CONDUCTING WALL AND PRESSURE PROFILE EFFECT

ON MHD STABILIZATION OF AXISYMMETRIC MIRROR

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Abstract

The MHD Energy Principle is used to examine the stabilization effect of a conducting wall located near to the plasma surface. The stabilization effect is maximized when the normal components of the perturbed magnetic field approaches zero ($Q_n=0$) at the plasma surface. Under this boundary condition, the eigen-equation of the plasma displacement is solved for a two-step flat pressure profile model, which can include both the non-hollow and a hollow pressure profile. Only the rigid $m=1$ mode is considered due to the FLR effect. For an isotropic pressure component it is found that a hollow profile has better stability than a uniform pressure when the integral of the radial pressure profile is fixed. The implication for plasma experiments and fusion reactors is discussed.

§1. Introduction

Recently Berk et al.[1,2] showed with a kinetic treatment that a conducting wall located near to the plasma surface has a strong stabilization effect on the m=1 mode in an axisymmetric mirror, which cannot otherwise be stabilized by finite Larmor radius effects. An MHD approach has been followed by several authors[3-5]. For simplicity references [2,3] assumed a flat pressure profile. However, the experiments on TARA[5] and Phaedrus[7] showed that a hollow plasma profile can be formed due to RF edge heating. On the other hand, the Mini MARS preliminary study[8] showed that a hollow edge stabilized plasma profile may be desirable due to its favorable average curvature. Therefore, it is interesting to see how the pressure profile affects the wall stabilization. It is known that a plasma cannot dig a MHD stable well for itself. A high β plasma reduces the local magnetic field, the effective mirror ratio increases (Fig. 1). Although a portion of the unfavorable curvature becomes favorable curvature, the remaining unfavorable curvature portion becomes worse than before. As a whole, plasma becomes less stable because the unfavorable curvature region is located at lower fields where the higher β value gives more weight than the favorable curvature region. However, the conducting wall will limit the displacement of the plasma surface and introduce a new weight factor. Particularly, when the wall is close to the plasma and the β value is high, the conducting wall can limit the displacement of the plasma surface in the unfavorable curvature region. We analyze the plasma conditions necessary for this stabilization to be effective. When we keep the integral $\int P(\psi)d\psi$ constant (where $P(\psi)$ is the plasma pressure profile as a function of the

magnetic flux ψ) and make the plasma hollow, the β value on the plasma surface increases. In the presence of a conducting all, this increase in β improves the stability of the plasma.

In order to illustrate the physics involved, we discuss the conducting wall effect first in §2, ·then, we evaluate the different aspects of the hollow plasma in §3. Finally, a brief discussion is given in §4.

§2. Conducting Wall Stabilization.

If a perfect conducting wall is put near to the plasma surface (Fig. 2), any displacement of the plasma surface will perturb the magnetic field in the vacuum region between the conducting wall and the plasma surface. This perturbation is related to the magnetic field line bending on the plasma surface. In the limit when the wall approaches the plasma surface, the magnetic field line bending term must be zero. This will in turn determine the displacement of the plasma surface.

We start from the MHD energy principle. The variation of the potential energy of the plasma-vacuum system may be written[9] as

$$\delta W = \delta W_v + \delta W_F + \delta W_s.$$ (1)

δW_v, δW_F and δW_s are the vacuum energy, plasma fluid energy and the surface energy respectively. δW_v is defined as

$$\delta W_v = \int_{(vacuum)} \frac{(\delta \vec{B}_v)^2}{2} \, d^3 r$$ (2)

here $\delta \vec{B}_v$ is the perturbed magnetic field in the vacuum region. Linearizing the Maxwell equation, we have

$$\nabla \times \delta \vec{B}_v = 0$$ (3)

$$\nabla \cdot \delta \vec{B}_v = 0$$ (4)

Equation (3) gives

$$\delta \vec{B}_v = \nabla \chi$$ (5)

where χ is a scalar potential in the vacuum region.

Eq. (4) gives

$$\nabla^2 \chi = 0 \tag{6}$$

which means that χ is a harmonic function. Later we will see that the behavior of the harmonic function makes wall stabilization effective only for low m-mode. Using Eq. (5) and (6), we change the volume integration in Eq. (2) into a surface integration.

$$\delta W_v = \frac{1}{2} \int \nabla\chi \cdot \nabla\chi \, d^3r$$

$$= \frac{1}{2} \int [\nabla \cdot (\chi \nabla \chi) - \chi \nabla^2 \chi] \, d^3r$$

Using its Euler-Lagrangian equation, Eq. (6), we have

$$(\delta W_v)_{min} = \frac{1}{2} \int_{surface} \chi \nabla \chi \cdot d\vec{A} \tag{7}$$

Since the conducting wall requires that the field lines near the wall remain unchanged, we only need to calculate the surface integration in Eq. (7) on the plasma-vacuum interface. Denoting the perturbed magnetic field by $\vec{Q}$, we have

$$Q_n = \nabla\chi \cdot \vec{n} \tag{8}$$

Here, n is the normal vector at the interface. We show in the appendix that

$$(\delta W_v)_{min} = \frac{2\pi}{m} \int_{-L}^{L} dZ \, R^2 Q_n^2 \, \frac{1 + \left(\dfrac{R}{R_w}\right)^{2m}}{1 - \left(\dfrac{R}{R_w}\right)^{2m}} \tag{9}$$

where m is the azimuthal mode number, R and R_w are the radius of the plasma and conducting wall respectively.

As we have seen from Eq. (2) δW_v is always positive and so is $(\delta W_v)_{min}$, and is therefore always stabilizing. The harmonic function χ is uniquely determined by its boundary value Q_n. Thus both $\nabla \chi$ and χ are proportional to Q_n, and $(\delta W_v)_{min}$ is proportional to $(Q_n)^2$. This characteristic is important because the variation of the plasma fluid energy δW_F may in certain cases be linearly proportional to Q_n.[4] Therefore, the vacuum energy increases much faster than the plasma fluid energy when Q_n is increasing. This is the essence of the Haas-Wesson effect.

From Eq. (9), it is clear that wall stabilization is weak for high mode number, m, since the required vacuum field perturbation decreases as m increases. Any perturbation on the plasma surface, $Q_n \neq 0$, will decay to zero on the conducting wall surface. The transition of the δB_v is determined by the harmonic behavior of χ, which has an associated radial and azimuthal dependence. The faster the χ changes azimuthally, the faster the χ changes radially. High m number means a rapid variation in azimuthal direction and therefore, a rapid decay behavior in the radial direction. Thus only a small vacuum region is affected by the perturbed plasma surface and wall stabilization effect decreases.

The expression of (9) gives an interesting conclusion when the wall approaches plasma, i.e. $R_w \rightarrow R$. In order to keep $(\delta W_v)_{min}$ finite, Q_n must be zero. In the vicinity of the plasma-vacuum interface,

there is a general expression for Q_n[10]

$$Q_n = (\vec{B} \cdot \nabla)(\vec{\xi} \cdot \vec{n}) - (\vec{\xi} \cdot \vec{n}) \vec{n} \cdot (\vec{n} \cdot \nabla)\vec{B} \qquad (10)$$

Here $\vec{\xi}$ is the displacement of the plasma-vacuum interface, $\vec{n}$ is its normal. In the long-thin approximation

$$Q_n \simeq B_z \frac{\partial}{\partial z} \xi_r - \xi_r \frac{\partial}{\partial r} B_r \qquad (11)$$

Since on the plasma-vacuum interface,

$$\frac{\partial r}{\partial z} = \frac{B_r}{B_z} \qquad (12)$$

and $B_\theta = 0$, $\nabla \cdot \vec{B} = 0$ gives

$$\frac{1}{r} \frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0 \qquad (13)$$

we have

$$r \frac{\partial B_r}{\partial r} = -r \frac{\partial B_z}{\partial z} - B_r = -r \frac{\partial B_z}{\partial z} - B_z \frac{\partial r}{\partial z}. \qquad (14)$$

Therefore,

$$Q_n \simeq B_z \frac{\partial}{\partial z} \xi_r + \xi_r \frac{1}{r} \frac{\partial}{\partial z}(r B_z)$$

$$= \frac{1}{r} \frac{\partial}{\partial z}(r B_z \xi_r) \qquad (15)$$

and $Q_n \rightarrow 0$ requires

$$r B_z \xi_r \Big|_{r=R} = \text{constant.} \qquad (16)$$

Or the displacement of plasma surface is

$$\xi_r \Big|_{r=R} \propto \frac{1}{B_z R} \qquad (17)$$

Notice that Q_n in the equation (9) is the perturbed magnetic field in the vacuum side of the interface. Now we calculate the quantity in Eq. (17) using the vacuum field B_v with the long-thin approximation. At the minimum of the vacuum field, B_z becomes small but the plasma radius R gets large. We need to consider how R varies with B_v for a given pressure profile. As we know

$$\frac{R^2}{2} = \int_0^{\psi_R} \frac{d\psi}{B} \tag{18}$$

here, ψ_R is the flux inside the plasma surface. Assuming a flat pressure profile, we have

$$\frac{R^2}{2} = \frac{1}{B} \psi_R = \frac{1}{\zeta B_v} \psi_R \tag{19}$$

Here ζ is the ratio of β corrected field, B, to the vacuum field B_v. In the long-thin approximation

$$\zeta = \sqrt{1 - \beta} \tag{20}$$

$$\beta = \frac{2 P_0}{B_v^2} \tag{21}$$

where P_0 is the plasma pressure. Thus

$$\xi_r^2 \propto \left(\frac{1}{B_v R}\right)^2$$

$$= \left(\frac{1}{B_v^2} \frac{\zeta B_v}{2\psi_R}\right) = \frac{1}{2\psi_R} \frac{\zeta}{B_v} \quad .$$

When B_v is decreasing, ζ is decreasing also. Thus we need to calculate the derivative with respect to z.

$$(\xi_r^2)' = \xi_r^2 \left(\frac{\zeta'}{\zeta} - \frac{B_v'}{B_v} \right) \tag{22}$$

Here, the prime denotes the derivative with respect to z. Assuming P_0 is constant along the field line, we have

$$\frac{\zeta'}{\zeta} = \sigma\tau \tag{23}$$

We introduce here the quantities σ and τ which are related to the β effect and the vacuum field variation respectively.

$$\sigma = \frac{\beta}{1 - \beta} \tag{24}$$

$$\tau = \frac{B_v'}{B_v} \tag{25}$$

$$(\xi_r^2)' = \xi_r^2 (\sigma - 1)\tau$$

$$= \xi_r^2 \frac{(2\beta - 1)}{(1 - \beta)} \tau \tag{26}$$

When β is greater than 1/2, $(\xi_r^2)'$ has the same sign of the $(B_v)'$. That is, ξ_r takes its minimum where B_v is minimum. When $\beta \to 1$, $\zeta \to 0$. Consequently, $\xi_r^2 \to 0$ at the $(B_v)_{min}$ where the curvature of magnetic field line is unfavorable. The significance of this behavior is that high β plasma may dig a well by itself with the MHD stability due to the conducting wall.

In the next section we want to calculate quantitatively how the high β plasma digs a well and avoids the unfavorable curvature in the $(B_v)_{min}$ region. At the same time we can see how the hollow pressure profile helps in reducing the necessary pressure integral value.

§3. Pressure Profile Effect.

To illuminate profile effects we will fix the pressure integral $\int P d\psi$, proportional to the plasma energy content, and consider an idealized profile illustrated in Fig. 3. Thus we can model either a uniform or a hollow cyclindrical plasma shell. We further assume the pressure is isotropic and thus independent of position along a field line. Inside the flux tube $\psi = \psi_1$, plasma pressure is assumed to be zero. The edge of the plasma is at $\psi = \psi_R$. The pressure is assumed to be a constant $P = P_k$ in the region $\psi_1 \leqslant \psi \leqslant \psi_R$. Therefore

$$
\int_0^{\psi_R} P(\psi)d\psi = P_k(\psi_R - \psi_1)
$$
$$
= \psi_R P_k (1 - f). \tag{27}
$$

Here, f is defined as ψ_1/ψ_R. We may keep the product $P_k (1 - f)$ constant and change f to examine the pressure profile effect. Reference [3] has derived a basic equation for the radial MHD displacement ξ_r

$$
2\xi \int_0^{\psi_R} d\psi \left(\frac{\rho\omega^2}{B} - P \frac{\partial}{\partial\psi} rr'' \right) + \int_0^{\psi_R} d\psi \left[\frac{1}{r} \frac{\partial}{\partial z} \frac{Q}{B} r^2 \frac{\partial}{\partial z} \frac{\xi}{r} \right.
$$

$$
\left. + rB \frac{\partial}{\partial z} \frac{Q}{r^2 B^3} \frac{\partial}{\partial z} rB\xi \right] = - RB_v \frac{\partial}{\partial z} \Lambda(z) \frac{\partial}{\partial z} RB_v\xi \tag{28}
$$

We have dropped the subscript on ξ. Here, ω is the frequency of the mode, ρ and P are the mass density and pressure of the plasma.

$$
Q = B^2 + P_\perp - P_\parallel = B_v^2 - 2P \tag{29}
$$

$$
P = \frac{1}{2} (P_\perp + P_\parallel) \tag{30}
$$

$$= \frac{1}{B_v} \psi_1 + \frac{1}{B} (\psi - \psi_1)$$

$$= \frac{1}{B} [\psi - (1 - \zeta)\psi_1]$$

$$= \frac{1}{B} D(\psi, \zeta, \psi_1) \qquad (39)$$

Here we introduce a new quantity

$$D(\psi, \zeta, \psi_1) \equiv \psi - (1 - \zeta)\psi_1 \qquad (40)$$

For a non-hollow plasma $\psi_1 \to 0$ and

$$D(\psi, \zeta, \psi_1) \to \psi . \qquad (41)$$

In this case $D(\psi, \zeta, \psi_1 = 0)$ is not a function of z. However, for the hollow plasma $\psi_1 \neq 0$, $D(\psi, \zeta, \psi_1)$ is a function of z through $\zeta(z)$. Hence,

$$rr'' = \frac{r^2}{2} \left\{ -\frac{1}{2} \left(\frac{D'}{D}\right)^2 - \left(\frac{D'}{D}\right) \left[\left(\frac{\tau^2}{2} - \tau'\right) + \tau^2 \left(\frac{1}{2} + \frac{2}{\zeta^2}\right) \right] \frac{1}{\tau} + \right.$$

$$\left. + \left(\frac{1}{2} \tau^2 - \tau'\right)(1 + \sigma) + \tau^2 \sigma \left[\frac{1}{2}(1 + \sigma) + \frac{2}{\zeta^2}\right] \right\} \qquad (42)$$

When $\beta = 0$ ($\sigma = 0$) and $\psi_1 = 0$, this reduces to the vacuum curvature result

$$r_v r_v'' = \frac{r_v^2}{2} \left(\frac{1}{2} \tau^2 - \tau'\right)$$

$$= \frac{r_v^2}{2} \left(\frac{3}{2} \tau^2 - \frac{B_v''}{B_v}\right) \qquad (43)$$

It is positive except in the region $\tau \approx 0$ and $B_v'' > 0$ which is just the region close to the $(B_v)_{min}$ – unfavorable curvature region.

When $\beta \neq 0$ and $\psi_1 = 0$, we have

$$rr'' = -\frac{r^2}{2\zeta^2}\left[\left(\frac{1}{2}\tau^2 - \tau'\right) + \frac{5}{2}\tau^2\sigma\right] \tag{44}$$

The plasma digs a well, and increases the mirror ratio. Some unfavorable curvature is converted into favorable curvature (Fig. 1). This effect is represented by the second positive-definite term in Eq. (44). We may call this a cusp-like effect, since the strong diamagnetic current in the $(B_v)_{min}$ region will form two cusp configurations in cooperation with the mirror coils in both sides. When $\beta \rightarrow 1$, this cusp-like effect becomes stronger. However, no matter how close to 1 the β is, the curvature becomes unfavorable near the point where $B_v = (B_v)_{min}$ (i.e. $\tau = 0$). Because the cusp-like effect bows out the field line, the unfavorable curvature at the $\tau = 0$ point becomes worse at high β, which is expressed by the factor ζ^{-2} in Eq. (44).

Now let us see the hollow plasma effect. When $\psi_1 \neq 0$, $D' \neq 0$, we have two additional terms in equation (42). Since

$$D' = \zeta'\psi_1 = \zeta\sigma\tau\psi_1 \tag{45}$$

and

$$\frac{r^2}{2}\frac{1}{D} = \frac{1}{B}, \tag{46}$$

$r^2/2 \, (D'/D)$ is independent of ψ in the model of Fig. 3 and the second term in Eq. (42) will disappear in calculating the finite integral Q_2 of Eq. (36). Hence,

$$Q_2 = (\psi_R - \psi_1) \int_{-L}^{L} dz \frac{2P_k}{(RB_v)^2} \frac{1}{B\zeta^2} \left\{ \frac{\zeta^3}{2} \left(\frac{\psi_1}{D_R}\right)(\sigma\tau)^2 + \right.$$

$$\left. + \left(\frac{1}{2}\tau^2 - \tau'\right) + \frac{5}{2}\tau^2\sigma \right\} \tag{47}$$

Thus we find a positive term due to the hollow effect ($\psi_1 \neq 0$).
Here D_R is the D value when $\psi = \psi_R$. Compared with the other
terms in Eq. (47), this hollow effect is always small. Since when β is
low, $\sigma < 1$, $(\sigma\tau)^2$ term is always smaller than $(\sigma\tau)$ terms.
When β is high, $\sigma > 1$, but the ζ factor ($\sim \sqrt{1 - \beta}$) makes it smaller.
Physically, the pressure gradients at the inner and outer surfaces of
the hollow plasma are always of opposite signs; therefore, any curvature
change will have opposite effects on the two surfaces. There is a
cancellation between them, so the net effect is small.

The more important factor is the weight factor in the integrand of
Q_2, i.e. the factor of $1/(RB_v)^2$ which comes from the displacement ξ and
the scale factor of the flux coordinate system. As we discussed in
§2, when β is higher than 0.5, $1/(RB_v)^2$ will decrease with B_v and take
the minimum at $(B_v)_{min}$. This weight factor suppresses the contribution
from the region near $(B_v)_{min}$ where the curvature is unfavorable.

Now let us check the stabilization effect due to Q_3. After integration
by parts, we have

$$Q_3 = -\int_{-L}^{L} dz \int_{0}^{\psi_R} d\psi \left\{ \frac{Q}{B} r^2 \left[\frac{\partial}{\partial z} \left(\frac{1}{RB_v r}\right) \right]^2 + \frac{Q}{r^2 B^3} \left(\frac{\partial}{\partial z} \frac{rB}{RB_v} \right)^2 \right\} \tag{48}$$

This is always negative, therefore stabilizing the plasma. The physical reason of this is the field line bending effect. Although the conducting wall forces the field line bending term to vanish on the plasma surface (i.e. $Q_n = 0$), the finite Larmor radius effect keeps the displacement constant inside the plasma. Therefore the line bending term will not vanish inside the plasma and in the vacuum hollow region, and can be expressed as Q_{31} and Q_{32}, respectively:

$$Q_{31} = - (\psi_R - \psi_1) \int_{-L}^{L} dz \; \frac{2}{(RB_v)^2} \frac{Q}{B} \; (\sigma\tau)^2 \left[1 - \frac{1}{2} \frac{D_1}{D_R} + \frac{1}{2} \left(\frac{D_1}{D_R}\right)^2 \right] \quad (49)$$

$$Q_{32} = - (\psi_R - \psi_1) \int_{-L}^{L} dz \; \frac{2}{(RB_v)^2} \; \frac{B_v^2}{B} \frac{1}{4} \; (\sigma\tau)^2 \; \frac{D_1}{D_R} \left[1 - \frac{D_1}{D_R} \right] \quad (50)$$

Here

$$D_1 \equiv D\Big|_{\psi = \psi_1} = \zeta\psi_1 \quad (51)$$

$$D_R \equiv D\Big|_{\psi = \psi_R} = \psi_R - (1 - \zeta)\psi_1 \quad (52)$$

Compared with the curvature drive term, they are small. The physical reason is as follows. In the high β case, the magnetic field inside plasma is small ($Q = B^2 \to 0$); in the low β case, the lines do not bend very much ($\sigma^2 \ll 1$), so the Q_{31} cannot be very large. For the vacuum field in the hollow region, its size is determined by ψ_1. When ψ_1 is large, the plasma shell is thin and thus the normal components of the perturbed field is small at the inner surface. Hence the line bending effect is small for the inner vacuum region also. When ψ_1 is

small, although the plasma shell is thick, the size of the hollow region
is small and this contribution is still small. However, if we keep the
pressure integral $\int P(\psi)d\psi$ constant, the hollow plasma still has a
big impact on the MHD stability. Since we keep

$$I_p = \int_0^{\psi_R} P(\psi)d\psi = P_k(\psi_R - \psi_1)$$

$$= P_k \, \psi_R \left(1 - \frac{\psi_1}{\psi_R}\right)$$

$$= \text{constant} \qquad (53)$$

P_k must increase while ψ_1/ψ_R is increasing. Hence the β value
will increase when plasma becomes hollow (keeping I_p constant).
Consequently, we know that the major effect due to the hollow is on the
drive term, i.e. the variation of the curvature at the outer surface of
the plasma. Combined with the conducting wall, a hollow plasma with the
same pressure integral, I_p, will have better MHD stability. In order
to have some quantitative result we need to assume a field configuration.
A sinosoidal configuration is selected since it ensures the boundary
condition (34).

$$B_v(z) = \frac{B_0}{2} \left[(M + 1) - (M - 1) \cos \frac{z\pi}{L}\right] \qquad (54)$$

Here M is the mirror ratio, B_0 is the vacuum magnetic field at the
bottom of the mirror cell, 2L is the distance between two mirror peaks.
A numerical integration code is used to calculate the following five
integrals.

$$\hat{Q}_1 = \int_{-L}^{L} dz \ \frac{2}{(RB_v)^2 B} \tag{55}$$

$$\hat{Q}_{21} = \int_{-L}^{L} dz \ \frac{2}{(RB_v)^2 B} \quad P_k \ \frac{1}{2} (\sigma\tau)^2 \ \frac{D_1}{D_R} \tag{56}$$

$$\hat{Q}_{22} = \int_{-L}^{L} dz \ \frac{2}{(RB_v)^2 B} \ \frac{1}{\zeta^2} \ P_k \left(\frac{\tau^2}{2} - \tau' \right) \tag{57}$$

$$\hat{Q}_{23} = \int_{-L}^{L} dz \ \frac{2}{(RB_v)^2 B} \ \frac{1}{\zeta^2} \ P_k \ \frac{5}{2} \ \sigma\tau^2 \tag{58}$$

$$\hat{Q}_{31} = \int_{-L}^{L} dz \ \frac{2}{(RB_v)^2 B} \quad Q \ (\sigma\tau)^2 \left[1 - \frac{1}{2} \ \frac{D_1}{D_R} + \frac{1}{2} \left(\frac{D_1}{D_R} \right)^2 \right] \tag{59}$$

$$\hat{Q}_{32} = \int_{-L}^{L} dz \ \frac{2}{(RB_v)^2} \ \frac{1}{B} \ B_v^2 \ \frac{1}{4} \ (\sigma\tau)^2 \ \frac{D_1}{D_R} \left[1 - \frac{D_1}{D_R} \right] \tag{60}$$

We have assumed an isotropic pressure such that

$$P_{\parallel} = P_{\perp} = p. \tag{61}$$

hence

$$Q = B_v^2 - 2p = B^2 \tag{62}$$

For the assumed sinosoidal configuration

$$\tau = \frac{B_v'}{B_v} = \frac{\left(\dfrac{\pi}{L} \right) \sin \left(\dfrac{z\pi}{L} \right)}{\left(\dfrac{M+1}{M-1} \right) - \cos \left(\dfrac{z\pi}{L} \right)} \tag{63}$$

$$\tau' = \frac{B_v''}{B_v} - \tau^2 \tag{64}$$

$$B_v'' = \frac{B_0}{2}(M-1)\left(\frac{\pi}{L}\right)^2 \cos\left(\frac{z\pi}{L}\right) \tag{65}$$

The sign of

$$S \equiv \frac{1}{\hat{Q}_1}(\hat{Q}_{21} + \hat{Q}_{22} + \hat{Q}_{23} + \hat{Q}_{31} + \hat{Q}_{32}) \tag{66}$$

will determine the MHD stability of the plasma. Here, $\hat{Q}_1$ is the inertial term; $\hat{Q}_{21}$ is the hollow effect on the drive term; $\hat{Q}_{22}$ is the only negative integral which represents the average unfavorable curvature and drives system unstable; $\hat{Q}_{23}$ is the major stabilizing term which comes from the cusp-like effect of the high β plasma; $\hat{Q}_{31}$ and $\hat{Q}_{32}$ are the line bending term of the plasma shell and the hollow region, respectively. Fig. 4 shows the results of the numerical calculations.

For every given pressure integral value $\int_0^{\psi_R} P(\psi)d\psi$,

we increase the hollow size ψ_1 until the S changes its sign. In Fig. 4 the abscissa is the ψ_1 normalized to ψ_R, and the coordinate is the

$\int_0^{\psi_R} P(\psi)d\psi$ normalized to $\frac{1}{2}B_0^2\psi_R$ which is the maximum of

this integral. We can see that for the same value of the integral $\int_0^{\psi_R} P(\psi)d\psi$

hollow plasma is more stable. The hollow plasma has greater stable region than the non-hollow plasma.

When the mirror ratio is decreasing, the stable region is increasing. This is due to the cusp-like effect which is stronger when the mirror ratio

is smaller. When the mirror ratio is 8, the stable limit of the integral I_p value is 0.82 for the non-hollow case ($f = 0$). This agrees well with Kesner's result[11].

It is noticed that although the stable I_p value is decreasing when f approaches 1, the stable β value is increasing. In Fig. 5 the stable region in β is shown as a function of f. It is clear that plasma pressure profile has a big impact on the MHD stability. This impact is due to the cusp-like effect of the high β plasma combined with the conducting wall.

§4. Discussion

Conducting wall stabilization is a kind of "surface stabilization." We have shown that a high-β plasma shell can be stable with a minimum of input power. Between the conducting wall and the high β plasma, there must be a low β region with a width of Larmor radius (Fig. 6). This tiny gap is important for it allows the displacement to grow from zero at wall to non-zero at the plasma interface. An ideal model for this gap is a vacuum region between wall and the plasma. In order to keep the magnetic energy in the vacuum region finite, the mode of the displacement of the plasma surface is forced to be $\sim \dfrac{1}{RB_v}$.

Because of the FLR effect, this displacement is transferred into the inner side of the plasma such that all the line bending terms become not important. For this mode, when the β is high, the displacement in the unfavorable curvature region is suppressed, and the cusp-like effect creates more favorable curvature region. As long as the β is high enough, the plasma is MHD stable.

The original theory of wall stabilization showed that the conducting wall makes plasma stable when the β value of the hot component satisfy:

$$\beta_c \sim \left(\frac{r_p}{L}\right)^2 \ll \beta_h \sim \left(\frac{L_h}{L}\right)^2 \ll 1. \tag{67}$$

Here β_c and β_h are the β value for the cold and hot components of the plasma. r_p is the radius of the plasma and L_h is the axial length of the hot components. Starting with the MHD equation,[3,4,5] we have tried to extend it to the higher β region with modified pressure profiles. Looking at the basic equation (28), one may take $(rr'') \sim \psi$ and conclude

that the drive term is proportional to the integral $\int P(\psi)d\psi$;
and therefore, the pressure profile $P(\psi)$ has no effect on the
instability. In fact, looking at Eq. (42), (39) and (40), one may see
that $(rr")\sim\psi$ is true only for the non-hollow plasma ($\psi_1 = 0$).
The more important effect is through the enhancement of β value which
is higher for hollow plasma with same pressure integral I_p. When the
conducting wall is close enough, the high β plays the dominant role for
plasma stabilization. Ref.[5] concludes that a hollow plasma is
less stable due to the added freedom of displacement at the inner
surface of the plasma. In that work rigidity of the mode is not
assumed due to the lack of FLR effect present in a square pressure
profile. On the contrary, we consider that the FLR effect makes the
plasma "rigid," consequently, there is no additional freedom of displacement.
In reality, the plasma will always have some pressure gradient which
ensures the FLR effect, except in the region where the pressure gradient
equals zero. Thus, although the flat pressure profile is
assumed for simplification, we still keep the "rigid" feature to include
the FLR effect. In fact in our calculation the displacement ξ is assumed
to be rigid even in the inner vacuum region. This is an approximation to
the real situation where the pressure gradient is enough to keep rigidity
but the pressure does not contribute significantly to the pressure integration
$\int P(\psi)d\psi$.

The cusp-like effect is seen to bow out the field lines and change
the curvature locally. This may lead to the invalidity of the long-thin
approximation (particularly, when the anisotropic pressure is used).
More work is needed in this line.

For the reactor start-up, we may initially use less power to heat the plasma surface only first, and then gradually heat the inner side of the plasma, in this way we may keep the plasma MHD stable during the start-up.

Acknowledgements

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Appendix: Azimuthal mode number dependence of
the vacuum field energy

The scalar potential, χ, in the vacuum region satisfies the harmonic equation:

$$\nabla^2 \chi = 0 \qquad (A-1)$$

In the axisymmetric case, it is

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \chi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \chi}{\partial \theta^2} + \frac{\partial^2 \chi}{\partial z^2} = 0 \qquad (A-2)$$

The general solution is

$$\chi(\vec{r}) = [C\, I_m(kr) + D\, K_m(kr)]\, e^{i(kz + m\theta)} \qquad (A-3)$$

Here, I_m and K_m are the modified Bessel functions, m and k are the azimuthal and axial mode number. C and D are two constants determined by the boundary condition. At the conducting wall surface, the normal component of the perturbed magnetic field should be zero, i.e.

$$\left. \frac{\partial \chi}{\partial r} \right|_{r = R_W} = 0 \qquad (A-4)$$

Hence

$$C\, I_m'(kR_W) + D\, K_m'(kR_W) = 0 \qquad (A-5)$$

Prime is the derivative with respect to the argument. Therefore

$$\frac{D}{C} = - \frac{I_m'(kR_W)}{K_m'(kR_W)} \qquad (A-6)$$

Assuming the normal component of the perturbed magnetic field at the plasma surface is Q_n, we have

$$Q_n = \left. \frac{\partial \chi}{\partial r} \right|_{r=R} = k \left[C \, I_m' \, (kR) + D \, K_m' \, (kR) \right] \qquad \text{(A-7)}$$

Therefore,

$$\chi(R) = \frac{Q_n}{k} \; \frac{I_m \, (kR)}{I_m'(kR)} \; \frac{1 - \dfrac{I_m' \, (kR_W) \, K_m \, (kR)}{I_m \, (kR) \, K_m' \, (kR_W)}}{1 - \dfrac{I_m' \, (kR_W) \, K_m' \, (kR)}{I_m' \, (kR) \, K_m' \, (kR_W)}} \qquad \text{(A-8)}$$

In the long-thin case,

$$kR_W \ll 1 \qquad \text{(A-9)}$$

Using the asymptotic expressions

$$I_m \, (kR) \sim (kR)^m \qquad \text{(A-10)}$$

$$K_m \, (kR) \sim (kR)^{-m} \qquad \text{(A-11)}$$

we have

$$\chi \, (R) = \frac{Q_n}{k} \; \frac{kR}{m} \; \frac{1 + \left(\dfrac{R_W}{R}\right)^{2m}}{1 - \left(\dfrac{R_W}{R}\right)^{2m}} \qquad \text{(A-12)}$$

Thus the vacuum energy is

$$\delta W_v{}^{min} = \int_{\substack{\text{plasma} \\ \text{surface}}} \chi \, \nabla \chi \cdot d\vec{A}$$

$$= 2\pi \int_{-L}^{L} R \left[-\chi \, \frac{\partial \chi}{\partial r} \right]_{r=R} dz$$

$$= \frac{2\pi}{m} \int_{-L}^{L} Q_n^2 \, R^2 \; \frac{1 + \left(\dfrac{R}{R_W}\right)^{2m}}{1 - \left(\dfrac{R}{R_W}\right)^{2m}} \; dz \qquad \text{(A-13)}$$

From Eq. (A-3), (A-10), (A-11) and (A-13), we see that the m-dependence

of the δW_V^{min} is due to harmonic behavior of the $\chi(r)$, the faster

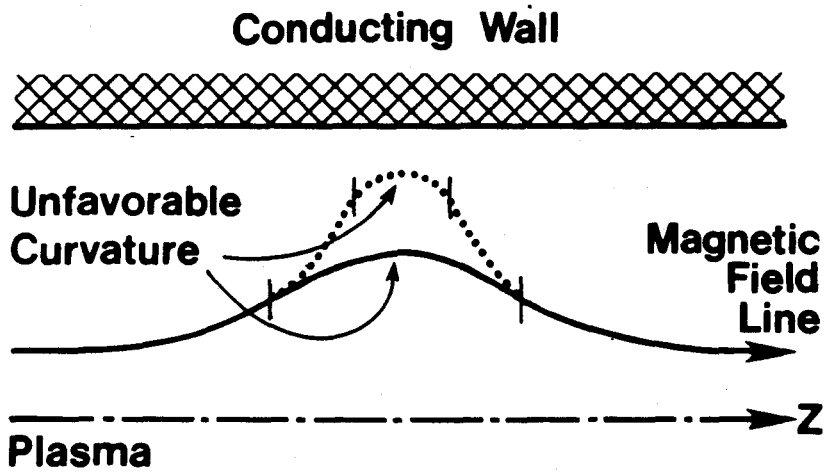the change in azimuthal direction is, the faster the change in radial direction.

Fig. 1 Plasma digs a well and switches a portion of
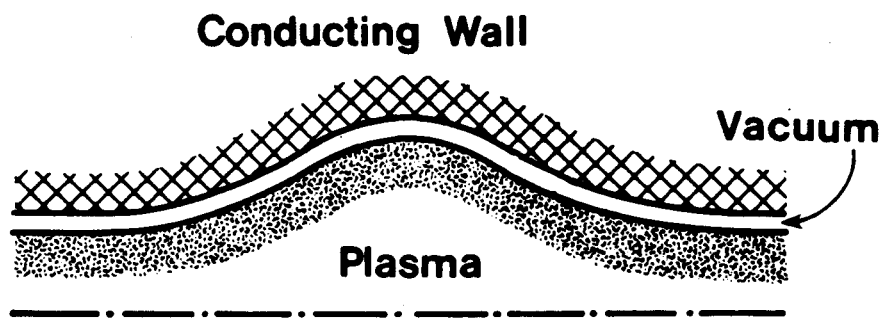unfavorable curvature into favorable curvature.

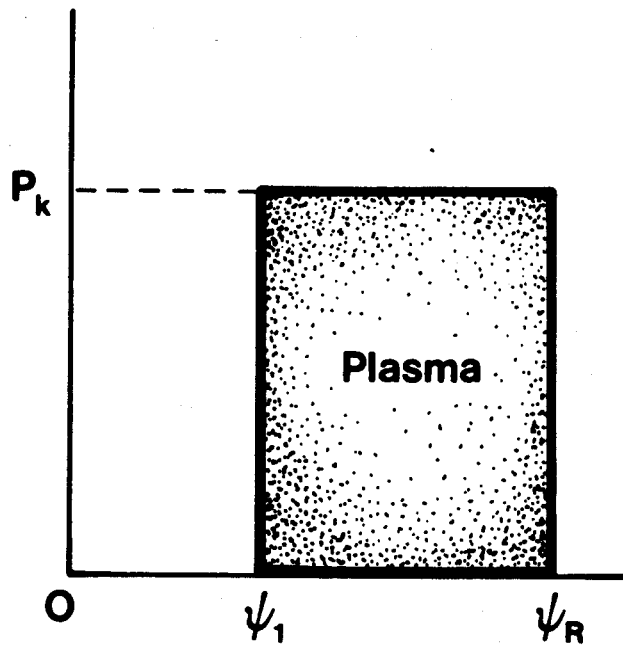Fig. 2 Conducting wall approaches plasma.
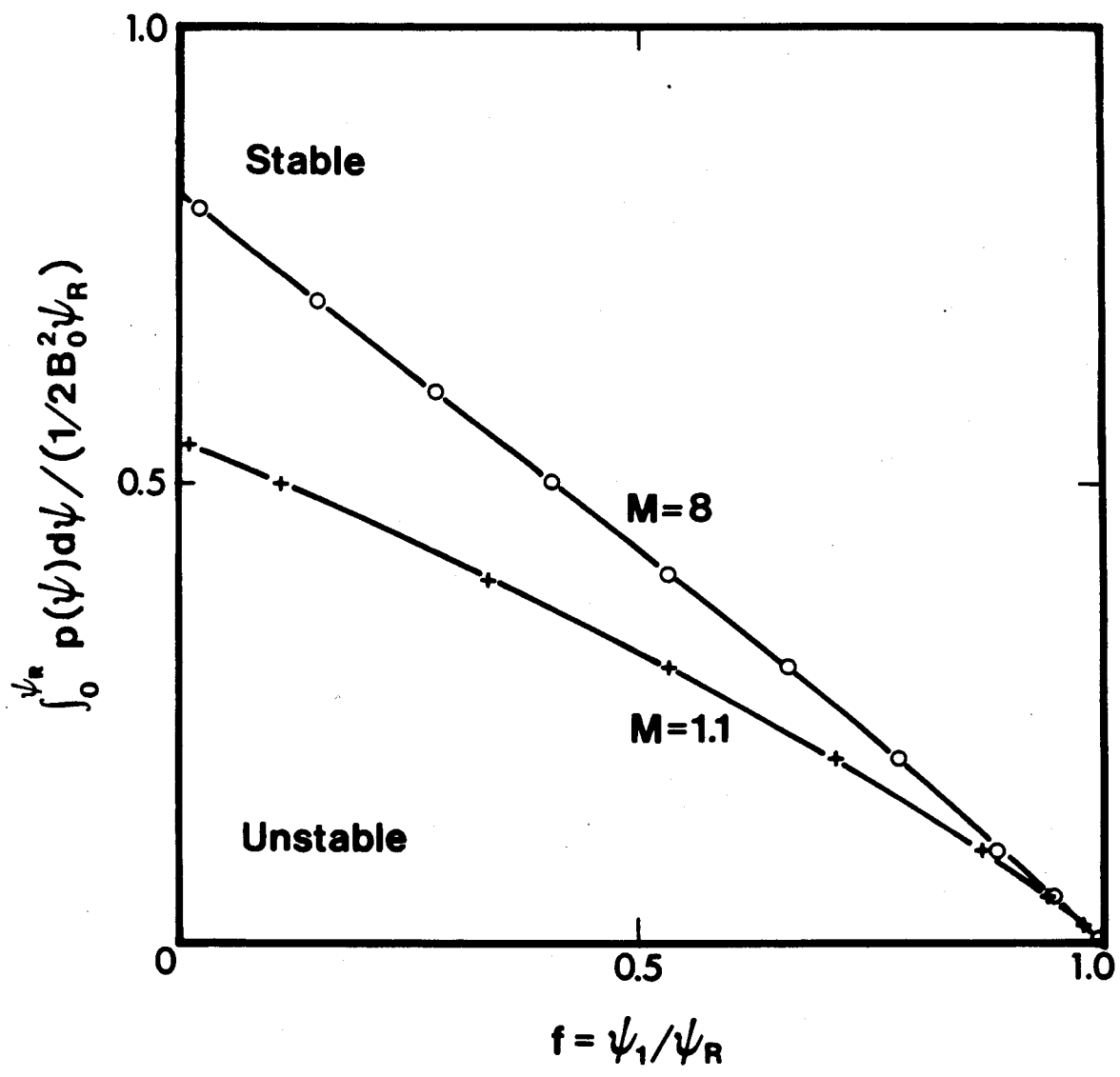
Fig. 3 Two step flat plasma profile.

Fig. 4 Profile effect on wall stabilization

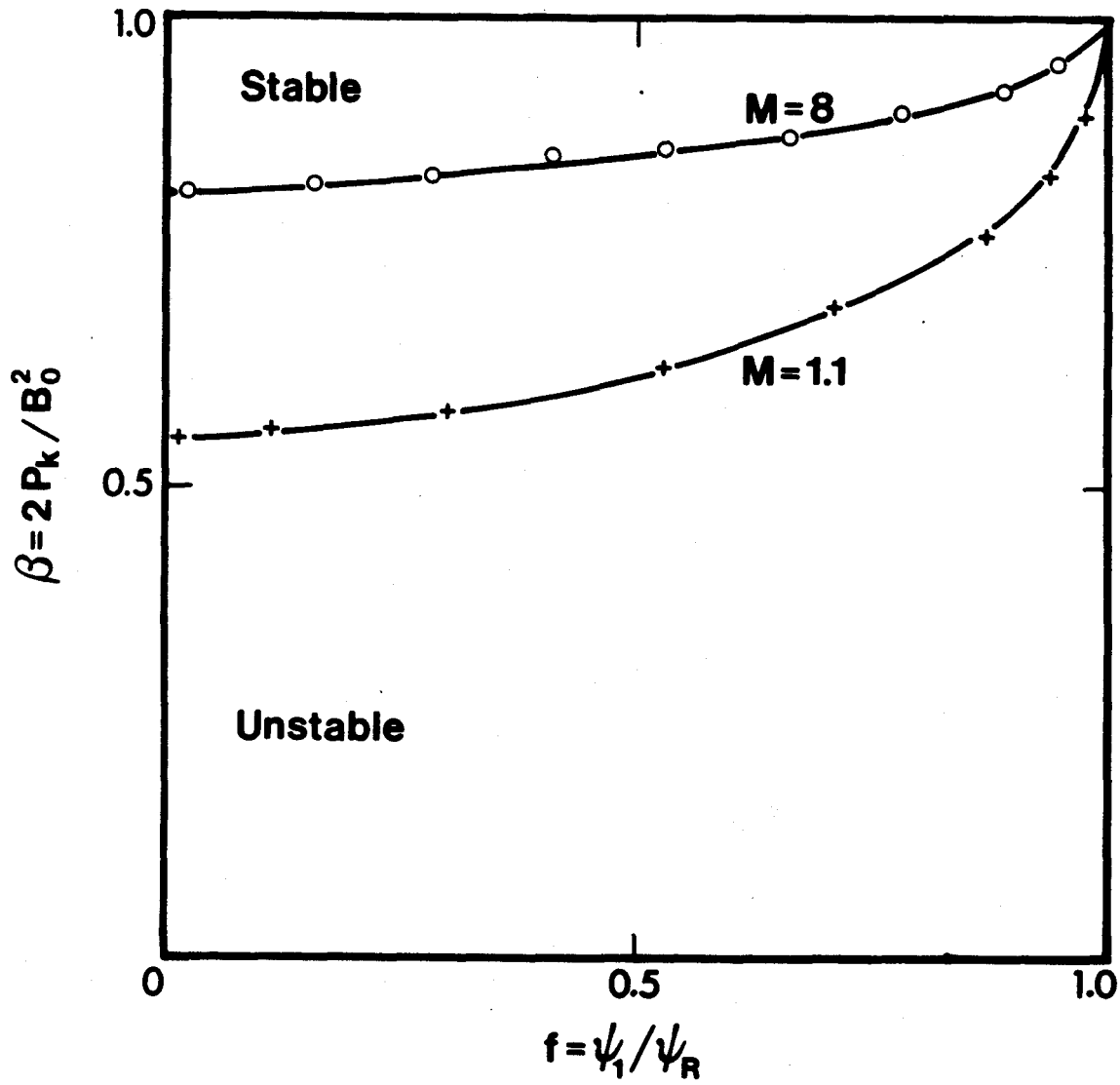(Pressure integral versus hollow size)

Fig. 5 Profile effect on wall stabilization

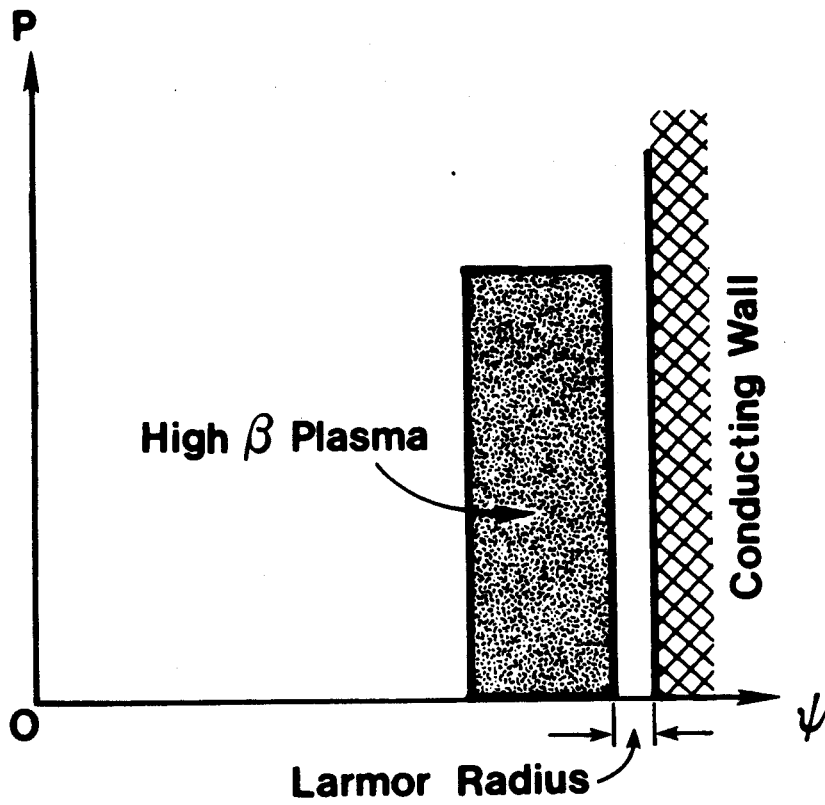(Plasma Beta versus hollow size)

Fig. 6   High β plasma "blanket" stabilization.