out to be true, the resulting impedance is too high to make an efficient plasma contactor, we will know that we should look at plasma contactors in which the electrons undergo

collisions (or are subject to turbulence which causes effective collisions) and diffuse into the anode, rather than going into the anode directly.

Using Eq.(27) for I_e , assuming Eq.(26) is barely satisfied, and using Wei and Wilbur's calculation[8] which relates r_{outer}/r_{inner} to I_e/I_i , we can find ϕ_0 and I_e for a given I_i and electron saturation current J_e^∞ . Since J_e^∞ depends only on the properties of the ionosphere in low earth orbit, both I_e and ϕ_0 are determined by I_i . These values really represent an upper limit for I_e and lower limit for ϕ_0 , since Eq. (26) is only a necessary condition, not a sufficient condition, for collisionless electrons to reach the anode, and since we ignored the requirement that $r_{inner} > r_{anode}$.

In Figure 1 we show the gain ξ against the argon ion current for a range of electron saturation current densities which span the range experienced in an equatorial low earth orbit (LEO). The gain is somewhat less than $(m_i/m_e)^{1/2} = 272$ for argon, and is weakly dependent on the ion current. Also shown on Figure 1 is the line where the contactor plasma passes from the nonignited phase to the ignited phase, as calculated in Sec. 3.6. This indicates that ignition will not occur in space if the electrons are collisionless, except possibly at very high ion currents where the collisionless electron assumption is in any case not likely to be valid. In Figures 1 through 5, the curves are dashed in the regime where Eq. (26) cannot be satisfied for a collisionless double layer with space charge limited current except by violating $r_{inner} \geq r_{anode}$. In Figure 2 we show the associated potential drop through the double layer, which is really a lower limit on the potential drop. Typical potential drops are in the range of thousands of volts for ion currents in the milliampere range.

In Figure 3 the inner and outer radii of the double layer are shown for space conditions. These radii are determined by imposing the Parker-Murphy condition Eq. (26). The double layer extends to about a meter for ion currents in the milliampere range. For comparison, the diameter of the CSU tank is shown on the figure. This indicates that finite tank effects would be important in experiments at realistic low earth orbit plasma densities, except for the very smallest ion currents. Note also that, except for the smallest ion currents, $r_{inner} < r_{anode}$, showing that a collisionless unmagnetized double layer with space charge limited current is not possible for most parameters of interest in low earth orbit. This conclusion does not depend on r_{anode} . Making $r_{anode} < 10$ cm would only make things worse, since, for a fixed ion current, r_{inner} would shrink faster than

r_{anode} . Making r_{anode} much greater than 10 cm would allow higher ion and electron currents while satisfying Eq. (26) and $r_{inner} > r_{anode}$. However, for $J_e^\infty \leq 2 \times 10^{-2}$ A/m², this could only be done if r_{anode} were nearly equal to r_{outer} , in which case the plasma contactor would serve no purpose.

In Figure 4 the total current is shown as a function of the electron saturation current density. The curve obtained for the collisionless double layer (really an upper limit) is shown for a fixed ion current of 10 mA. For comparison, we also show the total current for the isotropic quasineutral model described in Sec. 4, and for the anisotropic contactor model described in Sec. 5, for a fixed ion current of 1 Amp. This figure compares the realistic range of operation for the three models in typical ambient electron saturation current densities. A significant feature of this figure is that as the source varies by two orders of magnitude from 2×10^{-4} A/m² to 2×10^{-2} A/m², the total current (which is almost all collected electron current) varies by only a factor of 1.5, for the collisionless double layer model. This would seem to invalidate one of the conclusions in Ref. [1] which was that plasma contactors would not be useful on the nightside of an equatorial low earth orbit because the collected current would drop to almost nothing. Here the double layer moves out as the electron pressure drops so that the collected electron current is almost the same. On the other hand, if we took into account the actual requirements for electrons to reach the anode, rather than only using the Parker-Murphy condition, then it is likely that at low saturation current the double layer would be inhibited from moving out so far, and the collected electron current would be more sensitive to saturation current. Except for the upper end of the range of saturation current, the actual electron current that could be collected without collisions is certainly far less than the upper limit shown in Fig. 4. For the anisotropic collisional contactor model, which is more relevant for high current plasma contactors in low earth orbit, Fig. 4 shows that the total current is about 4 times higher, and the collected electron current is about 10 times higher, on the dayside ($J_e^\infty \approx 2 \times 10^{-2}$ A/m²) than on the nightside ($J_e^\infty \approx 2 \times 10^{-4}$ A/m²).

In Figure 5 the current voltage characteristic is shown for the range of electron saturation current densities. At constant current in the milliampere range the voltage is seen to vary by two or three orders of magnitude for one order of magnitude variation in electron saturation current, for the collisionless double layer. At constant voltage, the current is roughly linear with the electron

saturation current. Ampere range currents (which are mainly electrons) require tens of thousands of volts of potential drop, even for the highest value of the electron saturation current. These curves represent an upper limit on the electron current for a given potential, or a lower limit on the potential for a given electron current. For currents greater than about 50 mA, the space charge limited collisionless double layer model on which these curves are based cannot satisfy both Eq. (26) and $r_{inner} > r_{anode}$; the actual potential needed to collect such currents, in the absence of collisions, would be far greater than the lower limits shown in Fig. 5. Curves for the isotropic quasilinear model and anisotropic model discussed in Sec. 4 and 5 are shown for comparison.

With the use of these results we can calculate an upper limit for the current that could flow through a tether using a plasma contactor. The total potential drop ϕ_{total} across the contactor, tether, load, and electron gun is fixed by the length L of the tether, the earth's magnetic field $B_0 = 0.33 \times 10^{-4} T$, and the orbital velocity of the spacecraft $v_o = 8 \text{ km/s}$. For $L = 20 \text{ km}$, we find $\phi_{total} = v_o B_0 L = 5333 \text{ V}$. The potential across the load is $\phi_{load} = R_{load}(I_i + I_e)$, where R_{load} is the load impedance. The potential across the tether is $R_t(I_i + I_e)$, where we take the tether impedance $R_t = 200 \Omega$. We could include the radiation impedance[21] but this is typically only about 10Ω , so may be neglected compared to the tether impedance. If we assume a typical dayside ionosphere with $J_e^\infty = 2 \times 10^{-2} \text{ A/m}^2$, a good fit to the numerical results in figure 5 is $\phi_0 = b(I_i + I_e)^{2.08}$ where $b = 1.8 \times 10^5$. For a given load R_{load} , the current $I = I_i + I_e$ may be found by solving

$$\phi_{total} = R_{load}I + R_t I + bI^{2.08} \quad (28)$$

and we may then find the power across the load $P_{load} = R_{load}I^2$, and the efficiency $\eta = R_{load}I/\phi_{total}$, as functions of R_{load} . (This definition of efficiency neglects the energy needed to produce the ions, but that is justified since this energy, about 50eV, is much less than the potential drop across the double layer, unless $\eta \approx 99\%$.) Table 3 shows P_{load} , and ξ as functions of the efficiency $\eta = R_{load}I/\phi_{total}$.

The maximum power to the load is 400 W, but this occurs when the efficiency is only 60%. As noted in Ref. [1], in order for tethers to be competitive with other power systems in space it is necessary for them to operate at high efficiency, at least 80% or 90%. This is because all of the power has to be made up by periodically boosting the tether but only the load power can be

Table 3: Load power against efficiency of double layer contactor

η	$I_i(mA)$	ξ	$I(A)$	$P_{load}(W)$
0.1	7	26	0.18	100
0.3	6	27	0.16	260
0.5	5	35	0.14	380
0.7	2.5	44	0.11	400
0.9	0.8	75	0.06	290

usefully used. If we desire an efficiency of 85%, then the maximum load power we can maximum load power we can obtain is only 320 W. The maximum power will in fact be much less than this, since Eq. (26) is not a sufficient condition for electrons to get across the magnetic field to the anode[10], and is known to be far from sufficient in the regime where $r_{outer} \gg r_{inner}$, which is true at the maximum power. Also, as may be seen from Fig. 3, the requirement that $r_{inner} > r_{anode}$ is far from satisfied at the ion current needed for maximum power.

We conclude that it is not possible to design a high power contactor which draws electrons straight across a double layer without collisions. Instead we should consider designs where collisions (or, more realistically, effective collisions due to instabilities of some kind) transport electrons across the magnetic field to the anode.

3.6 Conditions for Ignited Plasma

The calculations so far with the double layer model have all been for a totally ionized plasma. For a partially ionized plasma it is possible to include the effect of ionization and to show when the plasma will ignite. If we assume that the neutral density varies with radius as $n_0(r) = n_0(r_{source})(r_{source}/r)^2$ and apply conservation of mass from r_{source} to r_{inner} then we obtain

$$I_e(r) = I_e(r_{inner}) \exp(\gamma(\Delta\phi) \left[\frac{r_{source}}{r} - \frac{r_{source}}{r_{inner}} \right]) \quad (29)$$

where $\gamma(\Delta\phi) = n_0(r_{source})r_{source}\sigma(\Delta\phi + T_{ea})$. From conservation of current we obtain the gain as

$$\xi = 1 + \frac{(\xi(r_{inner}) - 1) \exp(\gamma(1 - \frac{r_{source}}{r_{inner}}))}{1 + (\xi(r_{inner}) - 1)(1 - \exp(\gamma(1 - \frac{r_{source}}{r_{inner}})))} \quad (30)$$

where $\xi(r_{inner}) = I/I_i(r_{inner})$. The ion current at the source in terms of the ion current just inside the double layer is

$$\frac{I_i(r_{source})}{I_i(r_{inner})} = 1 + (\xi(r_{inner}) - 1)(1 - \exp(\gamma(1 - \frac{r_{source}}{r_{inner}}))) \quad (31)$$

In order to interpret the calculations in figure 1 with ionization present we must interpret the ion current in the ordinate as $I_i(r_{inner})$, and the gain as $\xi(r_{inner})$. The relationship in terms of the ion current emitted at the source is given above. It is apparent that there may be no positive solution of the source ion current for a given ion current at the double layer. Physically this will occur when there is so much neutral gas that the mixed gas-plasma flow ignites giving an avalanche of ion current. The ion current and collected electron current will continue to increase, and cannot reach a steady state until the collisionless double layer model is no longer valid. By setting the source ion current to zero we can obtain this critical neutral density for ignition as

$$n_{critical} = \frac{-\ln(1 - 1/\xi(r_{inner}))}{(1 - r_{source}/r_{inner})} \frac{1}{r_{source}\sigma} \quad (32)$$

If we relate the source neutral density to the ion flow rate and initial fractional ionization (f_i) we obtain ignition for

$$I_i(r_{inner}) > \frac{4\pi r_{source}^2 c_s e f_i}{1 - f_i} n_{critical} \quad (33)$$

In figure 1 we plot this critical ion current against gain for $r_{source} = 0.1$ m, $c_s = 4.89 \times 10^3$ m/s, $\sigma = \sigma_{max} = 3.21 \times 10^{-20}$ m⁻² (for ionization of Argon) and $f_i = 10^{-4}$ which is typical of hollow cathode devices. For ion current and gain pairs which fall on the curve on the figure the neutral flow would spontaneously ignite and set up a double layer structure with only a small seed ionization, if this occurred in a regime where the collisionless double layer model were valid. However, as shown in Sec. 3.5, the unmagnetized collisionless double layer model is not valid in space for ion currents greater than about 1 mA, and this is far from the ignition curve in Fig. 1. Ignition might be possible in the regime of higher ion current and lower gain typical of the anisotropic collisional contactor model described in Sec. 5.

4 Quasineutral Theory and Implications

4.1 Definition of Core Radius

A plasma source capable of producing a large enough contactor cloud to draw a high current in space will have a high density, $\omega_{pe} \gg \omega_{ce}$, for some distance out from the anode, and such a plasma is likely to be subject to instabilities which produce an effective electron collision frequency

$$\nu_e > \omega_{ce} \quad (34)$$

In the region where Eq.(34) is satisfied, the electrons will behave like a fluid, unaffected by the magnetic field. The electron fluid will still feel a $\mathbf{v} \times \mathbf{B}$ force, but this force is always small compared to the force due to the electric field, if Eq.(34) is satisfied, since the steady state radial velocity, at which the drag force $\nu_e m_e v_r$ balances the electric force eE , will be $eE/m_e \nu_e$, hence the ratio of the electric force to the magnetic force will be

$$\frac{E}{v_r B} = \frac{\nu_e m_e}{eB} = \frac{\nu_e}{\omega_{ce}} \gg 1$$

and the electrons will be unaffected by the magnetic field. Since there cannot be two different velocity components of electrons at the same place in this region, due to the high collision frequency, there can be no double layers, and quasineutrality will be satisfied everywhere. Since the effective collision rate due to instabilities tends to scale like ω_{pe} , it will decrease with distance from the source, and beyond some r Eq.(34) will no longer be satisfied, and the electrons will no longer behave like an unmagnetized fluid. Even beyond this radius, electrons can diffuse slowly across the magnetic field, and we will show in Sec. 5 that it may be possible to collect electron current out to a radius r_1 defined in Sec. 5. The electron current collected will be

$$I_e = 2\pi r_{core}^2 J_e^\infty \quad (35)$$

where r_{core} is at least as great as the r where Eq.(34) ceases to be satisfied, and will be equal to the r_1 defined in Sec. 5, if the model described there is applicable. We will consider both possibilities, to set upper and lower bounds on I_e . Also, of course, r_{core} must be smaller than the r at which the contactor ion density is equal to the ambient ion density, and must be smaller than an ion Larmor radius.

4.2 Numerical Solution of Equations

In order to evaluate a lower bound on r_{core} , we have solved the equations for the core region, given in Ref.[4], for the definition of r_{core} given by Eq. (34). The equations were solved by making a guess on the incoming electron current then marching forward in radius until the appropriate condition, Eq. (34) was satisfied. The electron saturation current across $r = r_{core}$ was then calculated and compared to the initial guess. If the two did not agree then a new guess on the incoming electron current was chosen and the process repeated. This iterative procedure was continued until the electron current entering the central anode was consistent with the electron saturation current crossing the core radius.

This model has been extensively discussed in Ref. [4]. Typical gains were close to 1; this low gain is due to the fact that the core region where Eq. (34) is satisfied is too small to collect much electron current. In the collisionless double layer model, much higher gains, over 100, are possible, but only because electrons are brought across the magnetic field by brute force, by having a very large potential drop in the double layer. This large potential drop reduces the efficiency of the tether, and it may be more efficient to produce ion current than to collect electron current across such a large potential. This is illustrated in Fig. 5, which compares the current voltage characteristic for the quasilinear model with that of the collisionless double layer model (limited by the Parker-Murphy condition at low potential, where the curves are fairly flat, and by $r_{inner} \geq r_{anode}$ at high potential). The quasilinear model always operates at low potential, tens of volts, and draws a much higher current than the collisionless double layer model at this potential, but the current is almost all emitted ion current or electron current produced by ionization of neutral gas. The latter increases sharply as the potential goes above the ionization energy. In Fig. 4, the total current is shown as a function of electron saturation current for the quasilinear model, with fixed ion current of 1 A, compared to the collisionless double layer model and anisotropic model. The current is very weakly dependent on the electron saturation current, in the range ($J_{e\infty} < 2 \times 10^{-2}$ A/m²) likely to be found in low earth orbit, because very little of the current is due to collected ambient electrons.

In the next section, we consider a model that is similar to the collisionless double layer model in the behavior of electrons along the magnetic field, but is collisional and quasilinear across the

magnetic field. This model has characteristics that are in between those of the collisionless double layer and collisional quasineutral model; modest gains, typically 2 to 10, are possible, at moderately low potential drops.

5 Anisotropic Contactor Model

In the region where the effective electron collision frequency ν_e is less than the electron cyclotron frequency ω_{ce} , the contactor cloud will be anisotropic, extending further in a direction along the magnetic field than across the magnetic field. We therefore use cylindrical coordinates z and r , where r now refers only to the distance across the magnetic field, not to the total distance from the anode as it did in previous sections. We assume that the plasma density in the cloud is still great enough to short out the electric field due to the orbital velocity, so the cloud will be cylindrically symmetric. (At still larger distances from the anode, the effects of the orbital motion induced electric field will become important, and the cylindrical symmetry will be broken.) In this region the electron velocity will be mostly azimuthal, at the drift velocity

$$v_d = \frac{e}{m_e \omega_{ce}} \frac{\partial \phi}{\partial r} - \frac{1}{m_e \omega_{ce}} \frac{\partial T_e}{\partial r} - \frac{T_e}{m_e \omega_{ce} n_e} \frac{\partial n_e}{\partial r} \quad (36)$$

For parameters of interest, this drift velocity is much greater than the radial flow velocity of the emitted ions, which are effectively unmagnetized since we assume that the scale lengths are all much less than an ion Larmor radius. The velocity difference between the electrons and ions will then be nearly in the azimuthal direction. This relative cross-field drift velocity of magnetized electrons and unmagnetized ions can give rise to a several instabilities, among them the ion acoustic instability (both $k_{\perp} \rho_e > 1$ and $k_{\perp} \rho_e < 1$ varieties), the Buneman instability, the electron cyclotron drift instability (also known as the beam cyclotron instability), the modified two-stream instability, and the lower hybrid drift instability. Which of these instabilities dominates depends on such parameters as T_e/T_i , v_d/c_s , v_d/v_e , β_e , ω_{pe}/ω_{ce} , and v_d/v_A , where v_d is the relative drift velocity, c_s is the sound speed, v_e is the electron thermal velocity, v_A is the Alfvén speed, and the other symbols have their usual meanings. These instabilities will give rise to turbulent azimuthal electric fields, which will exert an azimuthal drag force $\nu_e m_e v_d$ on the electrons, giving rise to an inward

radial drift at velocity

$$v_r = \frac{\nu_e}{\omega_{ce}} v_d \quad (37)$$

We will assume that the potential drop in the plasma cloud is very much greater than the ion temperature T_i , which is typically only a few eV. Since, as we will show later, T_e tends to be only a few times less than ϕ_0 , this implies that $T_e/T_i \gg 1$, except perhaps near the edge of the cloud. Also $c_s \ll v_d \ll v_e$. In these circumstances, we expect the $k_\perp \rho_e > 1$ ion acoustic instability to dominate (this is the same as the ion acoustic instability in an unmagnetized plasma). The effective collision frequency ν_e is for this instability in its nonlinear saturated state scales with density like ω_{pe} , and is independent of c_s/v_d for $c_s \ll v_d$, but there is some uncertainty as to its dependence on T_e/T_i and v_d/v_e . As a first cut at this problem, we will simply assume that

$$\nu_e \approx 10^{-2} \omega_{pe} \quad (38)$$

independent of the other parameters. The method we will use to find analytic expressions for $\phi(r, z)$ and the collected electron current may also be applied using more realistic expressions for ν_e .

The divergence of the radial flux of electrons due to ν_e and the radial electric field and temperature and density gradients must be balanced by an inward flux of electrons along the magnetic field, neglecting ionization and recombination:

$$\frac{1}{r} \frac{\partial}{\partial r} r n_e v_r + \frac{\partial}{\partial z} n_e v_z = 0 \quad (39)$$

At high densities, such as those in the experiment of Urrutia and Stenzel[23], with $\omega_{pe} \gg \omega_{ce}$, the mean free path of electrons will be short compared to the length of the contactor cloud, and the velocity v_z along the magnetic field may also be found by balancing the force from the electric field $e\partial\phi/\partial z$ with the drag force $m_e \nu_e v_z$. In this case Eq. (39) will generally not be separable in r and z , and it is necessary to solve a fully two-dimensional partial differential equation. The boundary conditions will be that $v_r = 0$ and $\phi = 0$ at the same surface, and the flux of electrons across this surface must be equal to the flux of the electron saturation current of the ambient plasma (along the magnetic field) outside the surface. The potential $\phi(r, z)$ would be quasineutral everywhere. Since the position of the $\phi = 0$ surface is not known in advance, this would be a difficult numerical

problem. The ambient plasma in low earth orbit has much lower density, $\omega_{pe} \leq \omega_{ce}$, and this would also be true in most of a space-based contactor cloud, which, as we will show, would extend along the magnetic field to a distance where the cloud density is comparable to the ambient density. In this case, the electrons will flow freely along the magnetic field, and a different model is needed. If the total potential drop ϕ_0 between the anode and the ambient plasma is greater than T_{ec} and T_{ia} , then double layers will form at a distance z_0 along the magnetic field in both directions, where

$$J_i = \frac{I_i g(z_0)}{2\pi z_0^2} = \left(\frac{m_e}{m_i}\right)^{1/2} J_e^\infty \quad (40)$$

for thin double layers, just as in the unmagnetized collisionless case (see Eq. (17)). Here $g(z)$ is a factor to take into account that the ions are focussed by the potential $\phi(r, z)$ if it is not spherically symmetric. Although the flow of electrons along the magnetic field is nearly collisionless, we will assume that there is enough drag to slow down the incoming electrons slightly, so that they will not escape out the other end, but will become trapped in the cloud. Only a small amount of drag is needed for this if $\phi_0 \gg T_{ea}$. At $z = \pm z_0$, the flux of electrons along the field must then satisfy the boundary condition

$$n_e v_z = \mp J_e^\infty / e \quad (41)$$

Because the flow of electrons across the magnetic field is collisional, no double layer exists in the radial direction. For fixed $|z| < z_0$, $\phi(r, z)$ must decrease smoothly to zero at some $r_1(z)$, satisfying quasineutrality all the way. For fixed r , along a given field line, as long as $\phi(r, z=0) > T_e(r)$, $\phi(r, z)$ will not go to zero for $|z| < z_0$. If ϕ_0 is at least a few times greater than T_e , then $\phi(r, z=0)$ will be greater than T_e for all r not too close to $r_1(z=0)$. It follows that r_1 is nearly independent of z . The contours of $\phi(r, z)$, and the flow of ions and electrons, is shown schematically in Fig. 6.

This means that Eq. (39) will be separable in r and z . The boundary conditions in r are

$$\phi(r = r_{anode}, z) = \phi_0 + T_e \ln(n_e(z)/n(z=0)) \quad (42a)$$

$$\phi(r = r_1) = 0 \quad (42b)$$

$$\frac{\partial \phi}{\partial r} = \frac{1}{e} \frac{\partial T_e}{\partial r} \quad \text{at } r = r_1 \quad (42c)$$

The last condition follows from the fact that $v_r = 0$ outside the contactor cloud, and there is no source or sink of electrons at $r = r_1$, hence v_r must vanish at r_1 just inside the contactor cloud. Eq. (36) (with $T_e = 0$), and Eq. (37) then yield Eq. (42c).

5.1 Electron Temperature

Before proceeding with the calculation of the potential profile $\phi(r)$, we will briefly consider whether we are justified in assuming that ϕ_0 is at least a few times greater than T_e . The electron temperature profile $T_e(r)$ is determined by the balance between convection, conduction, and ohmic heating (both perpendicular and parallel to the magnetic field). We neglect ionization and line radiation, which should only be important near the anode, and we neglect heat lost by electrons boiling out along the magnetic field.

$$\frac{-3}{2} v_r \frac{\partial T_e}{\partial r} + \frac{1}{r n_e} \frac{\partial}{\partial r} r \kappa \frac{\partial T_e}{\partial r} + e v_r \frac{\partial \phi}{\partial r} + \frac{J_e^\infty}{n_e z_0} \left(\phi - \frac{T_e}{e} \right) = 0 \quad (43)$$

Here κ is the cross-field thermal conductivity, which is dominated by turbulence just as the drag is. In general

$$\kappa = \frac{C n_e T_e \nu_e}{m_e \omega_{ce}^2} \quad (44)$$

where C is a constant which depends on the details of the ‘‘collisions’’ causing the heat transport. For electron thermal conductivity across a magnetic field due to Coulomb collisions[22], for example. $C = 4.7$.

The boundary conditions are

$$T_e = 0 \quad \text{at } r = r_1 \quad (45a)$$

$$\kappa \frac{\partial T_e}{\partial r} = \frac{Q}{4\pi r_{anode} z_0} - n_e v_r T_e \quad \text{at } r = r_{anode} \quad (45b)$$

where Q is the heat flux going into the anode. This is generally greater than the convective heat flux into the anode (the second term on the right hand side), because $\langle v_z^3 \rangle$ for a half-maxwellian is greater than $\langle v_z^2 \rangle \langle v_x \rangle$. So $\partial T_e / \partial r > 0$ at r_{anode} . Because $T_e = 0$ at $r = r_1$, $\partial T_e / \partial r$ must change sign between r_{anode} and r_1 , and we can estimate that the second term in Eq. (43) is of order $-\kappa T_e / n_e r_1^2$.

Using Eqs. (36), (37), and (44), we find

$$v_r = \frac{\kappa}{n_e T_e C} \left(e \frac{\partial \phi}{\partial r} - \frac{\partial T_e}{\partial r} \right) \quad (46)$$

Then the first term in Eq. (43) is of order $\pm \kappa e \phi / C n_e r_1^2$, and the third term is of order $+\kappa e^2 \phi^2 / C n_e T_e r_1^2$. From Eqs. (39) and (41), the fourth term in Eq. (43) is comparable to (and has the same sign as) the third term.

If $C \leq 1$, it follows that the second and/or the first term must balance the third and fourth terms, so T_e is of order $e\phi$. If $C \gg 1$, then the second term alone must balance the third and fourth terms, and $T_e \approx e\phi / C^{1/2} \ll e\phi$. Our assumption that T_e is at least a few times less than ϕ is thus valid if C is somewhat greater than one. This is true for Coulomb collisions; whether it is true for ion acoustic turbulence is an open question that is beyond the scope of this paper. If κ is dominated by an energetic tail of the electron distribution, perhaps electrons collected from the ambient plasma which have not yet thermalized, then $C \gg 1$.

5.2 Potential Profile and Cloud Radius

To find $\phi(r)$, we first integrate Eq. (39) over z from $-z_0$ to $+z_0$, and use Eq. (41) to eliminate v_z

$$\int_{-z_0}^{+z_0} dz \frac{1}{r} \frac{\partial}{\partial r} r n_e v_r = 2J_e^\infty \quad (47)$$

To obtain an expression for n_e , which appears explicitly in Eq. (47) and also implicitly through the dependence of v_e on ω_{pe} , we use quasineutrality

$$n_e = n_i = (4\pi)^{-1} I_i m_i^{1/2} e^{-3/2} (r^2 + z^2)^{-1} g(r, z) (\phi_0 - \phi)^{-1/2} \quad (48)$$

The expression for n_i in Eq. (48) comes from the fact that the ions are unmagnetized, and expanding spherically from the anode. The factor $g(r, z)$ takes into account the focussing of the ions by $\phi(r, z)$ which is not spherically symmetric. Using Eq. (37) for v_r , Eq. (38) for v_e , Eq. (48) for n_e , taking $B_0 = 0.3G$, defining the ion atomic weight $\mu = m_i/m_p$, and expressing I_i in amps, J_e^∞ in amps/m², and ϕ and ϕ_0 in volts, Eq. (47) becomes

$$\int_{-z_0}^{+z_0} dz \frac{1}{r} \frac{\partial}{\partial r} \left[r (\phi_0 - \phi)^{-3/4} (r^2 + z^2)^{-3/2} g(r, z) \frac{\partial \phi}{\partial r} \right] = -12 I_i^{-3/2} \mu^{-3/4} J_e^\infty \quad (49)$$

Because $(\phi_0 - \phi)$ and $\partial\phi/\partial r$ are fairly independent of z , and the integrand is most strongly weighted near $z = 0$, we replace ϕ and $\partial\phi/\partial r$ by their values at $z = 0$, so they can be taken out of the integral. Similarly, we can set $g(r, z) \approx 1$, because self-consistently there cannot be a strong focussing effect for $z < r$ where most of the contribution to the integral is. We then do the integration over z

$$\frac{\partial}{\partial r} \left[\frac{1}{r} (\phi_0 - \phi)^{-3/4} \frac{\partial\phi}{\partial r} \right] = -12r I_i^{-3/2} \mu^{-3/4} J_e^\infty \quad (50)$$

We integrate Eq. (50) over r , using the boundary condition Eq. (42c) to obtain the integration constant

$$\frac{1}{r} (\phi_0 - \phi)^{-3/4} \frac{\partial\phi}{\partial r} = 6 I_i^{-3/2} \mu^{-3/4} J_e^\infty (r_2^2 - r^2) \quad (51)$$

where

$$r_2^2 = r_1^2 + \frac{1}{6} r_1^{-1} \phi_0^{-3/4} e^{-1} \frac{\partial T_e}{\partial r} I_i^{3/2} \mu^{3/4} (J_e^\infty)^{-1} \quad (52)$$

We integrate over r again, using Eq. (42a) at $z = 0$ to obtain the integration constant

$$(\phi_0 - \phi)^{1/4} = 0.5 I_i^{-3/2} \mu^{-3/4} J_e^\infty (2r_2^2 r^2 - r^4) \quad (53)$$

Finally we use Eq. (42b) in Eq. (53) to obtain an equation for r_1

$$\phi_0^{1/4} = 0.5 I_i^{-3/2} \mu^{-3/4} J_e^\infty \left[r_1^4 + \frac{1}{3} r_1 \phi_0^{-3/4} e^{-1} \frac{\partial T_e}{\partial r} I_i^{3/2} \mu^{3/4} (J_e^\infty)^{-1} \right] \quad (54)$$

If, as we have been assuming, $T_e \ll e\phi_0$, then the second term in brackets may be neglected, and

$$r_1 = 1.2 \phi_0^{1/16} I_i^{3/8} \mu^{3/16} (J_e^\infty)^{-1/4} \quad (55)$$

Note that r_1 has an extremely weak dependence on ϕ_0 . For almost any reasonable ϕ_0 , say $10\text{V} < \phi_0 < 1000\text{V}$, for argon, and for $J_e^\infty = 2\text{ mA/m}^2$, which is between the typical dayside and nightside values,

$$r_1 \approx 15 I_i^{3/8} \quad (56)$$

and

$$I_e = 2\pi r_1^2 J_e^\infty \approx 2 I_i^{3/4} \quad (57)$$

In general the total current $I = I_i + I_e$ is

$$I = I_i + 8 (J_e^\infty)^{1/2} I_i^{3/4} \mu^{3/8} \phi_0^{1/8} \quad (58)$$

Table 4: Load power against efficiency of anisotropic contactor

η	$I_i(A)$	ξ	$I(A)$	$P_{load}(kW)$
0.1	1	4.99	4.99	2.64
0.3	1	4.83	4.83	7.65
0.5	1	4.6	4.6	12.1
0.7	1	4.22	4.22	15.4
0.9	1	2.32	2.32	10.6

A substantial ambient electron current can be collected for values of ϕ_0 and total current that are of interest for tethers. For 1 A of argon at $J_e^\infty = 2mA/m^2$, for example, we get a gain $I/I_i = 3$, while for 0.5 A of xenon, at a typical dayside electron saturation current $J_e^\infty = 20mA/m^2$, we obtain $I/I_i = 12$. These gains, although not as large as the gains that were found with a completely collisionless double layer model, can still make a significant contribution to operation of tethers for power generation.

In Fig. 4, the total current is shown for a fixed ion current of 1 A, as a function of electron saturation current, using Eq. (58), and is compared to the total current for the isotropic quasilinear model discussed in Sec. 4, and for the collisionless double layer model using an ion current of 0.01 A. Note that the current from Eq. (58) is much more sensitive to the electron saturation current than in the case of the collisionless double layer model. The reason is that the anisotropic contactor cloud, unlike the collisionless double layer cloud, cannot easily expand to larger radius to make up for a decrease in the ambient electron density. In Fig. 5, the current voltage characteristic is shown, from Eq. (58), for $J_e^\infty = 2mA/m^2$, and compared to the results from the isotropic quasilinear model, and from the collisionless double layer model for a range of electron saturation currents. For realistic potentials, less than 1000V, the current from Eq. (58) is at least an order of magnitude greater than for the collisionless double layer model.

Table 4 shows the load power P_{load} against efficiency, using the same ambient plasma and tether parameters as in Table 3, but using Eq. (58) to relate I and ϕ_{anode} . In this case, the maximum power obtained at $\simeq 80\%$ efficiency is 12kW, much higher than in in Table 3. Of course

Table 5: Load power against efficiency of emitting an ion beam

η	$I_i(A)$	ξ	$I(A)$	$P_{load}(kW)$
0.1	22.64	1	22.64	12.1
0.3	17.56	1	17.56	28.1
0.5	12.48	1	12.48	33.3
0.7	7.4	1	7.4	27.7
0.9	2.3	1	2.3	11.2

in a comparison with the collisionless double layer results the energetic cost of producing more ion current must be compared to the cost of the high potential associated with the space charge limited double layer.

Finally in Table 5 we show the power to the load for a quasineutral model which just emits an ion beam or a double layer with ionization so that a large current flows for very low potential drop ($\Delta\Phi_{contactor} \simeq 0$, $\xi = 1$). At 90% efficiency this configuration, which makes no use of the ambient plasma, can generate only slightly higher power than the anisotropic contactor, and requires substantially higher emitted ion current. This shows that the anisotropic contactor could make a significant contribution to the operation of tethers for power generation.

6 Conclusions

We have examined several models for electron collection by plasma contactors. The ground based experiments at currents below 1 A appear to be well described by a double layer model which treats the electrons as collisionless and unmagnetized. In those experiments, the double layer forms approximately at the radius where the plasma emitted from the contactor reaches the ambient plasma density. This radius is less than or comparable to both the electron Larmor radius, and the mean free path of the electrons, based on a model for effective collisions due to instabilities. In high power space applications, where the plasma cloud must have a radius of tens of meters, and the ambient electron Larmor radius is only a few cm, neither of these conditions applies. Still neglecting collisions, but taking into account the finite electron Larmor radius, we find that

ambient electrons can get across the double layer and reach the anode only if the Parker-Murphy condition[10] is satisfied (and even that is not a sufficient condition). For $r_{anode} \ll r_{outer}$ and ion current greater than 1 mA for dayside low earth orbit (even lower for nightside) the Parker-Murphy condition cannot be satisfied for a spherically symmetric double layer with space charge limited current, since the r_{inner} determined by Wei and Wilbur[8] would be less than r_{anode} , for any potential and r_{outer} satisfying the Parker-Murphy condition. This means that such collisionless double layers are not possible in space except at very low ion currents. (Collisionless double layers with higher ion currents are possible if r_{anode} is made big enough so that the bare anode could collect almost as much electron current as the contactor cloud, but the contactor cloud would then serve no purpose.) At higher ion currents and small anodes, if we assume the electrons are still collisionless, the collected electron current will not be space charge limited, as assumed by Wei and Wilbur, but will be limited to a lower value by the magnetic field. Neglecting the requirement that $r_{inner} > r_{anode}$, and considering only the Parker-Murphy condition, we found an upper limit to the collisionless electron current that could be collected, and a lower limit to the potential, as a function of ion current. We found that such a large potential is needed across the double layer in order to draw a reasonably large electron current that the available load power for a 20km long tether is never greater than 400 W. The maximum power is surely far less than this, since this figure was found for a configuration with $r_{inner} \ll r_{outer}$, and the Parker-Murphy condition is known to be far from sufficient in that limit; also, $r_{inner} > r_{anode}$ was known to be far from satisfied at the maximum power. The collisionless double layer model should be valid in space for emitted ion current sufficiently low ($I_i < 1$ mA for dayside low earth orbit, much lower for nightside) that a double layer can form with $\phi_0 < 5$ kV (the total tether voltage) allowing electrons to get across the magnetic field to the anode, and satisfying $r_{inner} > r_{anode}$. There is a further requirement for validity: the electrons must not be deflected from the anode by effective collisions, due to instabilities, as they are traversing the contactor. But this requirement is easily satisfied in space, where the ambient ω_{pe} is not too much greater than ω_{ce} .

Since a plasma contactor described by the collisionless double layer model cannot generate anything close to the desired power, we must use much higher emitted ion currents. Although

the transition from the collisionless double layer model to the collisional quasineutral model is not completely understood, we expect at sufficiently high ion current that there will be instabilities strong enough to produce a high effective electron collision frequency in the contactor cloud. Such a contactor can be described by a collisional quasineutral fluid model, in which electrons can flow across the magnetic field within a radius r_{core} of the anode. If r_{core} is defined conservatively as the radius within which the effective electron collision frequency, due to ion acoustic and Buneman instabilities, exceeds the electron cyclotron frequency, then we find that the contactor has a very low impedance, but draws very little electron current because r_{core} is rather small. The total contactor current is hardly enhanced at all above the ion current that it is emitting. Even for those cases of higher T_e where a modest gain in current occurs, that gain is due almost entirely to ionization of neutral gas emitted by the contactor, not to collection of electrons from the ambient plasma. In this case, the gas would probably be used more efficiently if it were ionized internally, in an ion source, rather than externally, where much of it can be lost.

If we include the anisotropic part of the contactor cloud where the effective electron collision frequency is less than the electron cyclotron frequency, then electrons can be collected out to a much larger radius, and an electron current a few times greater than the ion current can be drawn from the ambient plasma, even at fairly low potentials. In contrast to the upper limits derived for the collisionless double layer model, and to the quasineutral model based on the more conservative definition of r_{core} , the electron current has a significant dependence on the electron saturation current of the ambient plasma in this case, and is substantially higher, for a given ion current, on the dayside than on the nightside in equatorial low earth orbit. Analytic expressions for the potential profile and collected electron current can be obtained when the electron motion along the magnetic field is fairly collisionless, so that a double layer forms in that direction, but the electrons flow collisionally across the magnetic field. This is the regime that is relevant to high current plasma contactors in low earth orbit. Although the model which is solved analytically in Sec. 5 made the simple approximation that the effective electron collision frequency, due only to ion acoustic turbulence, is equal to $10^{-2}\omega_{pe}$, independent of T_e and the electric field, the same method should be applicable using more realistic expressions for the effective collision frequency.

Another approximation made in our analysis of this model is that there is sufficient electron thermal conductivity across the magnetic field to keep T_e much lower than ϕ_0 in the contactor cloud. The validity of this approximation must be examined using realistic turbulence models. If this approximation is at least marginally valid, then our results should be qualitatively correct.

One important conclusion of our analysis is that most of the present ground based experiments have limited relevance to space applications of plasma contactors, since they operate in a regime where the magnetic field and effective collisions are not important, or only marginally important. This is true of space-based contactors only at very low current and power levels. An exception is the experiment of Urrutia and Stenzel[23], which examined a plasma in which the electron Larmor radius was small compared to the scale of the potential, and anomalous transport of electrons across the magnetic field was important. Indeed, they found that the anode collected an electron current a few times greater than the saturation current of the flux tube that intersected the anode, even when the effective collision frequency was less than the electron cyclotron frequency. Urrutia and Stenzel attributed their cross field electron transport to ion acoustic instabilities that were excited by the azimuthal $E \times B$ drift of the electrons relative to the unmagnetized ions, which gave rise to azimuthal wave electric fields which cause radial $E \times B$ drifts. In this respect the experiment was similar to the anisotropic contactor cloud model considered in Sec. 5. However, this experiment differed in one important respect from the regime, appropriate to low earth orbit, that was considered in Sec. 5. In the experiment, the density was about $2 \times 10^{11} \text{ cm}^{-3}$ and $\omega_{pe}/\omega_{ce} \approx 50$, much higher than in low earth orbit, and as a result the anomalous parallel resistivity, due to Buneman and ion acoustic instabilities excited by the relative electron and ion flow velocity along the field, was high. The electrons did not flow freely along the magnetic field, but diffused along the field like a collisional fluid, so there were no double layers along the field. It would be desirable to do ground-based experiments in the regime where the electrons flow freely along the magnetic field but collisionally across the magnetic field, since this is applicable to high power plasma contactors in low earth orbit, and to compare the measured $\phi(r, z)$ and collected current to the expressions calculated in Sec. 5, or to similar expressions found with more realistic models for ν_e .

Another interesting feature seen by Urrutia and Stenzel is that the enhanced electron current

was not continuous in time but occurred in periodic bursts, as the instabilities periodically grew up, saturated, and decayed. It is not certain whether that is an inevitable feature of this kind of cross-field transport, or is a consequence of the particular conditions of the experiment, which could have been modified to produce steady enhanced transport. It is also not known whether similar behavior would occur in the regime of free electron flow along the magnetic field and collisional flow across the magnetic field, appropriate for low earth orbit. Theoretical and experimental studies are needed to answer these questions, which could have important implications for power systems based on electrodynamic tethers in space.

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Figure Captions

Figure 1 Gain vs. argon ion current for collisionless double layer with space charge limited current, marginally satisfying the Parker-Murphy condition with a 10 cm anode radius, for the range of electron saturation currents found in low earth orbit.

Figure 2 Lower limit on potential drop for collisionless double layer with space charge limited current, marginally satisfying the Parker-Murphy condition, as function of emitted ion current and electron saturation current.

Figure 3 Inner and outer radii of collisionless double layer with space charge limited current, marginally satisfying the Parker-Murphy condition, as function of emitted ion current and electron saturation current.

Figure 4 Total collected current vs. electron saturation current with the emitted ion current held constant, for the collisionless double layer model (upper limit), the isotropic quasineutral model and the anisotropic contactor model.

Figure 5 Total current vs. potential drop for the collisionless double layer model, the isotropic quasineutral model and the anisotropic contactor model.

Figure 6 Schematic picture of the anisotropic contactor model, showing equipotential contours and the flow of ions and electrons.

FIGURE 1

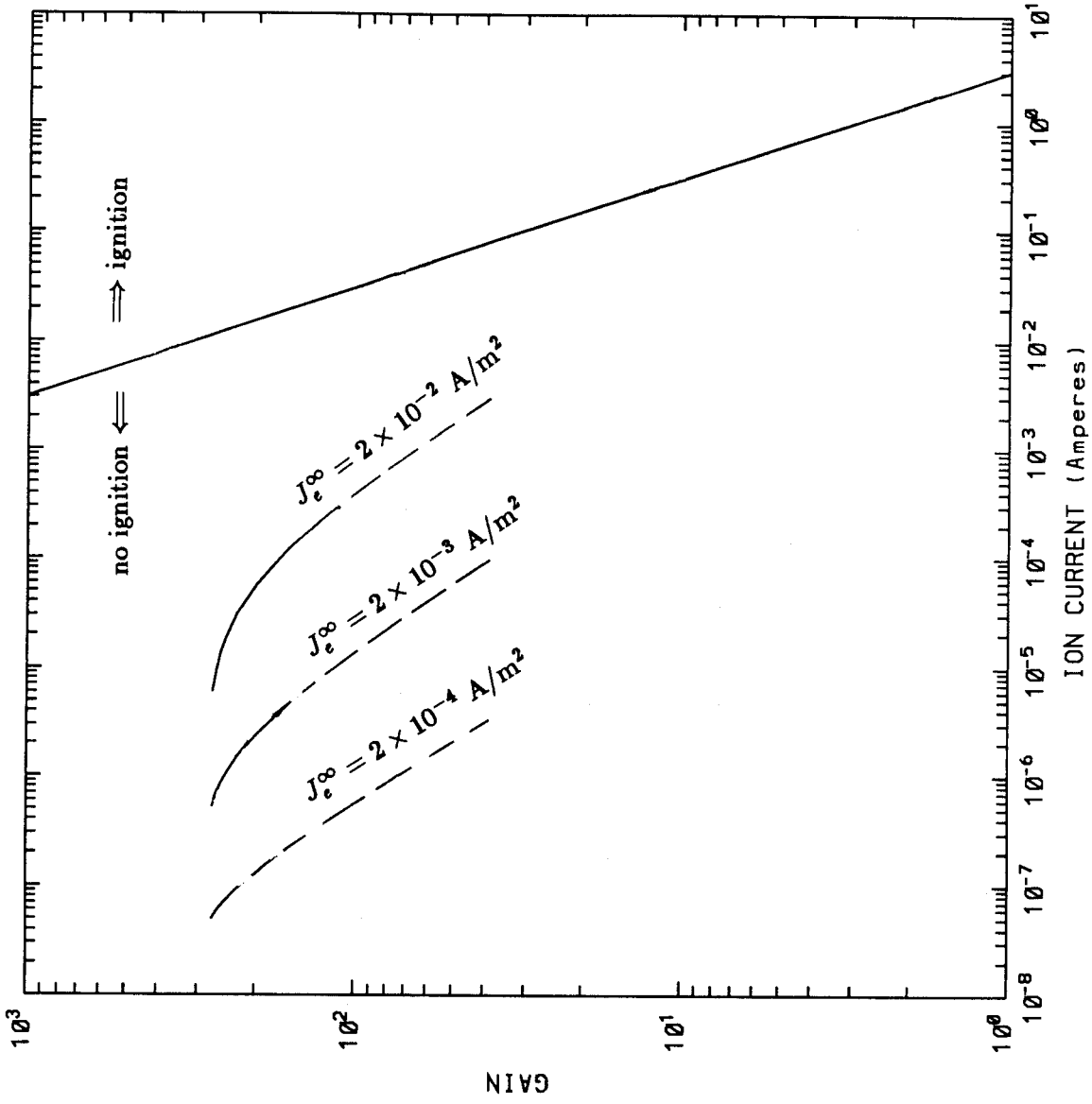


FIGURE 2

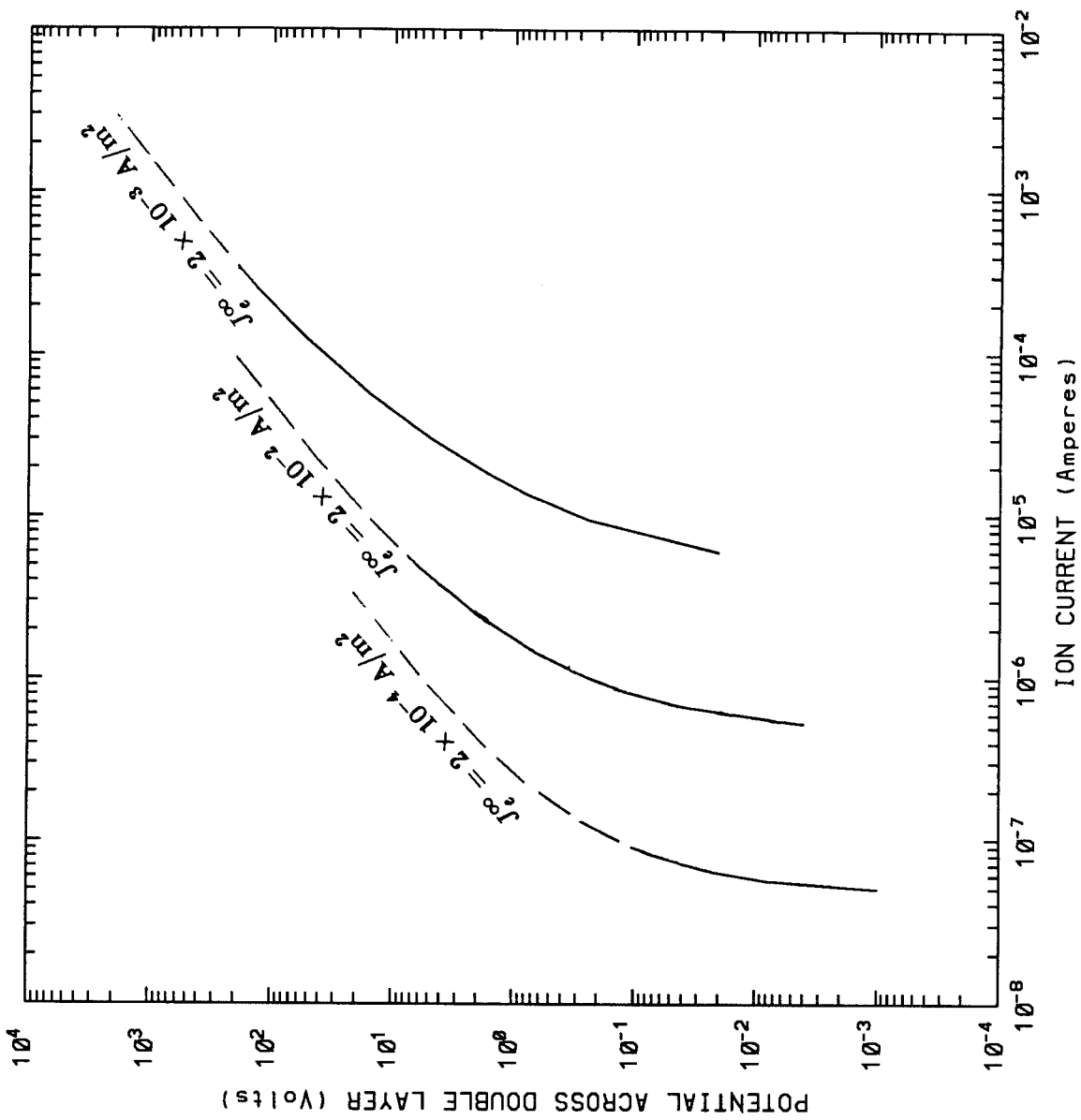


FIGURE 3

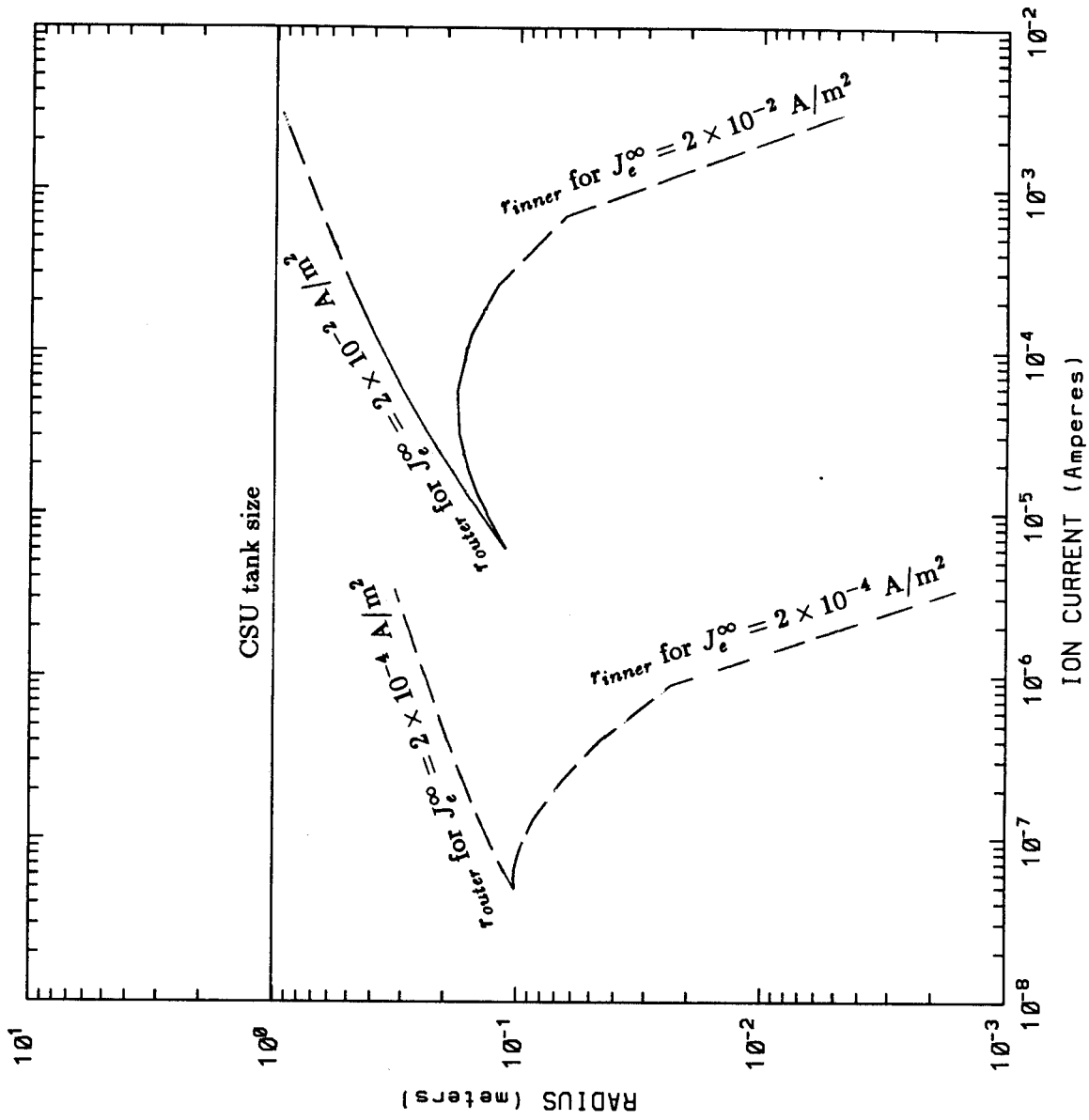


FIGURE 4

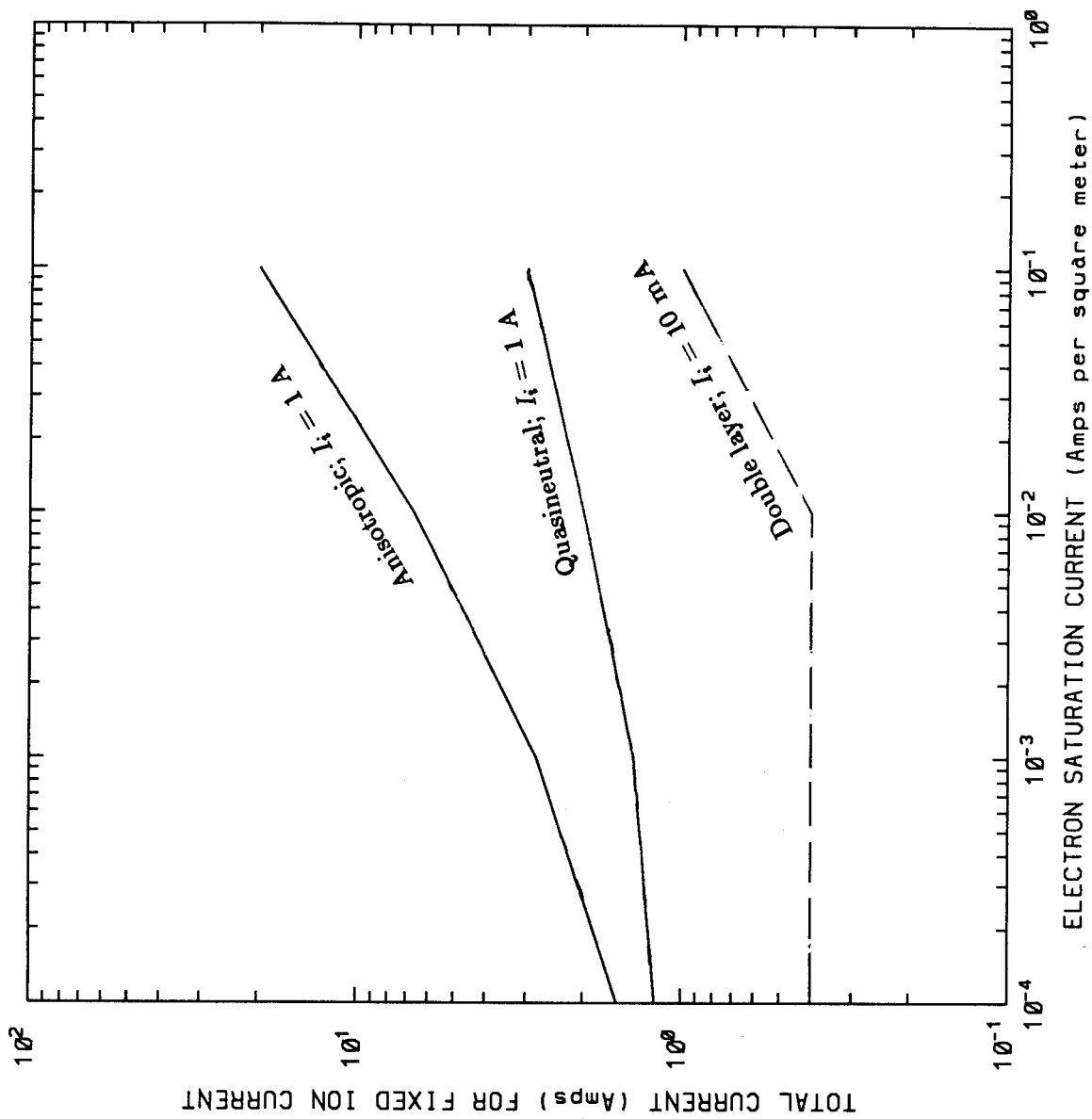
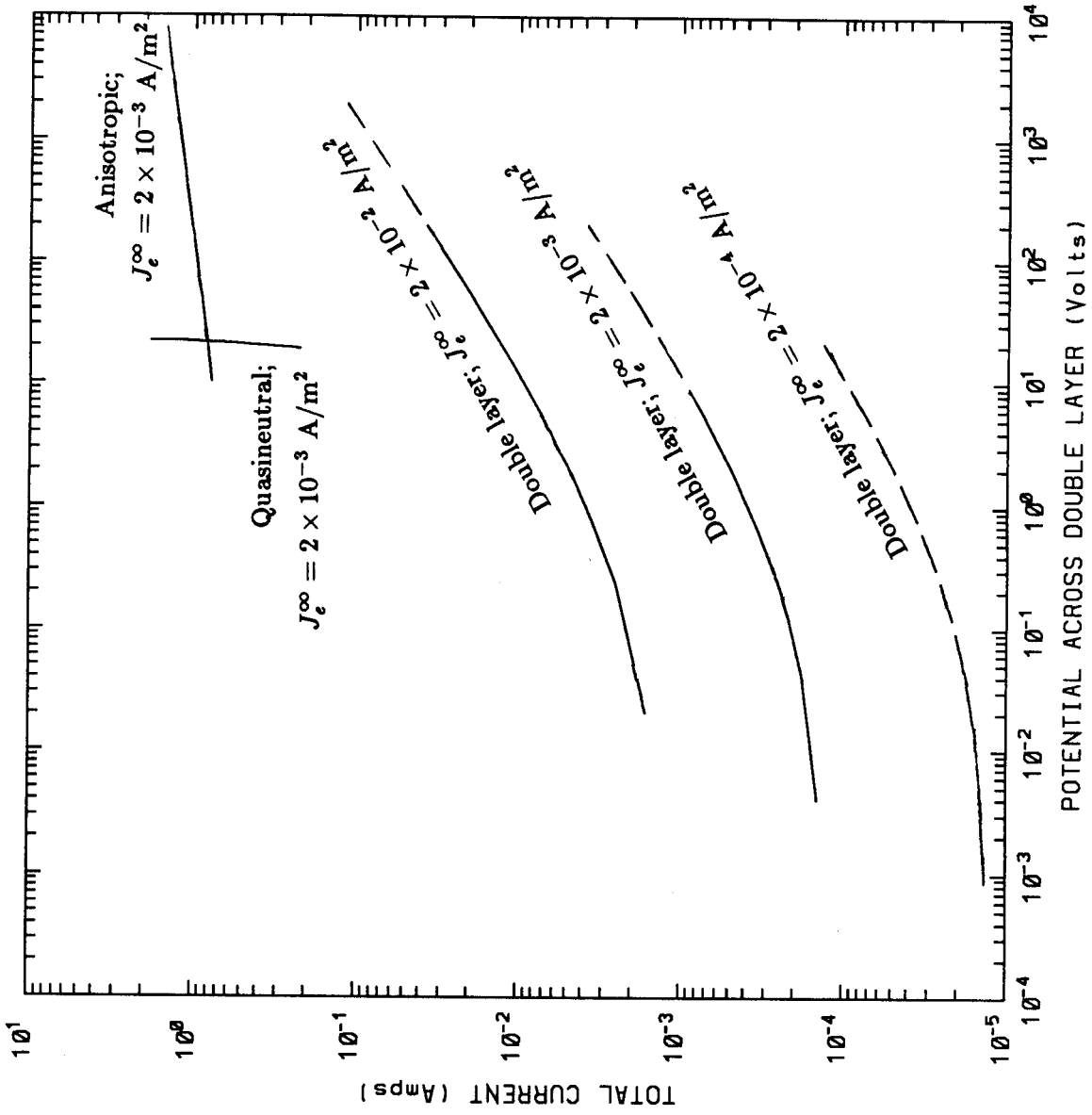


FIGURE 5



Anisotropic Contactor Model

