PSFC/JA-04-13

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January 2004

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This work was supported by the U.S. Department of Energy, Grant No. DE-FC02-01ER-54648.

*To be published in Computer Physics Communications (Elsevier, Amsterdam) (2004)*

# Ultrahigh Resolution Simulations of Mode Converted Ion Cyclotron Waves and Lower Hybrid Waves

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**Abstract**

Full Wave studies of mode conversion (MC) processes in toroidal plasmas have required prohibitive amount of computer resources in the past because of the disparate spatial scales involved. The `TORIC` code [1] solves the linear sixth order reduced wave equation for the ion cyclotron range of frequencies (ICRF), in toroidal geometry using a Fourier representation for the poloidal dimension and finite elements in the flux dimension. The range of problems that `TORIC` can do has been extended through both new serial algorithms and parallelization of memory and processing. The implementation of out-of-core memory management, FFT convolutions, and improved memory management brought MC studies just into range of the serial version of the code running on a NERSC Cray SV1. Some simple tests and arguments show that more resolution than is possible on a single processor system is needed to fully resolve these scenarios. By distributing the large linear system across many processors in conjunction with the out-of-core technique, the resolution limitations are effectively removed. `ScaLAPACK` is used to do the linear algebra operations and message passing interface (MPI) is used to distribute the significant amount of post-processing. The new parallel version of the code can easily do the most difficult MC problems on present day tokamaks (Alcator C-Mod and Asdex-Upgrade), with only 32 pc from a local Beowulf cluster. Using 48 or more processors admits us to problems in the lower hybrid range of frequencies.

*Key words:* TORIC ICRF FastWave IBW Bernstein LowerHybrid Simulation FullWave
*PACS:* 52.25.Mq 52.35.H 52.50.Sw 52.65.-y

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## 1 Introduction

We set out to solve a linear system. Electromagnetic waves are launched into a confined toroidal plasma by means of a plasma facing antenna mounted inside the vacuum vessel. The field and density perturbations produced by the plasma are small compared to background quantities and take place on a time scale that is much faster than diffusive or transport times with frequencies that are typically in the GHz range. (Although quasilinearly[2,3], the waves can create macroscopic changes in plasma quantities, and while of great interest in itself, this does not violate the assumption of linearity.) Several effects conspire to make a simple linear boundary value problem, much more difficult.

While Maxwell's equations form a straight-forward hyperbolic system. After Fourier transforming it in time, and introducing a single wave frequency, $\omega$, they become elliptic (Eq. (1a)). The inclusion of the plasma response in the form of the perturbed current transforms the problem into an integro-differential equation. One approximation is to reduce the perpendicular range of wave–particle interactions. That is, the dielectric response is only accurately retained for wavelengths larger than the ion gyroradius. When the wavelength is smaller than the ion gyroradius, the anti-Hermitian part of the dielectric that is responsible for damping the wave is modified to model the correct level of damping at those small scales, and this leads to the finite Larmor radius (FLR) approximation used in this paper. Recently developed codes[4], made possible with massively parallel processor (MPP) architectures, avoid this sacrifice, but at the cost of needing thousands of CPU hours for relatively modest resolutions. In the direction parallel to the equilibrium magnetic field, the waves interact with ions and electrons streaming along field lines, and this creates a strong coupling along this dimension. By using a Fourier basis in this direction, we are able to use the developments of homogeneous wave theory[5,6] in formulating the plasma response, and retain a degree of physical intuition. Finally, when exploring quasilinear effects and feedbacks on the equilibrium, it is necessary to evaluate the plasma dielectric response for non-Maxwellian particle distributions. The evaluation of the susceptibility expressions in reference [7] must now be done numerically. If in addition, we wish to "close the loop" with Fokker-Planck calculations of these distributions in the presence of the wave fields, then MPP calculations are necessary to evolve the weakly coupled system in a reasonable amount of time.

## 2 The `TORIC` Code and Physics Model

In this paper we discuss the FLR approach and the MPP modifications that have been made to increase the speed and resolution of the `TORIC` finite Larmor radius (FLR) full wave code. "Full wave", means that it solves Maxwell's equations in the presence of a plasma and wave antenna and so includes the effects of dispersion and mode-conversion, while FLR refers to the approximation that the Larmor radius is smaller than the perpendicular wave length and results in a finite order system, instead of an integral one. The system is solved for a fixed frequency with a linear plasma response [Eq. (1a)] in a mixed spectral-finite element basis [Eq. (1b)], where $\mathbf{J}^A$ and $\mathbf{J}^P$ are the antenna and plasma currents, $\sigma$ is the plasma conductivity, $m$ is the poloidal mode number, and $n_\phi$ is the toroidal mode number. The mixed Fourier-finite element basis introduces an algebraic parallel wave-number, $k_\parallel$, that depends on the local metric coordinates, $N_\tau$ and $R$, and $\Theta = \mathrm{atan} B_\tau / B_\phi$. $N_\tau$ reduces to the minor radius, $r$, in the limit of an orthogonal toroidal system and $\Theta$ is the angle between the poloidal and toroidal fields. The parallel wavenumber provides a connection to homogeneous theory that is useful in comparisons to the dispersion relations from plane-stratified theory and is exploited to modify the anti-Hermitian part of $\sigma$ in cases where the FLR approximation breaks down.

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2} \left\{ \mathbf{E} + \frac{4\pi i}{\omega} \left( \mathbf{J}^P + \mathbf{J}^A \right) \right\} \tag{1a}$$

$$\mathbf{E}(\mathbf{x}) = \sum_m \mathbf{E}_m(N_\tau) \exp\left(im\theta + in\phi\right) \tag{1b}$$

$$k_\parallel = \frac{m}{N_\tau} \sin\Theta + \frac{n_\phi}{R} \cos\Theta \tag{1c}$$

$$\mathbf{J}^P_m(N_\tau) = \sum_m \sigma\left(k_\parallel^m, N_\tau\right) \cdot \mathbf{E}_m(N_\tau) \tag{1d}$$

In `TORIC` the FLR approximation retains the second harmonic wave frequency and the second order ion gyro-radius effects, $(\rho_\mathrm{i} = v_\mathrm{ti}/\Omega_\mathrm{ci})$, for plasma interactions with the wave. This contributes terms from the dielectric that are of the same order or less as those that come from Maxwell's equations. It also retains the physics of the three ICRF waves: ion cyclotron waves (ICW)[8], ion Bernstein waves (IBW), and fast waves (FW). Near the mode conversion region, the FLR approximation breaks down and $k_\perp \rho_\mathrm{i} \sim 1$, where $k_\perp$ is the perpendicular wave number. In these regions, damping from a WKB approximation is used to modify the anti-Hermitian part of the conductivity operator in `TORIC` to capture the proper damping, while the real part describing propagation remains unchanged[1]. This approximation holds if the particles experience phase decorrelation within a few gyro-periods of their wave interactions.

## 3 Parallelizing `TORIC`

The computational problem in `TORIC` is one of matrix inversion. The discretization of the system of equations in Eqs. (1) leads to a block tri-diagonal system with

$3 \times N_r$ blocks. A weak variational formulation of the equations is used with cubic Hermite polynomials in the radial dimension, which for the FLR system, creates the sparse tri-diagonal structure. The Fourier decomposition decouples the toroidal dimension for axisymmetric devices so that it only enters parametrically as the toroidal mode number. Multiple simulations at different toroidal modes can be used to build up a complete three dimensional spectrum. The poloidal modes are coupled by the free space operators and the plasma response and create dense blocks which contain $(2 \times 3 \times N_m)^2$ elements (a factor of two for the real and imaginary parts of the three vector components.) Thus, the computational resources to solve for the fields quickly exceed the available memory of a single processor. For example, given an available memory of 2 Gigabytes of RAM, we would be limited to a maximum of approximately 150 radial elements by 128 poloidal modes, which is insufficient for the problems discussed above.

To take advantage of scalable architectures, the code has been parallelized in the power reconstruction and the matrix inversion by using the ScaLAPACK [9] library of parallelized linear algebra routines as well as direct use of the message passing interface (MPI). In this way, the limiting memory requirements of the large blocks of the block diagonal system are distributed across multiple processors, and so the problem size is limited in principal only by the available number of processors. The power and current deposition calculations are independent of flux surface, so by distributing those calculations along that dimension, those calculations are reduced from 50% to less than 2% of a typical calculation time.

## 4 Code Conversion and Mode Conversion

Using the spectral representation in Eq. (1b) and from the condition that $k_\perp \rho_\mathrm{i} \simeq 1$ we estimate the maximum poloidal mode number needed for mode conversion studies. Taking $k_\perp \sim \frac{m}{r}$, the required resolution is approximately $M_\mathrm{max} \equiv r/\rho_\mathrm{i} \equiv \rho^*$. In Fig.1, two plots of the power spectrum of the Fourier transform of the right circularly polarized component of the electric field are shown for a specified set of flux surfaces. The two plots demonstrate that in the bulk of the plasma and especially at mode-conversion layers ($\rho = 0.5$ for the cases shown) and at the outermost surfaces, several hundred poloidal modes are needed for convergence. Even at the outermost flux surfaces (around $r/a = 0.9$) the amplitude is down to  1%. Simulations at higher numbers of poloidal modes show that the spectrum begins to fall off much more quickly just past this resolution and the solution doesn't change appreciably. The parts of the spectrum shown at $|M| > 64$ in the left panel and at $|M| > 256$ in the right are the part of the spectrum above the Nyquist frequency. The poloidal resolution is twice the spectral resolution to avoid aliasing.

We may also graphically see the non-physical effects of insufficient resolution in the left panel of Fig. 2. At this low resolution the loss of horizontal localization is

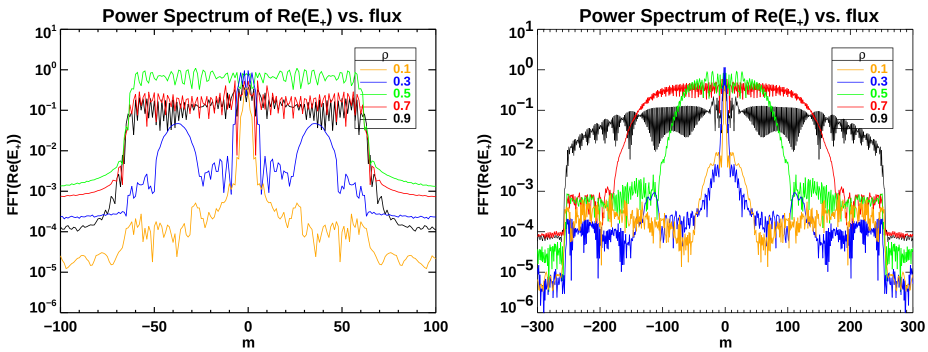


Fig. 1. The left panel is a D($^3$He) mode-conversion case in Alcator C-Mod with only 127 poloidal modes. The power-spectrum does not fall off at the largest modes and indicates insufficient resolution. Increasing to 511 modes in the right panel yields a well-converged spectrum, even at the largest radii.

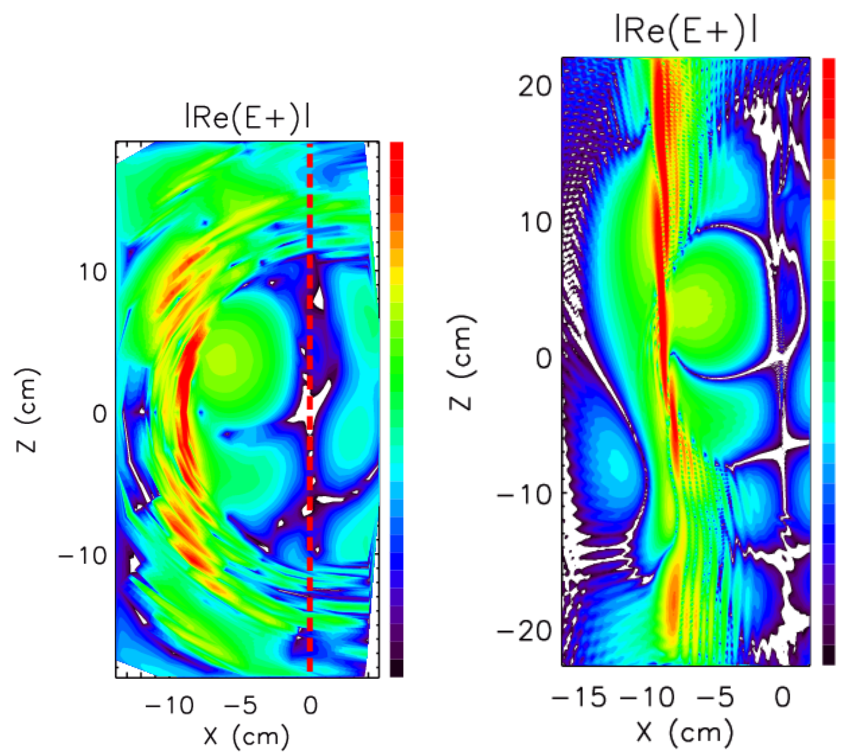


Fig. 2. The left panel shows a blow-up of the mode conversion region for the same scenario as in Fig. 1 with only 15 poloidal modes used to resolve the layer. The dashed red line indicates the (3He) cyclotron resonance. With 255 modes on the right, the vertical layer is well localized and the multiple scales of waves are observed.

exaggerated and electric field bleeds over along the flux surface into ion cyclotron resonances ($^3$He indicated by dashed red line), causing “spurious” ion power absorption at that location. In contrast, when the resolution is increased to the point where the spectrum was shown to converge, the mode conversion layer becomes well localized vertically and the spurious ion damping vanishes.

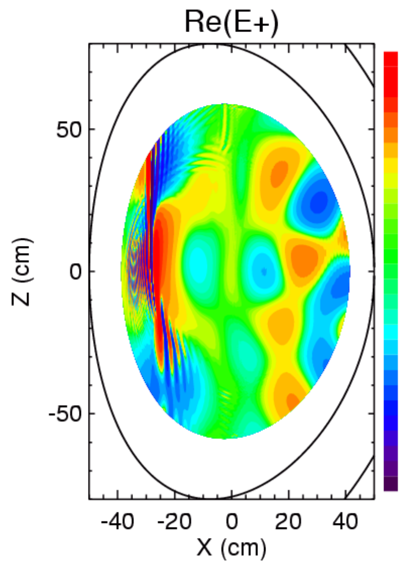


Fig. 3. The simulation of a D($^3$He) mode conversion scenario in Asdex-Upgrade shown here requires twice the resolution of the more central case in C-Mod.

New physics regimes are available at these higher resolutions. In the typical Asdex-Upgrade MC discharge in Fig. 3, the layer is far from the center of the device and requires much more poloidal resolution than the C-Mod case. While the C-Mod simulation converged acceptably at $N_m = 255$, Asdex-Upgrade requires at least $N_m = 511$. The complete poloidal cross-section of Fig. 3 also shows the three waves (FW, ICW, and IBW) all present simultaneously. The large structures on the right side are the FW propagating in from the antenna. The midrange waves off axis at about -20 cm are the ICW traveling backward to the right and the smallest wavelength mode in the midplane to the left of -20cm is the IBW. These cases were not possible with the serial version of the code and represent the extension of the `TORIC` code to a new class of problems.

We may progress to even higher resolution and run simulations in the lower hybrid range of frequencies (LHRF). The case in Fig. 4 has the proper parallel wavelength and frequency for lower hybrid waves, but couples only to the fast wave due to the antenna current strap being oriented perpendicular to **B**. Presently, a new antenna model is being developed in the code to couple properly to the lower hybrid slow wave polarization.

## 5 Conclusions

A new parallel algorithm for an FLR fast wave code has been developed and implemented. Well converged simulations of several mode-conversion scenarios in several devices have been demonstrated. Given the scaling of resolution requirements with $\rho^*$, simulations of burning plasma experiments will require a parallel

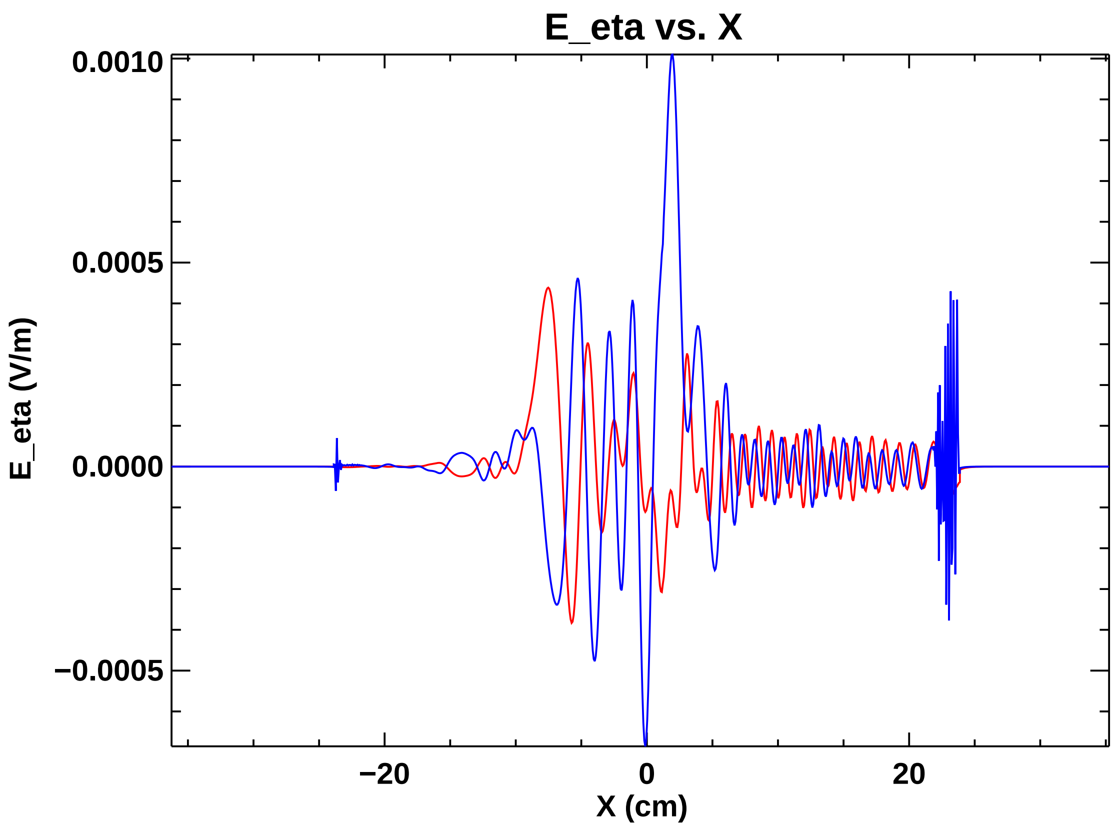


Fig. 4. The electric field from a LHRF simulation using TORIC. The coupled wave corresponds to the fast wave polarization.

code. New physics regimes such as lower-hybrid full wave simulations are now within reach and many interesting questions such as the role of wave focusing and diffraction in LH spectral broadening may yield new insights from the full wave model. The new speeds afforded by parallel algorithms will make coupling with generalized dielectric routines and Fokker-Planck calculations of perturbed distributions possible leading to “closed loop” RF calculations, and realistic modeling of wave propagation in plasmas with large fusion alpha populations.

## 6 Acknowledgments

Research sponsored by the Mathematical, Information, and Computational Sciences Division; Office of Advanced Scientific Computing Research; U.S. Department of Energy, under Contract No. DE-AC05-00OR22725 with UT-Battelle, LLC; and the DoE Wave-Particle Scidac Contract No. DE-FC02-01ER54648.

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