PFC/RR-93-4

Self-Similar Variables and the Problem
of Nonlocal Electron Heat Conductivity

S. I. Krasheninnikov, O. G. Bakunin

MIT Plasma Fusion Center
Cambridge, Massachusetts 02139 USA

October 1993

Self-similar variables and the problem of nonlocal electron heat conductivity

S. I. Krasheninnikov[1], O.G. Bakunin[2]

MIT Plasma Fusion Center, Cambridge, USA

Self-similar solutions of the collisional electron kinetic equation are obtained for the plasmas with one (1D) and three (3D) dimensional plasma parameter inhomogeneities and arbitrary Z_{eff}. For the plasma parameter profiles characterized by the ratio of the mean free path of thermal electrons with respect to electron-electron collisions, λ_T, to the scale length of electron temperature variation, L, one obtains a criterion for determining the effect that tail particles with motion of the non-diffusive type have on the electron heat conductivity. For these conditions it is shown that the use of a "symmetrized" kinetic equation for the investigation of the strong nonlocal effect of suprathermal electrons on the electron heat conductivity is only possible at sufficiently high Z_{eff} ($Z_{eff} \geq (L/\lambda_T)^{1/2}$). In the case of 3D inhomogeneous plasma (spherical symmetry), the effect of the tail electrons on the heat transport is less pronounced since they are spread across the radius r.

[1]Permanent address: Kurchatov Institute of Atomic Energy, Moscow, Russia.

[2]Kurchatov Institute of Atomic Energy, Moscow, Russia.

I. INTRODUCTION

It is well known that the Spitzer-Harm theory[1] for the electron heat conductivity along a magnetic field is only valid for comparatively small ratios of the thermal electron mean free path with respect to electron-electron collisions λ_T to the characteristic length of the plasma parameter variation, L: $\gamma = \lambda_T / L \leq 10^{-2}$.[2-6] This is due to the fact that the main role in the plasma heat conductivity is played by suprathermal particles. The mean free path of these suprathermals, $\lambda_* \simeq \lambda_T (\varepsilon_* / T_e)^2$ (ε_* is the suprathermal particle energy; T_e is the electron temperature; $\varepsilon_* \simeq (4 \div 9) T_e$) is much greater than λ_T.

At the same time, values $\gamma \geq 10^{-2}$ are rather typical of plasmas in the tokamaks scrape-off layer (SOL),[7,8] space plasmas,[9] plasmas produced on interaction of high energy fluxes with matter,[10] etc. So it seems rather attractive to generalize Spitzer-Harm's theory for relatively high values of γ: $\gamma \geq 10^{-2}$. Such attempts have been made by many authors. Various approximation methods for solving kinetic equations were used. For example, in Ref. 11 the terms proportional to γ^2 were taken into account in the expressions for the heat flux q; various modifications of the method of momenta were used in Refs. 12-16.

Let us consider the studies of Refs. 17,18 in detail, the results of which can be represented in rather compact form. These are often used for various applications.

In Ref. 17 the integral expression previously obtained for the heat flux $q(x)$ from a heuristic consideration[19] was

theoretically verified and compared with the results of numerical calculations:

$$q(x) = \frac{1}{2} \int dx' \frac{q_{SH}(x')}{\lambda_T(x')} \exp\left(-\left| \int\limits_x^{x'} dl / \lambda_T(l) \right| \right), \qquad (1)$$

where $q_{SH}(x)$ is Spitzer-Harm's heat flux.

In Ref. 18 a procedure for approximately solving the electron kinetic equation is proposed in the high effective plasma charge limit Z_{eff}, where the electron velocity distribution function $f_e(\vec{v},x)$ can be represented as the sum of the symmetric function $f_0(v,x)$ and the small asymmetric part $\mu f_1(v,x)$. In this case deviation of $f_0(v,x)$ from the Maxwellian function in the suprathermal region (responsible for the heat transfer) is causes deviation from Spitzer-Harm's theory. The main idea of the paper Ref. 18 is to convert the differential equation for $f_0(v,x)$,

$$\frac{\partial^2 f_0}{\partial \zeta^2} + \frac{1}{w^3} \frac{\partial}{\partial w} \left(f_0 + T_e(\zeta) \frac{\partial f_0}{\partial w} \right) = 0 , \qquad (2)$$

into an integral equation which is then solved by the iteration technique, using the Maxwellian distribution function as a zero approximation (here w is the electron energy; $\zeta \propto \int n_e(x)dx$; $n_e(x)$ is the electron density). This conversion of Eq.(2) into an integral equation was realized by means of the Green's function $G(\zeta,\zeta',w,w')$ for the operator $\partial^2 G/\partial \zeta^2 + w^{-3}\partial G/\partial w$:

$$f_0(w,\zeta) = - \int G(\zeta,\zeta',w,w') \frac{T_e(\zeta')}{w'^3} \frac{\partial^2 f_0}{\partial w'^2} dw' d\zeta' . \qquad (3)$$

The approach developed in Ref. 18 is generalized for medium

Z_{eff} in Ref. *20*. However, this technique (which seems natural at first sight) for approximately solving the kinetic equation *(2)* actually gives an incorrect result. Indeed, by direct calculation it can readily be shown that the distribution function obtained in Ref. *8* begins to differ from the the Maxwellian one at particle energies of $w^5 \geq \beta/\gamma^2$. Meanwhile, from the solution of Eq.*(2)* by means of perturbation theory it follows that this difference actually occurs already at $w^6 \geq \beta/\gamma^2$, and at $w^5 \geq \beta/\gamma^2$ the difference between $f_0(v,x)$ and the Maxwellian function becomes sufficiently large (see value below). This is explained by the fact that the Coulomb term in the kinetic equation for the distribution functions close to the Maxwellian one is in fact the small difference between two large operators: deceleration and heating of electrons *(the second and third terms in Eq.(2))*. Incorrect separation of these operators results in an artificial overestimation of the role of Coulomb electron-electron collisions and in a reduction of the deviation of $f_0(v,x)$ from the Maxwellian function.

Unfortunately, criteria of the applicability of the approximate solutions of the electron kinetic equation at high γ values are not available and it can be tested by means of numerical simulations in only a rather limited range of parameters because numerical solution of the kinetic equation is cumbersome. Moreover, all solutions of the electron kinetic equation mentioned above were found for the case of one dimentional (1D) plasma parameter inhomogeneity. It is still not clear what role nonlocal

effects play in the electron heat conductivity for the cases of two (2D) and three (3D) dimentional plasma parameter inhomogeneity.

In Ref. *21* a class of solutions of the collisional kinetic equation which allow representation in self-similar variables was found. The transition to self-similar variables makes it comparatively easy to find exact solutions of the kinetic equation. Analytical and numerical studies of these solutions[21,22] showed, in particular, that Eqs.*(1),(3)* do not describe the effect of a strongly anisotropic electron distribution function tail on the heat flux q, which under certain conditions can turn out to be decisive.

In the present paper solutions of the electron kinetic equation are found for the case of 1D and 3D (spherical symmetry) plasma parameter inhomogeneity for arbitrary plasma Z_{eff} by means of self-similar variables. Criteria of the strong effect of tail electrons on the electron heat transport are obtained.

Sections **II-IV** are devoted to investigating 1D inhomogeneous plasma. In Sec. **II** the main equations analyzed in the paper are given and the limits of their applicability are specified. In Sec. **III** solutions of the kinetic equation for moderate values of Z_{eff}, $\beta \equiv (1+Z_{eff})/2 < \gamma^{-1/2}$, are obtained. In Sec. **IV** the high Z_{eff} ($\beta > \gamma^{-1/2}$) limit is investigated. In Sec. **V** the case of 3D inhomogeneous plasma is considered. The results are discussed in Sec. **VI** and the main conclusions are summarized in Sec. **VII**.

II. SELF-SIMILAR VARIABLES FOR THE 1D KINETIC EQUATION

Let us assume ions to be at rest and consider the stationary kinetic equation for an electron distribution function that is inhomogeneous along the x-axis:

$$V\mu \frac{\partial f_e}{\partial x} - \frac{eE}{m} \left(\mu \frac{\partial f_e}{\partial V} + \frac{1-\mu^2}{V} \frac{\partial f_e}{\partial \mu} \right)$$

$$= \frac{2\pi e^4 \Lambda}{m^2} \left(St(\vec{V}, f_e) + \frac{Z_{eff} n_e(x)}{V^3} \frac{\partial}{\partial \mu} (1-\mu^2) \frac{\partial f_e}{\partial \mu} \right) ,$$

(4)

where

$$St(\vec{V}, f_e) = \frac{\partial}{\partial V_\alpha} \int \left(f_e(\vec{V}) \frac{\partial f_e(\vec{V}')}{\partial V'_\beta} - f_e(\vec{V}') \frac{\partial f_e(\vec{V})}{\partial V_\beta} \right) U_{\alpha\beta} \, d\vec{V}' ,$$

$$U_{\alpha\beta} = (u^2 \delta_{\alpha\beta} - u_\alpha u_\beta)/u^3 , \quad u_\alpha = V_\alpha - V'_\alpha ;$$

ϑ is the angle between the particle velocity vector $\vec{V}$ and the axis x; $\mu = \cos\vartheta$; $f_e(V,\mu,x)$ is the electron distribution function; e, m, $n_e(x)$ are its charge, mass and density; Z_{eff} is the effective ion charge; Λ is the Coulomb logarithm; $St(\vec{V}, f_e)$ is the collisional Coulomb operator; $E(x)$ is the ambipolar electric field.

As shown in Ref. 21 Eq.(4) allows solutions in the self-similar variables

$$f_e(\vec{V}, x) = N \, F(\vec{v})/[T_e(x)]^\alpha, \quad \vec{v} = \vec{V}(m/2T_e(x))^{1/2},$$

(5)

where N is the normalization factor; $\int F(\vec{v}) d\vec{v} = 1$; α is an adjustable parameter; the function $T_e(x)$ plays the role of a characteristic average electron energy. In this case the kinetic

equation *(4)* is converted in the following way:

$$\gamma v\mu \left(\alpha F + \frac{v}{2} \frac{\partial F}{\partial v} \right) - \frac{\gamma_E}{2} \left(\mu \frac{\partial F}{\partial v} + \frac{1-\mu^2}{v} \frac{\partial F}{\partial \mu} \right)$$

(6)

$$= \frac{1}{4} \left(St(\vec{v}, F) + \frac{Z_{eff}}{v^3} \frac{\partial}{\partial \mu} (1-\mu^2)\frac{\partial F}{\partial \mu} \right) ,$$

where the parameter $\gamma_E = eET_e/(2\pi e^4 \Lambda n_e)$ is found from the particle flux ambipolarity condition, which reduces to the relation $j_e \propto T_e(x)^{(2-\alpha)}J = const$, where $J = \int F(\vec{v})v\mu d\vec{v}$. This condition is automatically satisfied at $\alpha = 2$, and, as a result, the value of γ_E can be arbitrary in this case, while at $\alpha \neq 2$ the quantity γ_E is found from the relation $J = 0$. However, it will always be assumed that $j_e = 0$.

Here it is useful to note that Eq.*(6)* is similar in structure to the equation representing the electron runaway effect in a electric field, which allows one to use the approaches developed in Ref. *23-26* for it solution.

Solutions of type *(5)* correspond to a constant ratio of the mean free path of electrons with an energy of about $T_e(x)$ to the characteristic scale length of $T_e(x)$:

$$\gamma = - T_e^2(dlnT_e/dx)/(2\pi e^4 \Lambda n_e) = const. > 0 .$$

(7)

Taking account of the distribution function form *(5)*, relation *(7)* results in the following temperature and density profiles:

$$T_e^{(\alpha-1/2)}(dT_e/dx) = const, \qquad n_e \propto T_e^{(3/2-\alpha)}.$$

(8)

Considering the expression for the energy flux density, corresponding to Eq.(5),

$$q(x) = QT_e(x)^{(3-\alpha)}N(2/m)^2, \qquad Q = \int F(\vec{v})v^3\mu d\vec{v}, \qquad (9)$$

it is easy to see that the dependence on x in Eq.(9) vanishes at $\alpha = 3$. Note that the case $\alpha = 3$ corresponds to energy flux conservation for the classical[1] dependence of the electron heat conduction coefficient κ_e on temperature $\kappa_e \propto T_e^{5/2}$ as well (see Eq.(8)). In other cases one has $dq/dx \neq 0$, i.e. the presence of either temporal terms or energy sources or sinks not taken into account in Eqs.(4),(6). However, when $\gamma \ll 1$, the effect of $T_e(x)$ inhomogeneity on the distribution function in the range of thermal velocities $v \simeq (2T_e/m)^{1/2}$ is small and the function f_e in this range is close to the local Maxwellian one. Therefore for the case $\alpha \neq 3$ and $\gamma \ll 1$ it can therefore be assumed that Eq.(6) describes the effect of suprathermal particles on the heat flux $q(x)$ in the presence of the sinks or sources in the thermal velocity range.

In the limit of interest, $v \gg 1$, Eq.(6) transforms to

$$\gamma\mu\left(\alpha F + \xi\frac{\partial F}{\partial \xi}\right) - \gamma_E\left(\mu\frac{\partial F}{\partial \xi} + \frac{1-\mu^2}{2\xi}\frac{\partial F}{\partial \mu}\right)$$

$$\qquad\qquad (10)$$

$$= \frac{1}{\xi}\frac{\partial}{\partial \xi}\left(F + \frac{\partial F}{\partial \xi}\right) + \frac{\beta}{2\xi^2}\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial F}{\partial \mu},$$

where $\xi = v^2$; $\beta = (1+Z_{eff})/2$.

Further simplification of the kinetic equation (10) is related to fast symmetrization of the electron distribution function at high β, where $F(\vec{v})$ can be represented in the form of the sum of the symmetric part $F_0(v)$ and the small asymmetric part

$\mu F_1(v)$. In this approximation the equations for $F_0(v)$ and $F_1(v)$ have the forms:

$$\gamma \xi^{(1-\alpha)} \frac{d}{d\xi}(\xi^{\alpha} F_1) - \frac{\gamma_E}{\xi} \frac{d}{d\xi}(\xi F_1) = \frac{3}{\xi} \frac{d}{d\xi}\left(F_0 + \frac{dF_0}{d\xi}\right) ,$$

$$(11)$$

$$F_1 = -\gamma \frac{\xi^2}{\beta} \xi^{(1-\alpha)} \frac{d}{d\xi}(\xi^{\alpha} F_0) + \gamma_E \frac{\xi^2}{\beta} \frac{dF_0}{d\xi} .$$

Substituting the expression for $F_1(v)$ in the equation for $F_0(v)$, one obtains

$$\frac{\gamma^2}{\beta}\Bigg(\xi^{(1-\alpha)}\frac{d}{d\xi}\bigg(\xi^{\alpha}\Big[-\xi^{(3-\alpha)}\frac{d}{d\xi}(\xi^{\alpha} F_0) + \delta\xi^2\frac{d}{d\xi}(F_0)\Big]\bigg)$$

$$-\frac{\delta}{\xi}\frac{d}{d\xi}\bigg(\xi\Big[-\xi^{(3-\alpha)}\frac{d}{d\xi}(\xi^{\alpha} F_0) + \delta\xi^2\frac{d}{d\xi}(F_0)\Big]\bigg)\Bigg) \qquad (12)$$

$$= \frac{3}{\xi}\frac{d}{d\xi}\left(F_0 + \frac{dF_0}{d\xi}\right) ,$$

where $\delta = \gamma_E/\gamma$. Essentially Eqs.(11),(12) are a self-similar analog of the equation used in Ref. 18.

Analyzing Eq.(12), one can see that distortion of the Maxwellian distribution function starts at the energies $\xi^6 \geq \beta/\gamma^2$. The local "temperature" of the electrons with the energies $\xi^5 \simeq \beta/\gamma^2$ exceeds the temperature of the bulk electrons by a factor of two. On the other hand, in order for the condition $F_0 \gg F_1$ to be true, it is necessary, as seen from Eq.(11), that the electron energy be rather small, $\xi^3 \leq \beta/\gamma$. Hence, it follows that the applicability of Eqs.(11),(12) and of Eq.(2) derived in Ref. 18 is restricted by the relation $\beta \geq \gamma^{-1/2}$.

At energies $\xi^4 \geq \beta/\gamma^2$, i.e. where electrons have time to

diffuse at a distance of about γ^{-1} (measured in the mean free path) distortion of the Maxwellian function becomes very strong and $f_e(\vec{V},x)$ is determined by the integral $T_e(x)$ profile. Strong anisotropy in $f_e(\vec{V},x)$ and violation of the applicability conditions (11),(12) can be expected at $\xi^2 \geq \beta/\gamma$, where the mean free path becomes greater than the characteristic scale of $T_e(x)$.

In the opposite case $(\beta \leq \gamma^{-1/2})$, distortion of the Maxwellian distribution function is accompanied by strong anisotropization and one should consider the complete equation (10).

III. SOLUTION OF THE SELF-SIMILAR 1D EQUATION $(\beta < \gamma^{-1/2})$

Let us find solutions of Eq.(10) in various characteristic ranges of the dimensionless energy ξ with their further matching.

A. Low energy range, $\xi \leq (\beta/\gamma)^{1/3}$

In this energy range the distribution function $F(\vec{v})$ is quite close to Maxwellian and can be found by expansion in Legendre polynomials $P_i(\mu)$:

$$F(\vec{v}) = \sum_{i=0}^{\infty} F_i(\xi)P_i(\mu) \ . \tag{13}$$

Leaving the mean expansion terms only and substituting $F_i(\xi)$ in the form

$$F_i(\xi) = \pi^{-3/2} \ e^{-\xi} \sum_{k=0}^{\infty} (2\gamma\xi^3/\beta)^{(2k+i)}a_k^i \ , \tag{14}$$

where $a^0_0=1$, one finds from Eq.(10) the following recurrent relation:

$$a^i_k = \frac{\left(\dfrac{i}{2i-1} a^{i-1}_k + \dfrac{i+1}{2i+3} a^{i+1}_{k-1} \right)}{\left(i(i+1) + 6(2k+i)/\beta \right)} . \qquad (15)$$

It can be seen that $F_0(\xi)$ deviates noticeably from the Maxwellian function at energy $\xi \geq (\beta/\gamma^2)^{1/6}$, while the role of the asymmetric term $F_1(\xi)$ only becomes essential for higher energies $\xi \geq (\beta/\gamma)^{1/3}$.

B. Energy range $(\beta/\gamma)^{1/3} \leq \xi \leq \gamma^{-1/2}$

In this energy range it is convenient to introduce the variable $z = \xi\gamma^{1/2}$ and to represent the function $F(\vec{v})$ in the form

$$F(\vec{v}) = \pi^{-3/2} exp(-\varphi(z,\mu)). \qquad (16)$$

Expanding $\varphi(z,\mu)$ in a power series in $\gamma^{-1/4}$,

$$\varphi(z,\mu) = \varphi_0/\gamma^{1/2} + \varphi_1/\gamma^{1/4} + \varphi_2 + \ldots , \qquad (17)$$

and substituting this expansion in Eq.(10) after appropriate calculations, one finds

$$\varphi_0(z) = z - z^3/3 , \qquad (18)$$

$$\varphi_1(z,\mu) = 4\left(\frac{z^3}{\beta}\,(1-z^2)\right)^{1/2}\left(1-\left[\frac{1+\mu}{2}\right]^{1/2}\right)$$

$$+ \ \beta^{1/2}\int_0^z\left(\frac{z'}{(1-z'^2)}\right)^{1/2}dz' \ , \tag{19}$$

$$\varphi_2(z,\mu) = -\frac{z^2}{\beta}\left(\frac{3-5z^2}{1-z^2}\right)(1-\mu) + 4\left(\frac{z^2}{1-z^2}\right)\left(\frac{6+\beta-10z^2}{2\beta}\right)\left(1-\left[\frac{1+\mu}{2}\right]^{1/2}\right)$$

$$- \ \frac{\beta+6-10z^2}{\beta}\ln\left(2\left[1+\left(\frac{1+\mu}{2}\right)^{1/2}\right]^{-1}\right) - \frac{1}{4}\ln(2/(1+\mu)) \tag{20}$$

$$- \ \frac{1}{2}(\alpha-3/4)\ln(1-z^2) + \frac{\delta z^2}{2} - \frac{\beta}{4}\left(\frac{z^2}{1-z^2}\right) - \frac{3-\beta}{8}\ln\left(\frac{z^2}{1-z^2}\right) + C_1 \ ,$$

$$C_1 = -\ C_1^{\ 1}\beta + C_1^{\ 2}\beta^{1/2} + C_1^{\ 3}\beta\ln(z_*) + C_1^{\ 4} \ , \tag{21}$$

where $z_* \simeq \beta^{1/3}\gamma^{1/6}$ and $C_1^{\ 1} \simeq 1$ are the matching constants of the distribution functions (13), (17), realized at energies $z \simeq z_*$.

C. Energy range $z \simeq 1$

Let us introduce the variable $y = (z^2-1)/\gamma^{1/6}$ and represent the function φ as a power series in $\gamma^{-1/6}$:

$$\varphi(y,\mu) = \varphi^0/\gamma^{1/6} + \varphi^1 + \ldots + C_2. \tag{22}$$

Expanding the function φ^1 in Taylor series in the vicinity of $\mu=1$,

$$\varphi^1(y,\mu) = \sum_{k=0}^{\infty} \varphi^1_k(y) \frac{(1-\mu)^k}{k!} , \qquad (23)$$

and substituting Eqs.(22),(23) in Eq.(10), one finds

$$\varphi^0_0 = -\beta p/2 - \beta^2 p^4/4 , \qquad (24)$$

$$\varphi^0_1 = p , \qquad (25)$$

$$\varphi^1_0 = -\left(\alpha - \frac{(6+\beta)}{4}\right)\ln(p) + \frac{1}{2}\ln\left(\frac{1+2\beta p^3}{\beta p^3}\right) + \frac{\beta p^3}{6}\left(2-\beta-\frac{\beta^2 p^3}{2}\right), \qquad (26)$$

$$C_2 = C_1 + (2/3)\gamma^{-1/2} + (2\pi)^{3/2}\beta^{1/2}[\Gamma(1/4)]^{-2}$$

$$\qquad (27)$$

$$- (\alpha-(6+\beta)/4)\ln(\beta/2) - \frac{(\alpha-(6+\beta)/4)}{12}\ln\gamma + \frac{\delta}{2} - \frac{1}{8} - \frac{\ln 2}{2} ,$$

where $\Gamma(x)$ is the gamma function; C_2 is the matching constant of the solutions (17), (22) in the energy range $\xi \leq \gamma^{-1/2}$, where both expansions are applicable, and the function $p(y) = P(Y)/\beta^{1/3}$, where $Y = y/\beta^{1/3}$, is determined by positive roots of the equation

$$P^3 + YP - 1 = 0 . \qquad (28)$$

They are

$$P = 2(-Y/3)^{1/3}\cos\left(\frac{1}{3}\arccos\left[-\frac{2^{2/3}Y}{3}\right]^{-3/2}\right), \text{ for } \frac{2^{2/3}Y}{3} \leq -1 \qquad (29)$$

$$P = \frac{1}{2^{1/3}}\left(\left[\left(\frac{2^{2/3}Y}{3}\right)^3+1\right]^{1/2} + 1\right)^{1/3}$$

$$\qquad (30)$$

$$- \frac{1}{2^{1/3}}\left(\left[\left(\frac{2^{2/3}Y}{3}\right)^3+1\right]^{1/2} - 1\right)^{1/3} , \text{ for } \frac{2^{2/3}Y}{3} \geq -1$$

At the limit of applicability of the expansion (22) ($Y \leq Y_* =$

$\gamma^{-1/6}$), the function $\varphi(y_*, \mu = 1) = C_3$ turns out to be equal:

$$C_3 = C_2 + \frac{5\beta^2\gamma^{1/2}}{6} - \frac{\beta^3\gamma}{12} - \frac{\beta}{2} + \frac{\ln\beta}{2} + \frac{1}{6}\left(\alpha - \frac{6+\beta}{4}\right)\ln\gamma + \frac{\ln 2}{2} . \qquad (31)$$

Applicability of the expansions (17), (22) of the function $\varphi(\xi, \mu)$ can be verified by direct comparison between the magnitudes of successive terms in the series (17), (22). It is easy to show that the results of a given section are true at

$$\beta \leq \gamma^{-1/2} . \qquad (32)$$

D. Energy range $(2/\gamma)^{1/2} \leq \xi \leq (\beta/\gamma)^{1/2}$

This characteristic energy range only exists at rather high β-values. The distribution function $F(\vec{v})$ is found to be close to the spherically symmetric one, and one should use Eq.(12) to obtain it. Let us neglect the ambipolar electric field effect which is not essential here (the terms proportional to δ) Equation (12) is then transformed in the following way:

$$\frac{\gamma^2}{\beta} \xi^{(1-\alpha)} \frac{d}{d\xi}\left(\xi^3 \frac{d}{d\xi}(\xi^\alpha F_0) \right) = -\frac{3}{\xi}\frac{d}{d\xi}\left(F_0 + \frac{dF_0}{d\xi} \right) . \qquad (33)$$

The solution of Eq.(33) in the range $(2/\gamma)^{1/2} \leq \xi \leq (\beta/\gamma^2)^{1/4}$, taking account of its joining with the solutions in lower energy ranges, has the form

$$F_0 \simeq \pi^{-3/2}exp\left(-C_3 + \frac{3}{4}\beta\left((2/z)^4 - 1\right) \right). \qquad (34)$$

At higher energies $(\beta/\gamma^2)^{1/4} \leq \xi \leq (\beta/\gamma)^{1/2}$ the solution of Eq.(33) has the power law dependence

$$F_0 \simeq \pi^{-3/2} exp\left(- C_3 - \frac{3}{4}\beta\right) \left(\frac{\beta^{1/4}}{z}\right)^{\alpha}. \qquad (35)$$

The power law dependence $F_0(v) \propto v^{-2\alpha}$ (see Ref. 21) represents a "collectivization" of high-energy electrons practically insensitive to Coulomb collisions. In this case, the spectrum of such electrons is determined by just the global parameters of electron temperature and density profiles (α in the case under consideration). A similar dependence $F(v) \propto v^{-2\alpha}$ occurs at energies $\xi \gtrsim (\beta/\gamma)^{1/2}$, where $F(\vec{v})$ again becomes strongly anisotropic and the applicability condition of Eqs.(11),(12) is violated.

E. Energy range $\xi \gtrsim (\beta/\gamma)^{1/2}$

We seek the solution of Eq.(10) in this range in the form

$$F_0(\vec{v}) = \Phi_0(z,\mu) + \gamma^{1/2}\Phi_1(z,\mu) + \ldots . \qquad (36)$$

Substituting Eq.(36) in Eq.(10), one obtains the equation for $\Phi_0(z,\mu)$:

$$z^2\mu\ z^{(1-\alpha)}\frac{\partial}{\partial z}(z^{\alpha}\Phi_0) = z\frac{\partial\Phi_0}{\partial z} + \frac{1}{2}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial\Phi_0}{\partial\mu} . \qquad (37)$$

It is easy to see that at $\mu \gtrsim 0$ the main role in the parabolic type equation (37) is played by the terms on the left-hand side. The solution of Eq.(37) at $\mu \gtrsim 0$, with allowance for the corresponding matching, can therefore be represented in the form

$$\Phi_0(z,\mu) \simeq \pi^{-3/2}exp(- C_4)\left(\frac{\beta^{1/2}}{z}\right)^{\alpha} \tilde{\Phi}_0(\mu) , \qquad (38)$$

where

$$C_4 = C_3 + (3/4)\beta + (\alpha/4)\ln\beta \quad . \tag{39}$$

The function $\tilde{\Phi}_0(\mu)$ represents the angular dependence $F_0(\vec{v})$ at $\xi \geq (\beta/\gamma)^{1/2}$. Unfortunately, this dependence cannot be analytically defined. It can only be shown that $\tilde{\Phi}_0(-|\mu|) << \tilde{\Phi}_0(|\mu|)$ owing to the absence of a particle source at $v = \infty$ and $\mu \leq 0$. Therefore , taking into account that the angular asymmetry $F_0(\vec{v})$ at $\xi \simeq (\beta/\gamma)^{1/2}$ approaches unity, the following estimate can be assumed to be valid:

$$\int \tilde{\Phi}_0(\mu)\mu \, d\mu \simeq 1. \tag{40}$$

Thus the structure of the solution of Eq.(10) at $\beta < \gamma^{-1/2}$ can be represented in the following way (Fig.1). At low energies $\xi \leq (\beta/\gamma)^{1/3}$, the distribution function $F(\vec{v})$ is isotropic and approaches the Maxwellian one at $\xi \leq (\beta/\gamma^2)^{1/6}$; at energies $(\beta/\gamma)^{1/3} \leq \xi \leq \gamma^{-1/2}$, it is anisotropic and it deviates significantly from the Maxwellian function; at high β, at energies $(2/\gamma)^{1/2} \leq \xi \leq (\beta/\gamma)^{1/2}$, it differs considerably from the Maxwellian one but again becomes isotropic (at $\beta \simeq 1$ this range practically vanishes) and finally, at energies $\xi \geq (\beta/\gamma)^{1/2}$, it is far from being Maxwellian and is strongly anisotropic. Note that this so-called "collectivization" of electrons, where their spectrum is determined by the global parameters, the electron temperature and density profiles ($F(v) \propto v^{-2\alpha}$ in the case under consideration), takes place at energies $\xi \geq (\beta/\gamma^2)^{1/4}$.

IV. SOLUTION OF THE SELF-SIMILAR 1D EQUATION ($\beta > \gamma^{-1/2}$)

The solution of Eq.(12), representing the case of high Z_{eff}, where $\beta > \gamma^{-1/2}$, is readily found at $\alpha = 2$ since all the parts of the equation are complete differentials:

$$F_0(\xi) \propto EXP\left(-\int_0^\xi \frac{3\beta/\gamma^2 + (2-\delta)\xi'^3(\xi'-\delta)}{3\beta/\gamma^2 + (1-\delta)\xi'^4(\xi'-\delta)}\, d\xi'\right) . \qquad (41)$$

Unfortunately, the integral J, as will be shown later, diverges at $\alpha \leq 2$. Therefore we shall consider Eq.(41) as a formal expression with which one can, nevertheless, compare the solutions obtained in some indirect way.

Let us represent the function $F_0(\xi)$ in the form

$$F_0(\xi) = \pi^{-3/2}exp(-\psi(\xi)) . \qquad (42)$$

Since, as follows from Eq.(41) and from qualitative considerations given in the second section, a noticeable distortion of the Maxwellian distribution function (doubling of the local "temperature") occurs at energies $\xi \simeq (\beta/\gamma^2)^{1/5}$, let us introduce the variable $u = \xi(\gamma^2/\beta)^{1/5}$ and represent the function ψ as a power series in $s^{-1/2}$, where $s = (\gamma^2/\beta)^{1/5}$:

$$\psi = \psi_0/s + \psi_1/s^{1/2} + \psi_2 + \dots . \qquad (43)$$

Substituting Eqs.(42),(43) in Eq.(12), one finds

$$\psi_0 = 3 \int_0^u \frac{du'}{(u'^5+3)} , \qquad (44)$$

$$\psi_1 = 0 , \qquad (45)$$

$$\psi_2 = \frac{6}{5} \frac{\delta u^5}{u^5+3} + \frac{2}{5}(\alpha-1)\ln\left(\frac{u^5+3}{3}\right) . \qquad (46)$$

It is easily proved by direct verification that the applicability of the expression (41), and, as a result, of the expressions $(44)-(46)$, is violated at $\xi \geq (\beta/\gamma^2)^{1/4}$ $(u \geq (\beta/\gamma^2)^{1/20} \gg 1)$, i.e. at the time when strong distortion of the Maxwellian distribution function already occurs and "collectivization" of high-energy electrons starts (see expression (37) and estimates in Section II).

The solution of Eq.(12) in the energy range $(\beta^{1/2}/\gamma)^{1/2} \leq \xi \leq (\beta/\gamma)^{1/2}$ is similar to the solution (35) of Eq.(33). In the energy range $\xi \geq (\beta/\gamma)^{1/2}$, where the applicability of Eq.(12) is violated and it is necessary to consider Eq.(10), we obtain a solution of the type (36), (38):

$$F(\vec{v}) \simeq \pi^{-3/2}\exp(- C_5)\left(\frac{(\beta/\gamma)^{1/2}}{\xi}\right)^{\alpha} \hat{\Phi}_0(\mu) , \qquad (47)$$

where

$$C_5 \simeq \frac{3^{1/5}\Gamma(1/5)\Gamma(4/5)}{5} (\beta/\gamma^2)^{1/5} + \frac{2}{5}(\alpha-1)\ln\left(\frac{(\beta/\gamma^2)^{1/4}}{3}\right)$$
$$+ (\alpha/4)\ln\beta + (6/5)\delta - 3/4 . \qquad (48)$$

At the same time the function $\hat{\Phi}_0(\mu)$, like to $\tilde{\Phi}_0(\mu)$, represents the angular dependence $F(\vec{v})$ at high energies and the estimate (40) is also valid for it.

Thus the structure of the solution of Eq.(10), for $\beta > \gamma^{-1/2}$,

can be represented in the following way (Fig.2). The distribution function $F(\vec{v})$ is isotropic up to energies $\xi \leq (\beta/\gamma)^{1/2}$; at low energies, $\xi \leq (\beta/\gamma^2)^{1/6}$, it approaches the Maxwellian; at the energies $(\beta/\gamma^2)^{1/6} < \xi \leq (\beta/\gamma^2)^{1/4}$ considerable deviation from the Maxwellian function takes place; at energies $\xi \geq (\beta/\gamma^2)^{1/4}$, "collectivization" of the electrons starts and $F_0(v)$ greatly differs from the Maxwellian function and, finally, at energies $\xi \geq (\beta/\gamma)^{1/2}$ the function $F(\vec{v})$ becomes strongly anisotropic.

V. SELF-SIMILAR VARIABLES FOR 3D KINETIC EQUATION

Let us now consider a 3D inhomogeneous plasma. It will be shown that in the case of spherical symmetry of the plasma parameters self-similar variables can be introduced. The stationary kinetic equation for the electron distribution function $f_e(\vec{V},r)$ inhomogeneous along the radius r is

$$V\mu \frac{\partial f_e}{\partial r} - \frac{e\,E}{m} \left(\mu \frac{\partial f_e}{\partial V} + \frac{1-\mu^2}{V} \frac{\partial f_e}{\partial \mu} \right)$$

$$= \frac{2\pi e^4 \Lambda}{m^2} \left(St(\vec{V},f_e) + \frac{Z_{effi}n_e(r)}{V^3} \frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial f_e}{\partial \mu} \right) , \qquad (49)$$

where ϑ is the angle between the particle velocity vector $\vec{V}$ and the axis $\vec{r}$; $\mu = cos\vartheta$; $E(r)$ is the ambipolar electric field directed along the $\vec{r}$ axis.

It is easy to show that Eq.(49) allows solutions in the following self-similar variables:

$$f_e(\vec{V},r) = N\,F[\vec{v}\vec{r}/(vr),v]/[T_e(r)]^\alpha, \quad v = V(m/2T_e(r))^{1/2}, \qquad (50)$$

where, as in Eq.(5) N is the normalization factor; $\int F(\vec{v})d\vec{v} = 1$; α

is an adjustable parameter; and the function $T_e(r)$ plays the role of a characteristic average electron energy. Equation *(49)* is transformed to

$$\gamma v \left\{ \mu \left(\alpha F + \frac{v}{2} \frac{\partial F}{\partial v} \right) - (\alpha + 1/2)(1 - \mu^2) \frac{\partial F}{\partial \mu} \right\} - \frac{\gamma_E}{2m} \left(\mu \frac{\partial F}{\partial v} + \frac{1 - \mu^2}{v} \frac{\partial F}{\partial \mu} \right)$$

$$= \frac{1}{4} \left(St(\vec{v}, F) + \frac{Z_{eff}}{v^3} \frac{\partial}{\partial \mu} (1 - \mu^2) \frac{\partial F}{\partial \mu} \right) , \tag{51}$$

which differs from the corresponding self-similar equations derived in *[21,23]* for the case of 1D plasma parameter inhomogeneity by the term proportional to $\gamma \partial F / \partial \mu$ on the left-hand side of Eq.*(51)*. This term describes the spread of the electrons across the radius $\vec{r}$.

As in the case of 1D inhomogeneity the solution of Eq.*(51)* corresponds to a constant ratio of the mean free path for electrons with an energy of about $T_e(r)$ to the scale length of $T_e(r)$ and to the same dependencies of $T_e(r)$ and $n_e(r)$ on r.

In the limit of high energies, $v \gg 1$, Eq.*(51)* is transformed to

$$\gamma \left\{ \mu \left(\alpha F + \xi \frac{\partial F}{\partial \xi} \right) - (\alpha + 1/2)(1 - \mu^2) \frac{\partial F}{\partial \mu} \right\} - \gamma_E \left(\mu \frac{\partial F}{\partial \xi} + \frac{1 - \mu^2}{2\xi} \frac{\partial F}{\partial \mu} \right)$$

$$= \frac{1}{\xi} \frac{\partial}{\partial \xi} \left(F + \frac{\partial F}{\partial \xi} \right) + \frac{\beta}{2\xi^2} \frac{\partial}{\partial \mu} (1 - \mu^2) \frac{\partial F}{\partial \mu} . \tag{52}$$

For the high-β approximation the equations for $F_1(\xi)$ and $F_0(\xi)$ will have the forms:

$$F_1 = - \gamma \frac{\xi^2}{\beta} \xi^{(1-\alpha)} \frac{d}{d\xi}(\xi^\alpha F_0) + \gamma_E \frac{\xi^2}{\beta} \frac{dF_0}{d\xi} \; . \tag{53}$$

$$\frac{\gamma^2}{\beta} \xi^{(\alpha+2)} \frac{d}{d\xi}\left(-\left(\frac{\xi}{\xi_\alpha}\right)^2 \frac{d}{d\xi}(\xi^\alpha F_0) + \delta_E \xi^{(1-\alpha)} \frac{dF_0}{d\xi}\right)$$

$$- \frac{\delta_E}{\xi} \frac{d}{d\xi}\left(- \xi^{(4-\alpha)} \frac{d}{d\xi}(\xi^\alpha F_0) + \delta_E \xi^3 \frac{dF_0}{d\xi}\right) \tag{54}$$

$$= \frac{3}{\xi} \frac{d}{d\xi}\left(F_0 + \frac{dF_0}{d\xi}\right) \; ,$$

We shall not demonstrate here the solutions of Eqs. *(52),(54)* in all energy ranges, unlike in the case of 1D inhomogeneous plasma. We consider below just the tail electrons, since it can be shown that the difference between the 1D and 3D distribution functions at low energies is not as important as for tail electrons with energies $\xi \geq (\beta/\gamma)^{1/2}$.

The distribution function of these electrons is described by the equation

$$\mu\left(\alpha F + \xi \frac{\partial F}{\partial \xi}\right) - (\alpha+1/2)(1-\mu^2)\frac{\partial F}{\partial \mu} = 0 \; . \tag{55}$$

Assuming that the angular distribution function at $\mu \leq 1$ and at energies $\xi \simeq (\beta/\gamma)^{1/2}$ has the form $\tilde{\Phi}_0(\mu) \simeq exp(-C_\mu(1-\mu))$, where $C_\mu \simeq 1$ is the decay scale of the distribution function along the μ coordinate, the asymptotic solution of Eq.*(55)* for tail electrons, $\xi \gg (\beta/\gamma)^{1/2}$, can be represented in the following way:

$$F(\vec{v}) \propto \left[\frac{(\beta/\gamma)^{1/2}}{\xi}\right]^\alpha exp\left\{-C_\mu(1-\mu)\left(\frac{\xi}{(\beta/\gamma)^{1/2}}\right)^{(\alpha-1/2)}\right\} \; . \tag{56}$$

The normalization constant for Eq.*(56)* is practically the

same as in the case of 1D inhomogeneous plasma.

Thus, the main effect of 3D inhomogeneity or, in other words, the main effect of of the electron spread across the radius $\vec{r}$ is the beam-like distribution function of the tail electrons, which to some extent leads to suppression of the influence of tail electrons on the heat flux.

VI. DISCUSSION

First of all the presence of a strongly anisotropic power law tail in the distribution function at energies $\xi \geq (\beta/\gamma)^{1/2}$ in the above derived equations should be emphasized. This result is quite natural if one takes into account that electrons are collectivized at such energies and their spectrum should be determined by global characteristics of the temperature and density profiles $T_e(x)$, $n_e(x)$, but not by their local values. Since we consider the distribution function $f_e(\vec{V},x)$ and the dimensionless energy ξ in the form (5), the only dependence f_e on v satisfying this condition, will be $f_e \propto 1/v^{2\alpha}$.[21]

Formally, such a power law dependence of $f_e(V)$ results in a divergence of the dimensionless heat flux Q in the expression (9) at $\alpha \leq 3$ (for 1D inhomogeneity) owing to the contribution of suprathermal particles for any values of γ. Relation $\alpha \leq 3$ corresponds to a rather steep dependence $T_e(x)$: $d\ln T_e/d\ln x \geq 2/7$. At $\alpha = 3$ the divergence is a logarithmic, while at $\alpha < 3$ it obeys the power law. At a much steeper dependence of $T_e(x)$, where $\alpha \leq 2$, the dimensionless particle flux J diverges and it turns out to be

impossible to determine the quantity δ, while at $\alpha \leq 3/2$ the quantity N turns out to be divergent. Here it should be remembered that the case $\alpha = 3$ corresponds to the constant heat flux q according to Spitzer's electron heat conduction law.

It should be noted that the Luciani-like nonlocal expression for the heat flux, Eq.(1), for the electron temperature and density profiles under consideration does not make any difference between the cases with $\alpha > 3$ and $\alpha < 3$ and gives $q_L = q_{SH}(x)/[1-\gamma^2(3-\alpha)^2]$.

However, the emergence of a great number of suprathermal electrons is related to their transport from some hotter zones (electric field effect, $E \propto dT_e/dx$ is inessential here). Therefore in real limited systems, where the maximal temperature is limited to $T_e \leq T_{max}$ the formally emerging divergences can be eliminated by cutting off the divergent integrals at the energy level $mV^2/2 \leq T_{max}$ (we assume that the distribution function f_e approaches a self-similar one in the energy range $mV^2/2 \leq T_{max}$). Assuming that $\xi \leq \xi_{max} \simeq T_{max}/T_{min}$, in Eq.(9), one finds the contribution of tail electrons to the integral Q ($\alpha < 3$):

$$\Delta Q_{tail} \simeq \frac{\pi^{-3/2}}{(3-\alpha)} \left(\frac{\beta}{\gamma}\right)^{\alpha/2} \xi_{max}^{(3-\alpha)} \begin{cases} exp(-C_4) \ , & \beta < \gamma^{-1/2} \\ exp(-C_5) \ , & \beta > \gamma^{-1/2} \end{cases} . \qquad (57)$$

Taking into account the fact that the contribution of electrons to the integral of Q, where $\xi \leq (\beta/\gamma)^{1/2}$ is of the order of γ, the effect of tail electrons on the heat flux becomes essential when the temperature difference is rather large:

$$\left\{\frac{T_{max}}{T_{min}}\right\}^{(3-\alpha)} > \gamma\left(\frac{\gamma}{\beta}\right)^{\alpha/2}\begin{cases} exp(\tilde{c}_4/\gamma^{1/2}) & , \ \beta < \gamma^{-1/2} \\ \\ exp(\tilde{c}_5(\beta/\gamma^2)^{1/5}) & , \ \beta > \gamma^{-1/2} \end{cases} . \qquad (58)$$

In formulating Eq.(58) we left the main term only in the constants c_4 and c_5; here $\tilde{c}_4 \simeq 2/3$, $\tilde{c}_5 \simeq 3^{1/5}\Gamma(1/5)\Gamma(4/5)/5$.

In the case of smooth $T_e(x)$ profiles, where $\alpha > 3$, the effect of tail electrons on the heat flux only becomes noticeable at rather high values $\gamma \simeq 1$.

Note, however, that the estimate (58) and other generalizations made in this section are based on self-similar solutions of the collisional kinetic equation that were obtained for the electron temperature and density profiles characterized by γ. It is obvious that they do not spread to the zones with uniform electron temperature in the step-like temperature profile. Moreover, a definite effect on the heat flux production is also exerted by some boundary effects (e.g. interaction between the plasma and tokamak divertor plates). The study of such effects is beyond the scope of this paper.

In the case of 3D inhomogeneous plasma, owing to the beam-like character of the tail electron distribution function (56), the effect of tail electrons on the heat flux starts to become pronounced at much steeper effective temperature profiles ($\alpha < 7/4$) than in the case of the 1D inhomogeneous plasma.

Let us now consider when the high-Z approach,[18] which is based on representation of the distribution function in the form of the sum of the symmetric part and the small asymmetric one,

can be applied for investigating the effect of nonmaxwellian suprathermal particles on the electron heat conduction. It is necessary to note that if inequality (58) is not fulfilled the energy range responsible for the electron heat conduction is[10] $\varepsilon_1 < \varepsilon < \varepsilon_2$, where $\varepsilon_1 \simeq (3\div5)T_e$, $\varepsilon_2 \simeq (7\div9)T_e$.

Let us assume the electron temperature profile to be characterized by the magnitudes of $\lambda_T/L = \gamma < 1$ and $\beta < \gamma^{-1/2}$. It was shown above that in this case the electron distribution function deviates noticeably from Maxwellian and is still symmetric in the range $(\beta/\gamma^2)^{1/6} < \xi < (\beta/\gamma)^{1/3}$; at higher energies $\xi > (\beta/\gamma)^{1/3}$ the distribution function becomes strongly anisotropic. The effect of this deviation becomes noticeable for the heat conductivity when $(\beta/\gamma^2)^{1/6} < \varepsilon_2/T_e$ and becomes very strong when $(\beta/\gamma^2)^{1/6} \lesssim \varepsilon_1/T_e$. But this effect can only be described by high-Z approach when the distribution function in the range of interest is still symmetric, that is $\varepsilon_2/T_e < (\beta/\gamma)^{1/3}$. Both inequalities ($(\beta/\gamma^2)^{1/6} \lesssim \varepsilon_1/T_e$ and $\varepsilon_2/T_e < (\beta/\gamma)^{1/3}$) can be fulfilled if Z_{eff} is extremely high, $Z_{eff} \gtrsim 260$, but no elements with this charge exist. Therefore for $\beta < \gamma^{-1/2}$ the analysis of strong deviation of the electron heat conduction from the Spitzer-Harm theory must take into account the energy range $\xi > (\beta/\gamma)^{1/3}$ where the distribution function can not be described by high-Z approach. Thus, high-Z approach[18] can only be applied for the investigation of the nonlocal effects under the condition $\beta \gtrsim \gamma^{-1/2}$.

VII. CONCLUSIONS

i. Solutions of the collisional electron kinetic equation are found for 1D and 3D (spherical symmetry) inhomogeneous plasmas with arbitrary Z_{eff} by means of self-similar variables.[21]

ii. The criterion (58) is obtained for the plasma parameter profiles characterized by the ratio of the mean free path for thermal particles, λ_T, to the electron temperature scale length L. It determines the effect that tail particles with motion of the non-diffusive type have on the electron heat conductivity.

iii. For these conditions it is shown that the use of a "symmetrized" kinetic equation of the type of Ref. 18 for the investigation of the strong nonlocal effect of suprathermal electrons on the electron heat conductivity, when the electron heat conduction can not be described by Spitzer-Harm theory, is only possible at sufficiently high Z_{eff} $(Z_{eff} \geq (L/\lambda_T)^{1/2})$.

iv. In the case of 3D inhomogeneous plasma (spherical symmetry), the effect of tail electrons on the heat transport is less pronounced since the tail electrons are spread across the radius $\vec{r}$.

[1] Spitzer L., Harm R., Phys. Rev. **89**, 977 (1953).

[2] Gurevich A.V., Istomin Ya.N., JETP **77**, 933 (1977) (in Russian).

[3] Bell A.R., Evans R.G., Nicholas D.J., Phys. Rev. Lett. **46**, 243 (1981).

[4] Mason R.J., Phys. Rev. Lett. **47**, 652 (1981).

[5] Matte J.P., Virmont J., Phys. Rev. Lett. **49**, 1936 (1982).

[6] Khan S.A., Rognlien T.D., Phys. Fluids **24**, 1442 (1981).

[7] Igikhanov Yu.L., Krasheninnikov S.I., Kukushkin A.S., Yushmanov P.N. *"Specific features of transport processes in a tokamak edge plasma."* in: Rev. of Science and Technology, Plasma Physics, edited by V.D.Shafranov (Moscow, USSR Inst. of Scientific and Technological Information, 1990), Vol.11, P.5 (in Russian).

[8] Nedospasov A.V., Uspekhi Fiz. Nauk **152**, 479 (1987) (in Russian).

[9] Galeev A.A., Natanzon A.M., Fizika Plazmy **8**, 1151 (1982) (in Russian).

[10] Hares J.D., Kilkenny J.D., Key M.H., Lunney J.G., Phys. Rev. Lett. **42**, 1216 (1979).

[11] Kishimoto Y., Mima K., J. Phys. Soc. of Japan **52**, 3389 (1982).

[12] Hassam A.B., Phys. Fluids **23**, 38 (1980).

[13] Shirasian M.H., Steinhauer L., Phys. Fluids **24**, 843 (1981).

[14] Zawaideh E., Kim N.S., Najmabadi F., Phys. Fluids **31**, 3280 (1989).

[15] Belyi V.V., Demaulin W., Paiva-Veretennikoff I., Phys. Fluids **B1**, 305 (1989).

[16] Belyi V.V., Demaulin W., Paiva-Veretennikoff I., Phys. Fluids
B1, 317 (1989).

[17] Luciani J.F., Mora P., Pellat R., Phys. Fluids **28**, 835 (1985).

[18] Albritton J.R., Williams E.A., Bernstein I.B., Swartz K.P.,
Phys. Rev. Lett. **57**, 1887 (1986).

[19] Luciani J.F., Mora P., Virmont J., Phys. Rev. Lett. **51**, 1664
(1983).

[20] Murtazs G., Mirza M., Qaisar M.S., Phys. Lett. **141**, 56 (1989);
144, 164 (1990).

[21] Krasheninnikov S.I., Sov. Phys. JETP **67**, 2483 (1988).

[22] Krasheninnikov S.I., Dvornikova N.A., Smirnov A.P.,
Contributions to Plasma Physics **30**, 67 (1990).

[23] Gurevich A.V., JETP **39**, 1296 (1980) (in Russian).

[24] Lebedev A.N., JETP **48**, 1392 (1965) (in Russian).

[25] Gurevich A.V., Zhivlyuk Yu.N., JETP **49**, 214 (1965)
(in Russian).

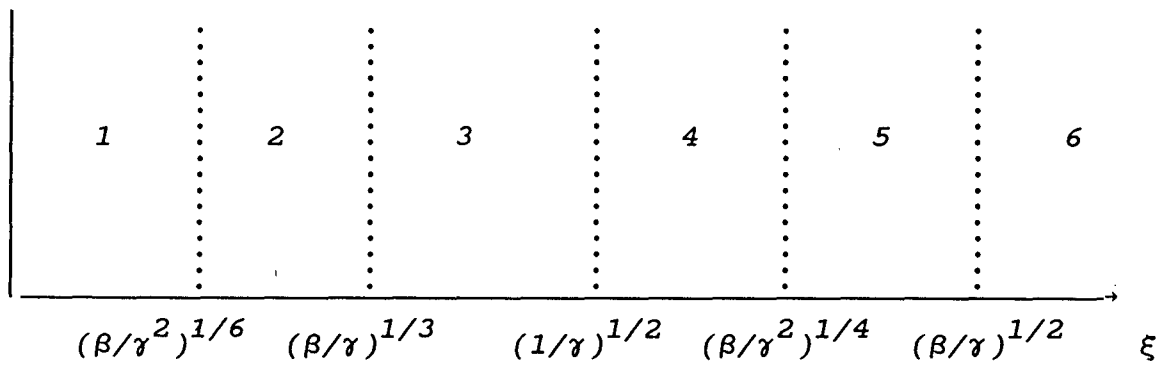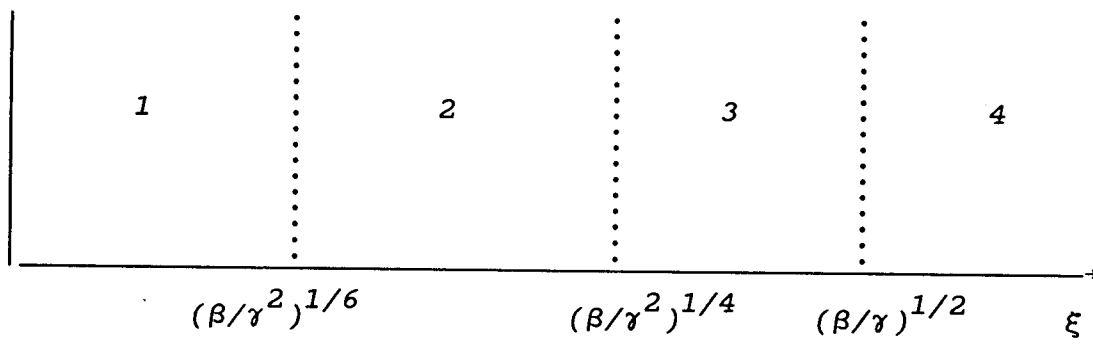[26] Connor J.W., Hastie R.J., Nucl. Fusion **15** (197) 415.

$$(\beta/\gamma^2)^{1/6} \qquad (\beta/\gamma)^{1/3} \qquad (1/\gamma)^{1/2} \qquad (\beta/\gamma^2)^{1/4} \qquad (\beta/\gamma)^{1/2} \qquad \xi$$

Fig. 1.

Fig. 2.

Figure captures

Fig.1   Characteristic energy zones of electron distribution function change at moderate Z_{eff}: $\beta \equiv (1+Z_{eff})/2 \leq 1/\gamma^{1/2}$. Region 1 - linear deviation $F(\vec{v})$ from Maxwellian function, $F(\vec{v})$ is symmetric; 2 - weak deviation $F(\vec{v})$ from Maxwellian function, $F(\vec{v})$ is symmetric; 3 - essential deviation $F(\vec{v})$ from Maxwellian function, $F(\vec{v})$ is asymmetric; 4 - strong deviation $F(\vec{v})$ from Maxwellin function, $F(\vec{v})$ is symmetric; 5 - "collectivization" cf electrons, $F(\vec{v})$ is symmetric; 6 - "collectivization" of electrons, $F(\vec{v})$ is asymmetric.

Fig.2   Characteristic energy zones of electron distribution function change at moderate Z_{eff}: $\beta \equiv (1+Z_{eff})/2 \geq 1/\gamma^{1/2}$. Region 1 - linear deviation $F(\vec{v})$ from Maxwellian function, $F(\vec{v})$ is symmetric; 2 - strong deviation $F(\vec{v})$ from Maxwellian function, $F(\vec{v})$ is symmetric; 3 - "collectivization" of electrons, $F(\vec{v})$ is symmetric; 4 - "collectivization" of electrons, $F(\vec{v})$ is asymmetric.