PFC/JA-87-39

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October 1987

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This work was supported in part by NOAA Sea-Grant Contract NA86AA-D-SG089, NSF Grant No. ECS-8515032, and DOE Contract No. DE-AC02-78ET-51013.

To appear in: Physical Review Letters

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ABSTRACT

The Karman vortex-street in the wake of a circular cylinder is shown to be due to an absolute instability of the flow in the near wake. A new means of instability analysis is used, involving mappings from the complex k-plane to the complex ω-plane.

PACS numbers: 47.20.Ft, 47.25.Gk, 52.35.Py

The onset and formation of coherent vortex structures in shear flow is a problem of long-standing and current interest in the dynamics of fluids and plasmas. For example, they appear in the flow past a cylinder (Karman vortex street), the Kelvin-Helmholtz instability in neutral plasmas, and the diocotron instability in non-neutral plasmas. The appropriate, from first principles, nonlinear dynamic equations, for the fluid or the plasma, are much too difficult to solve, even numerically, to describe the detailed evolution of such structures. We show here that the formation of the Karman vortex street can be understood in a new and rather simple way: from a *linear, space-time stability analysis of the average flow.* The comparison between our experimental results and the relevant experimental features is excellent, thus indicating that this approach may be useful in a variety of other, similar physical problems.

Vortex streets are known to form in the wake of circular cylinders for a wide range of Reynolds numbers (roughly, from 40 to 300,000). The mechanism of vortex street formation has been the subject of many investigations, owing to its importance for flow-induced vibration problems [1]. In this letter we report a new approach, in which the vortex formation process is treated as a hydrodynamic instability of the cylinder wake[2], and the distinction between absolute and convective instabilities[3] is used to elucidate experimental observations of the phenomenon. Namely, the appearance of a vortex street in a cylinder wake, seen as a self-sustained oscillation of the wake, is shown to be related to the issue of whether the wake instability is absolute or convective. In an absolutely unstable wake, any initial disturbance will grow in time, and, after non-linearities have saturated its growth, will evolve into a self-sustained oscillation of the wake. In a convectively unstable wake, all disturbances will be carried away, leaving finally the wake undisturbed. The results of the stability analysis suggest that the vortex street behind the circular cylinder is the non-linear evolution of the "preferred instability" mode determined from a linear stability analysis. Interestingly, by comparing predictions of the linear stability analysis[2] with experiments, it appears that the frequency and wavelength of the "preferred instability" mode found in the linear problem, remain unchanged throughout the non-linear evolution of the instability, for both laminar[4] and turbulent[5] wakes. Furthermore, calculated shapes of the impulse response of the wake indicate that the wake instability develops in time without significant interactions with the cylinder itself.

Within the context of linear theory, the distinction between absolute and convective instabilities for a spatially homogeneous medium can be made by studying the dispersion relation $D(\omega,k)=0$ of the medium, where ω is the frequency, and k the wavenumber. In general, both ω and k are complex. Let $G(x,t)$ be the response of the medium at a location x and time t to an impulsive excitation applied at the origin. The response $G(x,t)$ is expressed by the Fourier-Laplace integral:

$$G(x,t)=\frac{1}{(2\pi)^2}\int_L d\omega \int_F dk \frac{e^{i(kx-\omega t)}}{D(\omega,k)} \tag{1}$$

where L and F are appropriate integration contours in the complex ω and k planes, respectively. For most physical problems, the double integral in (1) cannot be easily evaluated for all t. In order to distinguish between absolute and convective instabilities, however, we only need to know the asymptotic behaviour of $G(x,t)$ for large times. This time-asymptotic behaviour of $G(x,t)$ can be determined using a well-known method of analytic continuation, in which the Laplace contour L is deformed towards the lower half of the complex ω-plane. If L can be deformed below the real-ω axis, the instability is convective. Otherwise, $G(x,t\to\infty)$ is dominated by the "pinch-point" singularity having the largest temporal growth rate[3]. This is the case of an absolute instability, where the real parts of the wavenumber and frequency of the pinch-point specify the "preferred instability mode".

The procedure described above, requires obtaining from the dispersion relation the wavenumber k as a function of the frequency ω. However, for the dispersion relations resulting in the stability analysis of parallel shear flows, it is easier to determine ω as a function of k, than the other way around. We consider, therefore, that the dispersion relation has been solved to yield ω as a function of k, and we seek an inversion of the previously described analytic continuation, that does not depend on mapping from the ω-plane into the k-plane. This is done by deforming the F-contour off the real-k axis in such a way that its image in the ω-plane progresses downward from the highest branch of the map of the real-k axis (Figure 1). Double roots of the dispersion relation , (ω_0,k_0), are easily detected by the local angle-doubling property of the map : $(\omega-\omega_0) \sim (k-k_0)^2$. In the simplest cases, absolute instabilities occur when the deformed F-contour maps into the complex ω-plane as shown in Figure 1, where the point ω_0 is found to lie in the upper-half ω-plane, beneath a single unstable branch of the image of the real-k axis. The point ω_0, connecting two Riemann sheets of the multi-sheeted ω-plane, is only covered by the image of the real-k axis on one of these two sheets. Thus, if the L-contour , deformed to pass through ω_0, is mapped in the k-plane, its image will pinch the deformed F-contour at k_0. Consideration of this simple topology is sufficient for the stability analysis of symmetric shear flows. The procedure for cases leading to mappings of higher topological complexity can be found in [6].

We consider the stability of the time-average flow in the wake of a circular cylinder in steady flow. We define the x axis to be parallel, and the y axis normal to the oncoming flow. The mean flow and the disturbances are assumed to be two-dimensional. Outside the cylinder's boundary layer the flow can be considered inviscid. For any inviscid parallel shear flow, the dispersion relation consists of the Rayleigh equation[7]:

$$(kU(y)-\omega)(f''(y)-k^2f(y))-kf(y)U''(y)=0 \qquad (2)$$

subject to the boundary conditions:

$$f(y) \rightarrow 0 \quad \text{for} \quad |y| \rightarrow \infty \tag{3}$$

where f(y) is the stream function of the disturbance, U the mean flow in the wake, and an upper prime stands for differentiation with respect to y. For any given k, equations (2) and (3) define an eigenvalue problem for ω, and vice-versa. Strictly speaking, the flow in the wake is not parallel, as use of Rayleigh's equation implies, but slowly diverging. However, as experimental measurements of the average velocity distribution in wakes show[4],[5], the rate of change of the velocity profile in the x direction is small. Therefore, we can assume that, at each location behind the cylinder, the flow is locally parallel, and, consequently, that the mean velocity U is a function of y only. Within the limits of this assumption, a local stability analysis can separately be performed at each location behind the cylinder. The distribution of the mean velocity U(y) in the wake is symmetric about the x axis. Therefore the eigenvalue problem defined by equations (2),(3) can be decomposed into two parts, or modes, symmetric and antisymmetric. For the symmetric mode, we have: $f(y)=f(-y)$, and for the antisymmetric mode: $f(y)=-f(-y)$. Superposition of the vorticity of a symmetric mode to the initial antisymmetric vorticity distribution leads to a staggered vortex street; conversely, the antisymmetric mode would lead to a symmetric vortex street. Thus, decomposition of the disturbance stream function into symmetric and antisymmetric parts proves very helpful in explaining why vortex streets are always staggered.

For an arbitrary U(y), the eigenvalue problem must be solved numerically. In [2] a fourth-order-accurate finite difference scheme was used to approximate the derivatives of the stream function in Rayleigh's equation. Thus, together with the boundary conditions and the symmetry or anti-symmetry of the stream function, the problem was reduced to a generalized matrix eigenvalue problem, that was solved to yield ω as a function of k. The stability of experimentally measured velocity profiles of cylinder wakes was made using data for laminar wakes provided in Kovasznay[4], and data for turbulent wakes provided in Cantwell[5]. The results of the stability analysis are summarized below.

We first discuss the results for laminar wakes. Kovasznay[4] has provided extensive measurements of the average velocity in the wake of a circular cylinder at Reynolds numbers 34 and 56. When the Reynolds number is equal to 56, the wake of the cylinder is unstable, and a laminar vortex street is formed. When the Reynolds number is equal to 34, the wake of the cylinder is unstable, but no vortex street is formed. This qualitatively different behaviour of two apparently similar situations can be explained by examining the physical character of the wake instability. When the Reynolds number is equal to 34, the results of the stability analysis indicate that the wake instability is convective at any location behind the cylinder. Thus, in agreement with Kovasznay's observations[4], in absence of a permanent external excitation, all randomly excited disturbances are convected away, leaving the wake undisturbed. When the Reynolds number is equal to 56, however, the stability analysis indicates that the near wake, i.e. the wake immediately behind the cylinder, is absolutely unstable in the symmetric stream function mode (the one that produces a staggered vortex street). Further away from the cylinder, the instability gradually changes into convective again. Therefore, the following mechanism of vortex street formation is suggested: Disturbances in the near wake, which are absolutely unstable, lead eventually to the development of a self-sustained oscillation. The self-sustained oscillation of the near wake serves as an oscillatory source of excitation for the rest of the wake, which is only convectively unstable, and merely responds to the excitation provided by the near wake. Thus the frequency of the vortex street is selected in the near wake, whereas the wavelength of the vortex street varies along the wake, as the local dispersion relation requires at each location. The theoretically predicted frequency from the detailed stability analysis[2] yields a Strouhal number equal to 0.13. The experimentally recorded value of the Strouhal number for this Reynolds number is also 0.13 [8].

We now discuss the results for turbulent wakes. Cantwell[5] has provided measurements of the time average velocity distribution in the wake of a circular cylinder at Reynolds number equal to 140,000. At this Reynolds number a turbulent vortex street is formed. Following [9], it was assumed that the turbulent vortex street results from the instability of the time-average (or "pseudo-laminar") flow in the wake. The direct effect of the small scale turbulence on the evolution of the instability was neglected. The presence of the small scale turbulence was acknowledged, however, indirectly, as it affects the form of the time-average velocity profile. By the instability analysis described above, the physical mechanism of vortex-street formation in turbulent wakes is found to be the same as the one in laminar wakes. Namely, it is found that the time-average flow in the near wake presents an absolute instability in the symmetric function mode, which excites the rest of the wake. The detailed stability analysis [2] predicted a frequency of vortex street formation giving a Strouhal number equal to 0.21. Cantwell[5] has reported a Strouhal number value, uncorrected for blockage effects, equal to 0.18. Roshko[10], who has summarized the results of several investigations, gives a Strouhal number equal to 0.20. Therefore, as for laminar wakes, theory and experiment are in good agreement.

In the stability analysis of the cylinder wake[2], the effect of the presence of the cylinder itself on the development of the instability was neglected. A justification for this approach can be sought in the way that the impulse-response of the wake evolves in time. As shown in [3], the time-asymptotic shape of the response is self-similar, and can be determined by finding the imaginary part, ω_i, of the pinch-point frequency, as seen by observers moving at various speeds. The time-asymptotic shape of the disturbance has been calculated[11] for Reynolds numbers 56 and 140,000, and the results are shown in Figures 2a and 2b respectively. In both figures it can be seen that the response propagates mainly downstream of the cylinder, with only a very small portion of the response propagating towards the cylinder at a very low speed. Consequently, in laminar and turbulent wakes alike, the instability of the wake of the cylinder develops downstream of the cylinder, without significant interactions with the cylinder itself. This result supports the assumption made earlier by Abernathy & Kronauer[12], that formation of vortex streets in a wake occurs independently of the object producing the wake.

In conclusion, linear stability analysis of the time-average flow in the wake of a circular cylinder offers a relatively simple way of understanding the dynamics of the wake. In particular, by investigating whether the instability is absolute or convective, the ability of the wake to develop self-sustained oscillations is determined. Absolute instabilities are shown to be established unequivocally by mapping, through the dispersion relation, from the k-plane into the ω-plane. For fluid-mechanics problems, this procedure is much easier to implement than the usual reverse mapping. The vortex street in the wake of a circular cylinder is found to form as a result of an absolute instability in the near wake. The unstable disturbances in the near wake propagate mainly downstream of the cylinder and excite the rest of the wake. The frequency of the preferred instability mode predicted by a linear stability analysis is in good agreement with the experimentally recorded frequency for low and high Reynolds numbers. The good agreement between the results of the linear stability analysis and experimental observations suggests that the present methodology could, potentially, be applied to a variety of phenomena in flow transition and turbulence.

This work was supported in part by NOAA Sea-Grant Contract No. NA86AA-D-SG089, NSF Grant No. ECS-8515032, and DOE Contract No. DE-AC02-78ET-51013.

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Figure Captions

Figure 1 : Curve 1: image of the real-k axis in the ω plane; Curves 2 through 6: images of lines parallel to the real-k axis. The pinch-point is located at the cusp of curve 6.

Figures 2a, 2b : Time-asymptotic form of the unstable disturbance for Reynolds numbers 56 and 140,000, respectively[11]. Note, $\omega_i t \sim ln|G(x,t \to \infty)|$ [3].

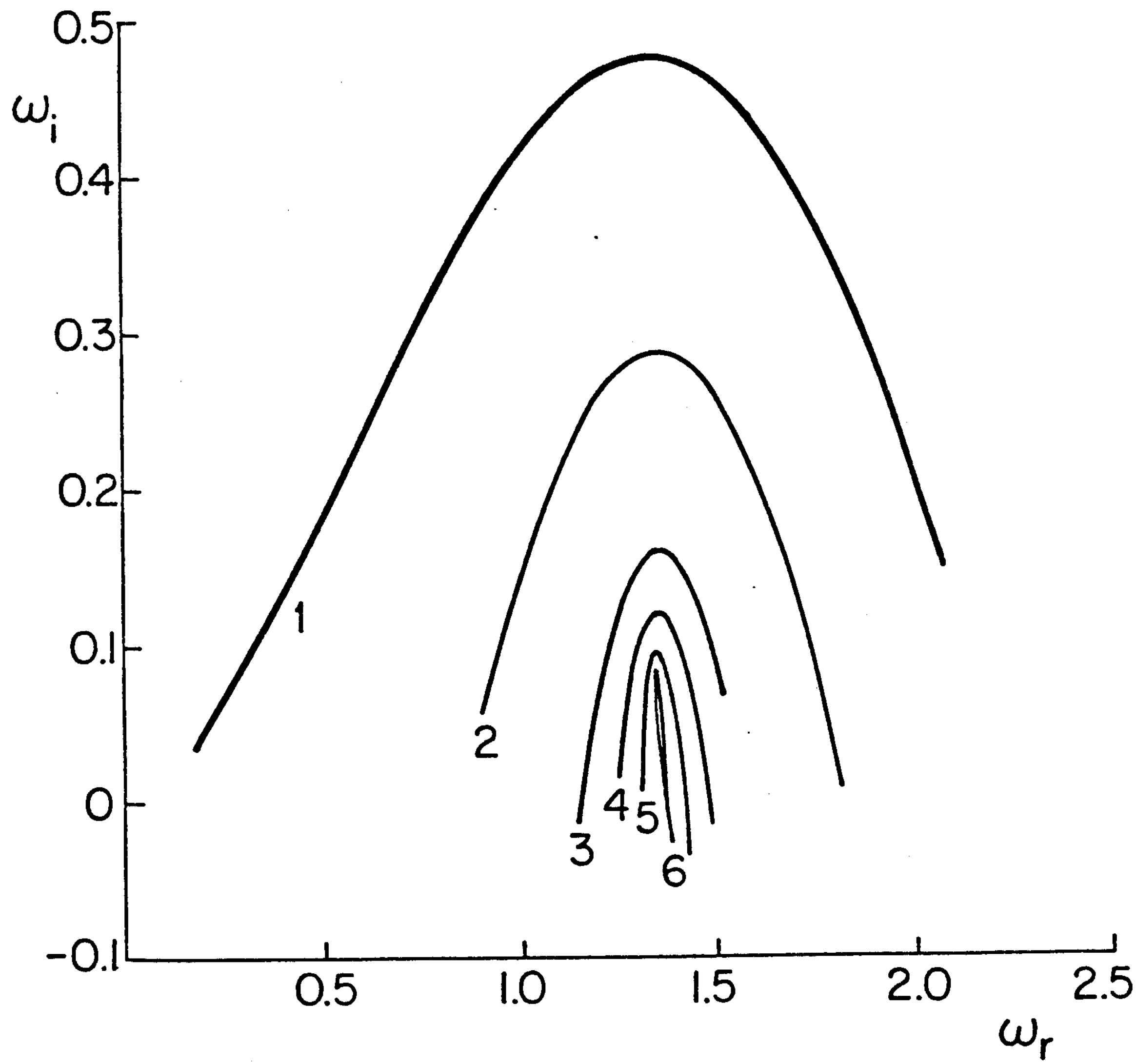

Figure 1

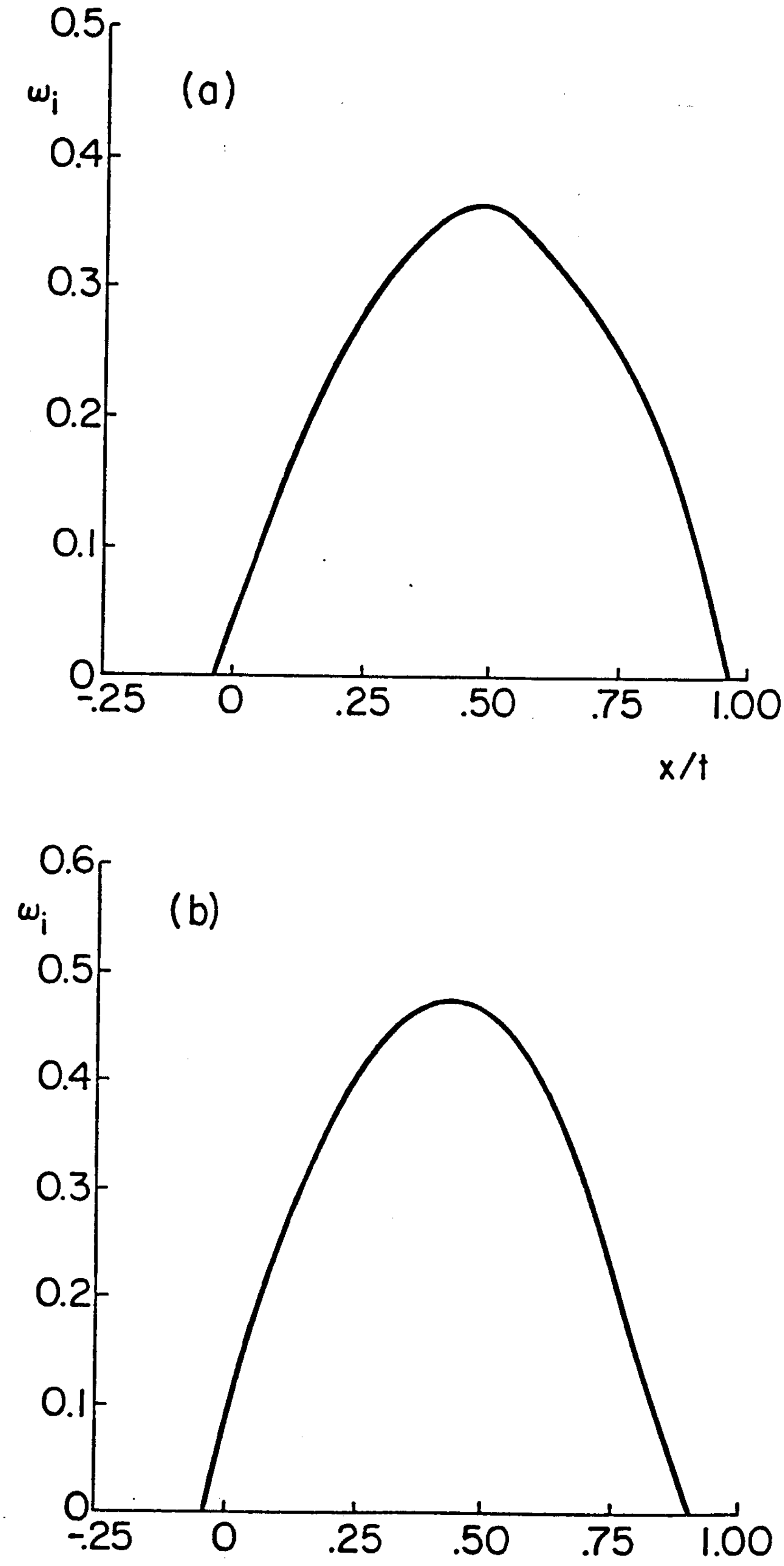
0.5
ωi
(a)
0.4
0.3
0.2
0.1
0
-.25
0
.25
.50
.75
1.00
x/t
0.6
ωi
(b)
0.5
0.4
0.3
0.2
0.1
0
-.25
0
.25
.50
.75
1.00
x/t

Figure 2, A, B