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Radial Electric Field Evaluation and Effects in the Core and Pedestal

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Radial Electric Field Evaluation and Effects in the Core and Pedestal

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Abstract. We highlight the necessity for evaluating and evolving the global axisymmetric radial electric field in a tokamak consistent with conservation of toroidal angular momentum for both δf and full f gyrokinetic codes, where f is the full distribution function and δf its departure from Maxwellian. We also consider the effects of a strong radial electric field in a subsonic pedestal in the banana regime, finding that it acts to reverse the poloidal flow, increase the bootstrap current, and enhance the residual zonal flow regulation of turbulence. We then generalize the concept of intrinsic ambipolarity to an H mode pedestal.

1. Introduction

We have recently obtained a complete gyrokinetic description for evaluating and evolving the global axisymmetric radial electric field in a turbulent tokamak as well as for solving for the remaining components of the electric field [1-3]. In addition, we have developed procedures for evaluating strong radial electric field caused modifications to the banana regime ion heat flow, ion and impurity flows in a flux surface, the bootstrap current, and the zonal flow residual in a subsonic pedestal of a tokamak [4-7]. In what follows, we provide more insight into these detailed calculations in a relatively equation free manner in order to focus on physics issues not discussed in detail in these earlier publications. In particular, we explain why the global axisymmetric radial electric field in a tokamak is so difficult to determine and why it cannot be determined from quasineutrality, and we describe how the concept of intrinsic ambipolarity [8-9] must be generalized in a subsonic pedestal where, unlike the core, the transport coefficients depend on the radial electric field.

In section 2 we illustrate the impracticality of determining the global axisymmetric radial electric field from quasineutrality by considering the general form for the flux surface average of total toroidal angular momentum conservation for a turbulent tokamak [3]. The discussion highlights the difficult and subtle issues associated with extending gyrokinetics to the transport time scales at which the turbulent and neoclassical fluxes evolve the flux surface averaged density, temperature, and electrostatic potential profiles. We demonstrate that any analytic or numerical error in the radial current and/or quasineutrality can easily generate a torque that will generate a non-physical global axisymmetric radial electric field. The need to evaluate the global radial electric field from toroidal angular momentum conservation implies that δf gyrokinetic descriptions are sufficient and there is no necessity for a full f description.

Sections 3 to 5 focus on the role of the pedestal radial electric field. In a subsonic pedestal the **E**×**B** drift and the ion diamagnetic drift must cancel to lowest order so their difference is small and comparable to ion temperature gradient terms in the ion flow on a flux surface [10-12]. In this situation the ions are electrostatically confined to lowest order and a large radial electric field exists. The associated large **E**×**B** can compete with the poloidal component of the parallel streaming to modify ion trajectories. These altered orbits thereby introduce electric field modifications into the ion temperature gradient terms of the ion heat flux and ion and impurity flows [5], and the bootstrap current [7] as briefly discussed in section 3, as well as modify [4,6] the residual zonal flow response of Rosenbluth and Hinton [13] as noted in section 5. In spite of these changes we explain in section 4 how the plasma remains intrinsically ambipolar in the more general sense that the particle flux depends on the radial electric field (unlike in the core), but cannot be determined by the setting the flux surface averages of the radial ion and electron currents equal to lowest order.

To perform calculations in the pedestal, where there are strong radial density and electron temperature gradients on the order of a poloidal ion gyroradius ρ_{pi}, it is convenient to employ canonical angular momentum as the radial gyrokinetic variable [10]. To simplify further we assume $B_p/B << 1$. The ion temperature pedestal is always at least B/B_p wider than ρ_{pi} because of the constraint that the entropy production must vanish in a banana regime pedestal [10].

2. Global axisymmetric radial electric field in the core

Parra and Catto [14] point out the impracticality of using quasineutrality or radial ambipolarity to determine the global axisymmetric core radial electric field of a turbulent tokamak. To verify this conclusion in the electrostatic limit consider the flux surface average (denoted by $\langle ... \rangle$) of total toroidal angular momentum conservation:

$$\langle \mathbf{J}\cdot\nabla\psi\rangle - ce\langle (Zn_i - n_e)\partial\Phi/\partial\zeta\rangle = c\partial/\partial t\langle R^2 M n_i \mathbf{V_i}\cdot\nabla\zeta\rangle + (c/V')\partial/\partial\psi[V'\langle R^2\nabla\zeta\cdot\vec{\vec{\pi}}_i\cdot\nabla\psi\rangle], \quad (1)$$

where $\mathbf{B} = I\nabla\zeta + \nabla\zeta\times\nabla\psi$ with ζ, ϑ and ψ the toroidal angle, poloidal angle and poloidal flux variables, $B = |\mathbf{B}|$, $|\nabla\psi| = RB_p$, $V' = \oint d\vartheta/\mathbf{B}\cdot\nabla\vartheta$, $\mathbf{J}$ is the current density, n_e, n_i, $\mathbf{V}_i$ and M are the electron and ion densities, the ion mean velocity and mass, R is the major radius, c is the speed of light, e is the magnitude of the charge on an electron, and $\vec{\vec{\pi}}_i$ is the full ion stress tensor including Reynolds stress, and gyro and perpendicular viscosity. For a quasineutral plasma $n_e = Zn_i$ and $\nabla\cdot\mathbf{J} = 0$ requiring ambipolarity: $\langle \mathbf{J}\cdot\nabla\psi\rangle = 0$.

To see the difficulty we allow an error departure from ambipolarity by estimating $\langle \mathbf{J}\cdot\nabla\psi\rangle \sim RB_pJ_{error}$ rather than setting it to zero. Then, we estimate the size of the off diagonal stress tensor by assuming a gyroBohm scaling to find $\langle R^2\nabla\zeta\cdot\vec{\vec{\pi}}_i\cdot\nabla\psi\rangle \sim RB_pD_{gB}\nabla(RMn_i\mathbf{V}_i) \sim p_iR^2B_p(\rho_i/a)^3(B/B_p)$, where $p_i = n_iT_i$ is the ion pressure, a is the minor radius, and $\rho_i = v_i/\Omega_i$ is the ion gyroradius with $v_i = (2T_i/M)^{1/2}$ and $\Omega_i = ZeB/Mc$ the ion thermal speed and cyclotron frequency. The B/B_p factor comes from the ion flow $\mathbf{V}_i \sim v_i(\rho_i/a)(B/B_p) << v_i$. For our gyroBohm diffusivity estimate, $D_{gB} \sim (\rho_i/a)\rho_iv_i$. The preceding estimates give $J_{error} \sim en_iv_i(B/B_p)^2(\rho_i/a)^4$ as the ambipolarity error that will cause radial momentum transport to be incorrectly evaluated. This error can be generated by analytic simplifications (such as a not going to high enough order in the ρ_i/a expansion - a problem with typical Hamiltonian gyrokinetic treatments in addition to their neglect of collisions) or with numerical noise or algorithm shortcomings.

The ion diamagnetic current is $J_{dia} \sim en_iv_i(B/B_p)(\rho_i/a)$. It is $(R/a)^{1/2}$ larger than the bootstrap current that is the same order as the Ohmic and driven currents generating the poloidal magnetic field when $(R/a)^{1/2}(4\pi p_i/B_p^2) \sim 1$. Noticing that the ratio $J_{error}/J_{dia} \sim (\rho_i/a)^3(B/B_p) << 1$, we see that an error in the correction to the lowest order ion Maxwellian distribution function f_{Mi} as small as $(B/B_p)^2(\rho_i/a)^4 \sim J_{error}/en_iv_i$ will lead to an unphysical torque on the plasma if it results in a non-ambipolar radial current. Estimating $\rho_i/a \sim 1/300$ and $B/B_p \sim 10$ this requires evaluating f/f_{Mi} to better than order 10^{-8}, which is $(B/B_p)(\rho_i/a)^3$ smaller than the ion diamagnetic correction required to properly evaluate ion neoclassical heat transport. Such accuracy, for diffusive gyroBohm transport, would require a very large number of particles per cell. To see why this is the case we use the radial diffusivity due to noise estimate from equation (9) of [15]. Taking the correlation time $\tau \sim a/v_i$, the grid cell size $\Delta y \sim \rho_i$, and the fluctuating noise potential associated with discretization $\langle\Phi\rangle_{rms} \sim T/eN^{1/2}$, where N is the number of particles per cell, gives $D_{noise} \sim (\tau/B^2)(\langle\Phi\rangle_{rms}/\Delta y)^2 \sim av_i/N$. Noise induced transport competes with anomalous transport when $D_{noise} \sim D_{gB} \sim (\rho_i/a)\rho_iv_i$ or for $N \sim (a/\rho_i)^2 \sim 10^5$ particles per cell. This estimate seems to be conservative but within an order of magnitude of what is observed when momentum transport effects are ignored since a recent full f kinetic electron PIC simiulation of TEM turbulence in FT-2 ran with $\sim 10^4$ particles per cell for $(a/\rho_i)^2 \sim 10^4$ [16]. Of course, many more particles per cell are required to avoid having the noise impact toroidal angular momentum conservation. Estimating $J_{noise} \sim eD_{noise}n_i/a$ as the error current $J_{error} \sim (B/B_p)^2(\rho_i/a)^4en_iv_i$ that has an impact gives $N \sim (B_p/B)^2(a/\rho_i)^4 \sim 10^8$ particles per cell. As already indicated, the precise estimate of the number of particles per cell is somewhat sensitive to the details of the PIC code used, but the 10^3 increase in the number of cells required should be a reasonable estimate of the increased number of particles per cell required to avoid introducing noise errors into conservation of toroidal angular momentum. Moreover, runs $(a/\rho_i)^2 \sim 10^5$ times longer than run times of tens to hundreds of transit times $\sim a/v_i$ will be required to evolve turbulence on transport time scales.

To estimate the associated quasineutrality error that will generate the same unacceptable torque we use charge conservation with $\partial/\partial t \sim v_i/a$ to find $(Zn_i - n_e)_{error}/n_i \sim J_{error}/en_iv_i$. As a result, we find $(Zn_i - n_e)_{error}/n_i \sim (B/B_p)^2(\rho_i/a)^4 \sim 10^{-8}$ as the quasineutrality error that will lead to an unacceptable torque. For such an error in the drift ordering, $c\partial\Phi/\partial\zeta \sim RB_pv_i\rho_i/a$, the second term on the left of (1) gives $ce\langle(Zn_i - n_e)_{error}\partial\Phi/\partial\zeta\rangle << \langle\mathbf{J}_{error}\cdot\nabla\psi\rangle$. Consequently, a direct evaluation of the gyrokinetic charge densities via quasineutrality requires an accuracy much greater than $(B/B_p)^2(\rho_i/a)^4$. Such an evaluation is a hopelessly difficult and unnecessary task.

Any attempt at a direct solution of a kinetic equation and quasineutrality to evaluate radial momentum transport or equivalently, the global axisymmetric radial electric field, will be dominated by the shortcomings of the kinetic description as illustrated in [17], as well as numerical inaccuracies. These problems can be avoided by using a hybrid or extended fluid - gyrokinetic description that exploits the smallness of the ion gyroradius in strongly magnetized plasmas in the presence of turbulence [1-3,18]. Hybrid or extended descriptions use the conservation of number, momentum, and energy equations, as well as a few other moments of the exact Fokker-Planck equation rather than a reduced kinetic description [19]. Even so, a direct evaluation of $\langle R^2\nabla\zeta\cdot\vec{\pi}_i\cdot\nabla\psi\rangle$ using the gyrokinetic f requires f/f_{Mi} to order $(B/B_p)(\rho_i/a)^3$ - still an impractical task. Fortunately, the use of higher moments of the Fokker-Planck equation allows $\langle R^2\nabla\zeta\cdot\vec{\pi}_i\cdot\nabla\psi\rangle$ to be evaluated using an f/f_{Mi} that need only be known to order $(\rho_i/a)^2(B/B_p) \sim 10^{-4}$ [1]; more than a $(B_p/B)(a/\rho_i)^2 \sim 10^4$ decrease in the accuracy required over full f gyrokinetic treatments. Moreover, gyrokinetic equations need only be solved occasionally since profile evolution is handled by the conservation equations. Implementing a hybrid fluid - gyrokinetic description differs for δf and full f gyrokinetic simulations [1], but in both cases the hybrid description is formulated to employ the least accurate f required to evaluate the off-diagonal elements of the ion stress tensor to determine the global axisymmetric radial electric field.

It is tempting to form conservation of toroidal angular momentum from a conservative form of the gyrokinetic equation by taking its canonical angular momentum = $\psi_* \equiv \psi - (Mc/e)R^2\vec{v}\cdot\nabla\zeta$ moment, just as energy conservation can be formed from the total energy = $E = v^2/2 + Ze\Phi/M$ moment. However, this method is only valid when the distinction between $\vec{R} \approx \vec{r} + \Omega_i^{-1}\vec{v}\times\vec{n}$ and $\vec{r}$ variables is unimportant to lowest order. Unlike radial ion heat flux, the lowest order radial flux of toroidal angular momentum (the lowest order Reynolds stress) has strong spatio-temporal variation, that must coarse grain average to zero in an up-down symmetric tokamak in the absence of flow shear (as generated by the Coriolis pinch due to a toroidally directed flow). Indeed, this lowest order vanishing of the Reynolds stress is expected to be a robust property even in asymmetric tokamaks provided the mean flow is diamagnetic in order [1]. Consequently, even seemingly small corrections to the lowest order Reynolds stress matter. Only for sonic flow will the symmetry breaking flow shear terms matter more than these diamagnetic drift ordered gradient driven corrections that were evaluated in [1].

Retaining these drift ordered terms becomes extremely difficult when toroidal momentum conservation is formed using the ψ_* moment of the gyrokinetic equation because higher order corrections in the gyroradius, ρ_i/a, and fluctuation amplitude, $e\delta\Phi/T$, expansions of the gyrokinetic equation are required that carefully retain the necessary distinction between $\vec{R}$ and $\vec{r}$ in momentum conservation when transforming from guiding center phase space back to physical space [1,2]. In particular, the expansions in $\rho_i/a \sim e\delta\Phi/T$ must be performed concurrently to second order (at least) by keeping modifications due to products of ρ_i/a and $e\delta\Phi/T$ [including the distinction between $\vec{B}(\vec{R})$ and $\vec{B}(\vec{r})$, and collisional turbulent and neoclassical effects]. Moreover, when rewriting ψ_* in gyrokinetic variables the gyrokinetic corrections to the ψ have to be retained to one order higher than in the $Iv_{\|}/\Omega_i$ term since to lowest order $\psi_* = \psi$, while to next order $\psi_* \approx \psi + \Omega_i^{-1}\vec{v}\times\vec{n}\cdot\nabla\psi - (Iv_{\|}/\Omega_i)$ with $B = B(\vec{R})$ and $\vec{n} = \vec{n}(\vec{R})$. These higher order corrections are required to insure that the gyrokinetic ψ_* is equal to the exact ψ_* order by order (unless the exact ψ_* is used as the radial gyrokinetic variable as in [10]).

3. Effects of the pedestal radial electric field

In a subsonic banana regime density pedestal of poloidal ion gyroradius width ρ_{pi}, the $\mathbf{E}\times\mathbf{B}$ and ion diamagnetic flows must cancel to lowest order. This behavior is seen in the helium discharges on DIII-D where the background ion temperature can be measured directly [20]. The lowest order cancellation keeps the ion flow subsonic and means that the ions are electrostatically confined with the radial electric field satisfying $(Ze/T_i)d\Phi/dr \approx -d\ln p_i/dr \sim 1/\rho_{pi} >> 1/a$ [10]. The associated subsonic $\mathbf{E}\times\mathbf{B}$ drift competes with poloidal component of parallel streaming when $B_p << B$ in an axisymmetric tokamak since it allows $c\mathbf{E}\times\mathbf{B}/B^2 \sim$

$(B_p/B)v_i$. This competition modifies the collisionless orbits [4-7] by introducing finite orbit effects as well as orbit squeezing, and makes it necessary to retain the distinction between surfaces of constant magnetic flux and drift surfaces on which the canonical angular momentum remains constant. This nonlocality is removed by assuming the inverse aspect ratio $\varepsilon << 1$; thereby allowing us to obtain analytic results since the trapped and barely passing ions remain localized with their departure from a flux surface of order $\varepsilon^{1/2}\rho_{pi} << \rho_{pi}$. The strong radial electric field alters the shape of the trapped - passing boundary and moves it on to the tail of the ion Maxwellian thereby exponentially reducing ion heat transport [5] and neoclassical polarization effects in the zonal flow residual [4,6,20]. Perhaps more importantly, the poloidal ion and impurity [21] flows,

$$V_i^{pol} = \frac{7cIB_p}{6e\langle B^2\rangle}\frac{\partial T_i}{\partial\psi}J_b(U^2) \quad \text{and} \quad V_z^{pol} = V_i^{pol} - \frac{cIB_p}{en_i\langle B^2\rangle}\left(\frac{\partial p_i}{\partial\psi} - \frac{n_i}{Z_z n_z}\frac{\partial p_z}{\partial\psi}\right), \tag{2}$$

are modified by the radial electric field because it affects momentum conservation during ion-ion collisions [5], with $u = cI\Phi'/\langle B^2\rangle^{1/2}$ and $U = u/v_i$, where $\Phi' = \partial\Phi/\partial\psi$. A subscript z denotes the impurity, and J_b is given by

$$J_b(U^2) = \frac{6}{7}\left[\frac{5}{2} + U^2 - \frac{\int_o^\infty dy e^{-y}(y+2U^2)^{3/2}(y\nu_\perp + 2U^2\nu_\parallel)}{\int_o^\infty dy e^{-y}(y+2U^2)^{1/2}(y\nu_\perp + 2U^2\nu_\parallel)}\right], \tag{3}$$

with $J_b(0) = 1$. Measurements of the poloidal Boron impurity flow in Alcator C-Mod indicate the importance of retaining this finite radial electric field effect in the pedestal [12] that acts to change the ion poloidal flow direction [22].

Finite orbit effects indirectly affect the electrons through their friction with the ions. This friction depends on the parallel ion flow and thereby on the poloidal ion flow that in turn is proportional to the radial ion temperature gradient. As a result, finite orbit effects modify the coefficient of the ion temperature gradient term in the bootstrap current. This modification is found to enhance the bootstrap current and the effect can easily be incorporated into existing expressions [7]. For example, using total pressure $p = n_e(T_i+T_e)$, the Z = 1 expression for the bootstrap current [21] becomes

$$J_\parallel^{bs} = -2.4\frac{\sqrt{\varepsilon}cIB}{\langle B^2\rangle}\left[\frac{\partial p}{\partial\psi} - 0.74n_e\frac{\partial T_e}{\partial\psi} - 1.17J_b(U^2)n_e\frac{\partial T_i}{\partial\psi}\right]. \tag{4}$$

The bootstrap current is enhanced over the conventional expression [23] for a finite radial electric field because J_b is monotonically decreasing (becoming negative when $U^2 > 1.4$).

The expression for the ion heat ion heat flux in the banana regime has also been evaluated [5], as have all plateau regime results [24]. In both regimes flows and currents are independent of orbit squeezing since they are insensitive to the localized portion of the ion distribution function. In the banana regime the ion heat diffusivity is reduced by the strong pedestal radial electric field, while only depending algebraically on orbit squeezing [5]. The plateau ion heat diffusivity increases with electric field strength before falling off exponentially and does not depend on orbit squeezing [24].

4. Intrinsic ambipolarity in a pedestal

The appearance of the electric field dependent J_b factor in the coefficient of ion temperature gradient terms and other electric field factors in the ion heat flux [5,24] and particle fluxes [24] leads to the question as to whether intrinsic ambipolarity in a axisymmetric, turbulence-free tokamak [8,9,25] is modified in the pedestal. For isothermal ions or in the weak radial electric field limit all our results reduce to the conventional ones [23,26]. In addition, the radial electric field has not introduced any new transport forces. The ion kinetic equation still has only an ion temperature gradient drive and the electron kinetic equation has the usual drives except for the J_b coefficient in the ion temperature gradient term that comes from the electron friction with the ions. Consequently, the altered neoclassical expression for banana regime radial particle transport for Z = 1 and large aspect ratio from equation (11.43) [23] becomes

$$\langle n_e \vec{V}_e \cdot \nabla\psi \rangle = -2.2 \frac{\sqrt{\varepsilon}\, I^2}{m_e \Omega_e^2 \tau_{ei}} \left[\frac{\partial p}{\partial \psi} - 1.4 n_e \frac{\partial T_e}{\partial \psi} - 1.17 J_b(U^2) n_e \frac{\partial T_i}{\partial \psi} \right], \tag{5}$$

where τ_{ei} the Braginski electron-ion collision time and we have neglected the pinch term for simplicity. In spite of the radial electric field entering the particle flux through J_b, conservation of momentum in like particle collisions insures a vanishing lowest order radial ion particle flux and intrinsic ambipolarity in the pedestal to next order since the toroidal component of the species momentum equations give [8,9]

$$\langle n_i \vec{V}_i \cdot \nabla\psi \rangle = -(cI/Ze)\langle B^{-1} \int d^3v M v_{\|} C_l^{ie} \rangle = (cI/Ze)\langle B^{-1} \int d^3v m v_{\|} C_l^{ei} \rangle = Z^{-1} \langle n_e \vec{V}_e \cdot \nabla\psi \rangle, \tag{6}$$

where the C_l^{jk} are the unlike linearized collision operators and we continue to neglect the pinch or induced electric field effects for simplicity. Then the only change in the neoclassical radial electron particle flux is through the J_b coefficient of the ion temperature gradient term. As a result, employing $\langle \mathbf{J} \cdot \nabla\psi \rangle = 0$ cannot determine the radial electric field - that is, the plasma remains intrinsically ambipolar even though the radial electric field dependent coefficient J_b appears in the radial electron particle flux.

To determine Φ' total toroidal angular momentum conservation must be employed, just as in the core, to evaluate the departure from radial Maxwell-Boltzmann ions, that is, the departure from $d\Phi/d\psi + (Zen_i)^{-1} dp_i/d\psi = 0$. The only difference is that in the pedestal typically $Zen_i d\Phi/d\psi \approx -T_i dn_i/d\psi >> - n_i dT_i/d\psi$. Indeed, the proof in [10] that the ion temperature T_i must vary slowly compared to the poloidal ion gyroradius scale of the pedestal relies on the observation that the T_i dependent puesdo-density $\eta \equiv n_i \exp(Ze\Phi/T_i)$ must also vary slowly. If the plasma is toroidally rotating sonically then the toroidal rotation frequency $\omega = -c[d\Phi/d\psi + (Zen_i)^{-1} dp_i/d\psi]$ must also vary slowly so there are still only rather weak departures from a generalized Maxwell-Boltzmann ion relation. This behavior is required to make the Vlasov operator vanish to lowest order since the ion distribution function can only depend on the constants of the motion $E = v^2/2 + Ze\Phi/M$ = total ion energy and $\psi_* \equiv \psi - (Mc/e)R^2 \vec{v} \cdot \nabla\zeta = \psi + \Omega_i^{-1} \vec{v} \times \vec{n} \cdot \nabla\psi - (Iv_{\|}/\Omega_i)$ = canonical angular momentum. And, in addition, it must also be Maxwellian, $f_{Mi} = \eta (M/2\pi T_i)^{3/2} \exp(-ME/T_i)$, to make the entropy production integrated over the pedestal vanish, where to lowest order this requires $T_i(\psi_*) \approx T_i(\psi)$ and $\eta(\psi_*) \approx \eta(\psi)$, with f_{Mi} independent of poloidal angle in the banana regime. The fact that the ion temperature and psuedo-density must vary slowly means that the pedestal differs surprisingly little from the core, with the key difference being that for the typical case of a subsonic pedestal the ions are electrostatically confined to lowest order so that the **E**×**B** drift can alter results. We also remark that the slower variation of the ion temperature and psuedo-density mean that the electrons must be playing the key role in setting the strong variation of the plasma density (and thereby the potential) and electron temperature.

In a turbulent tokamak pedestal the ambipolarity argument is only slightly more involved than in the core. We have demonstrated that the neoclassical contributions to the radial particle fluxes remain intrinsically ambipolar. Consequently, the turbulent portions must as well, as long as direct orbit and ripple losses remain small and neutrals do not play a significant role. This observation might seem physically reasonable since the turbulent fluxes are expected to be associated with **E**×**B** drifts. A more detailed and convincing proof follows by realizing that the toroidal angular momentum conservation equation (1) remains valid. As a result, as long as quasineutrality continues to hold in the pedestal then $\langle \mathbf{J}\cdot\nabla\psi\rangle = 0$. Therefore, for steady state turbulence only $\langle R^2\nabla\zeta\cdot\vec{\pi}_i\cdot\nabla\psi\rangle$ can change, but its vanishing must continue to determine the axisymmetric radial electric field! We remark, however, that the form of $\langle R^2\nabla\zeta\cdot\vec{\pi}_i\cdot\nabla\psi\rangle$ will differ in the pedestal since the finite **E**×**B** drifts can alter particle trajectories. This change could increase the size of the off diagonal stress terms to a Bohm diffusivity. The resulting a/ρ_i increase in size would mean that momentum flows, as in the sonic case, are no longer in flux surfaces to lowest order [1]. Consequently, in the pedestal larger error currents and quasineutrality errors may be allowed, but not as large as $J_{error}/en_iv_i \sim (\rho_i/a)^3 \sim (Zn_i - n_e)_{error}/n_i$.

5. Pedestal electric field effects on the zonal flow residual

The altered neoclassical polarization of the plasma due to the strong radial electric field of the pedestal modifies the Rosenbluth and Hinton zonal flow residual [13]. The derivation of this modification is given in detail in references [4,6] so here we simply present the final result. The residual is defined as the ratio of the final ($t \to \infty$) over the initial ($t = 0$) potential perturbation $\tilde{\Phi}$ applied without altering the density

$$\frac{\tilde{\Phi}(t\to\infty)}{\tilde{\Phi}(t=0)} = \frac{1}{1+\Re}\ , \tag{7}$$

with

$$\Re = \Re_{RH}\left[\frac{\Gamma(U^2)}{\sqrt{S}} + i\frac{\Lambda(U,S)}{\langle k_r\rho_{pi}\rangle}\right] \tag{8}$$

where $\Re_{RH} = 1.6q^2/\sqrt{\varepsilon}$ is the Rosenbluth and Hinton result,

$$\Gamma(U^2) = (4/3\sqrt{\pi})\exp(-U^2)\int_0^\infty dy(y+2U^2)^{3/2}\exp(-y) \tag{9}$$

and

$$\Lambda(U,S) = 2S^{-1/2}U\left[S\Gamma(U^2) + 4\pi^{-1/2}(1-S)\exp(-U^2)\int_0^\infty dy(y+2U^2)^{1/2}\exp(-y)\right]\ , \tag{10}$$

where $S = 1 + (cI^2\Phi''/\langle\Omega_i B\rangle)$. The important differences between the pedestal and the core are as follows. At large U the trapped region shifts to the tail of the distribution function causing an exponential decay of Γ and Λ, thereby reducing $\Re$. As a result, the zonal flow residual can approach unity and more strongly regulate pedestal turbulence. The imaginary term Λ represents a spatial phase shift in $\bar{\Phi}$ introduced by u. Orbit squeezing effects do not enter Γ and only enter Λ algebraically.

The form of (10) suggests that once the U goes beyond a critical value a further increase in the global pedestal radial electric field enhances the zonal flow residual and thereby is expected to reduce anomalous transport. This feedback mechanism may play a role in pedestal formation and the low to high mode transition as the pedestal density profile steepens.

6. Discussion

We have explained why great care is required when evaluating the global axisymmetric radial electric field in a tokamak by considering the size of the allowed error terms in the conservation of total angular momentum equation. These errors can be due to numerical noise or algorithm shortcomings, or approximations associated with the asymptotic gyrokinetic treatment (in particular, not retaining magnetic geometry effects to high enough order in a turbulent plasma). We stress that a $(B_p/B)(a/\rho_i)^2$ less accurate ion distribution function is required if a hybrid fluid - gyrokinetic treatment is employed [1,2,25].

We also discuss the effects of a strong pedestal radial electric field in a weakly collisional plasma and generalize the notion of intrinsic ambipolarity to demonstrate that it remains valid. We note that in the banana regime this strong pedestal radial electric field acts to reverse the poloidal flow [5] and increase the bootstrap current [7], and enhance the residual zonal flow regulation of turbulence [4,6]. The influence of the pedestal electric field is stronger in the banana regime than in the plateau regime [24]. The effect of the electric field on the poloidal impurity flow is clearly observed in the Alcator C-Mod pedestal [22] and is agreement with our predictions [5,24].

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