# ACTIVE BURN CONTROL OF NEARLY IGNITED PLASMAS

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*Abstract*

The stabilization of thermal runaway in nearly ignited plasmas by actively controlled auxiliary heating is considered. The use of a variable amount of auxiliary heating for burn control can greatly increase the values of the power multiplication factor $Q$ (fusion power/auxiliary heating power) relative to those values permitted with constant auxiliary heating. A one-dimensional calculation is used to determine the maximum allowable deviation from thermal equilibrium as a function of the equilibrium temperature and $Q$. The results are applied to tokamak plasmas. The effect of the auxiliary heating deposition profile upon $Q$ is determined. For fixed deviation from thermal equilibrium, significantly higher values of $Q$ can be obtained with central ion heating relative to those obtained with edge ion heating. Central electron heating has about the same order of effectiveness as edge ion heating. For central temperatures of $\sim$ 25 keV, $Q \sim 25$ can be obtained if the deviation in the ion temperature is held below 15%.

## I. Introduction

It is well known that an ignited plasma can be unstable against temperature perturbations.[1] This instability results from an increase of fusion power production with temperature which is greater than the increase of energy loss.

The stabilizion of the thermonuclear burn by nearly ignited operation with steady state auxiliary heating has been studied. The values of $Q$ (fusion power/auxiliary heating power) which were obtained were found to be relatively modest.[2]

It has also been shown that variable auxiliary heating power may be used for burn stabilization.[3] In this paper, the use of variable auxiliary heating is investigated as a means to obtain higher values of $Q$. A one-dimensional model is employed to determine the tradeoffs between the allowable deviation from equilibrium and $Q$. The results are applied to tokamak plasmas. The effect of the auxiliary heating profile on $Q$ is determined. The use of ion heating is compared with the use of electron heating.

## II. One Dimensional Thermal Equilibrium and Stability Model

The ion and electron power balances are given by

$$\frac{3n}{2}\frac{\partial T_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \chi_i \frac{\partial T_i}{\partial r} - \frac{3n(T_i - T_e)}{2\tau_{ie}} + P_{\alpha i} + P_{ext,i} \qquad (1)$$

and

$$\frac{3n}{2}\frac{\partial T_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r \chi_e \frac{\partial T_e}{\partial r} + \frac{3n(T_i - T_e)}{2\tau_{ie}} + P_{\alpha e} - P_{Br} + P_{ext,e} \qquad (2)$$

where $n$ is the plasma density, $T_e$ and $T_i$ are the electron and ion temperatures, $\chi_e$ and $\chi_i$ are the electron and ion thermal conductivities. $P_{Br}$ is the Bremsstrahlung power density. $P_{ext,e}$ and $P_{ext,i}$ represent the auxiliary heating power imparted to the electrons and ions. Losses due to neutral particles and synchrotron radiation have been neglected. The ohmic input power has also been ignored. $P_{\alpha e}$ and $P_{\alpha i}$ are the alpha particle heating power densities of the electrons and ions. It is assumed that the alpha particles deposit their energy in the flux surface where they are born. This approximation is valid for reactor size tokamaks.[4]

Since the particle dynamics are slower than the thermal dynamics of the plasma, the plasma density in (1) and (2) is taken to be constant in time. A parabolic plasma density profile is assumed throughout.

At equilibrium,

$$\frac{\partial T_e}{\partial t} = \frac{\partial T_i}{\partial t} = 0$$

and (1) and (2) are reduced to a set of nonlinear ordinary differential equations.

The plasma $Q$ or energy multiplication factor is defined by

$$Q = \frac{fusion\ energy\ released}{input\ energy\ to\ plasma} = \frac{\frac{1}{4}\overline{n^2 <\sigma v>}(E_\alpha + E_n)}{\overline{P_{ext,i} + P_{ext,e}}} \qquad (3)$$

Here $E_\alpha$ and $E_n$ are the energy of the alpha particle (3.5 MeV) and the neutron (14.1 MeV) released in the fusion reaction, respectively. The horizontal bars in (3) refer to radial averaging. At ignition $P_{ext,e} = P_{ext,i} = 0$ and $Q = \infty$.

It is assumed that the ion thermal conductivity is described by the neoclassical theory.[5] The electron thermal conductivity is taken to be constant in space and given by

$$\chi_e = 6.5\ 10^{19}\ /m\ s \tag{4}$$

With this value of electron thermal conductivity and with parabolic density and temperature profiles, the electron energy confinement time given by (4) is in agreement with the empirical scaling law[6]

$$\tau_e = \frac{1}{2}\ \tau_E = 1.9\ 10^{-21}\ n_o\, a^2.$$

where $n_o$ is the central plasma density in $m^{-3}$ and $a$ is the plasma minor radius in m.

Thermal equilibrium temperature profiles have been previously determined by following the time evolution of the power balance equations[7]; the assumed initial condition of the plasma, although not in thermal equilibrium, is close to it. This approach is appropiate for thermally stable operation and was motivated by the assumption of trapped ion scaling for the ion thermal conductivity which results in thermally stable operation at relatively low temperatures. However, the power balance equations (1) and (2) result in unstable equilibria for central temperatures $T_{io} < 50$ keV when $\chi_i$ is given by the neoclassical scaling and $\chi_e$ by (4).[8] (Possible thermal stabilization by plasma expansion against an appropiate vertical field[9] has not been considered here). Because the equilibrium is unstable for $T_{io} < 50$ keV, the thermal equilibirum temperature profiles are determined by setting all time derivatives $\frac{\partial}{\partial t} = 0$; for the assumed electron and ion thermal conductivity scalings the temperature profiles are found to be significantly more peaked than parabolic profiles.

The stability analysis is done in the following manner: equations (1) and (2) are replaced by a set of difference equations. The plasma is thus divided into several radial zones. These difference equations are linearized about the equilibrium with respect to the electron and ion temperatures and alpha particle population in each zone. The alpha dynamics are included in the analysis by modelling the alpha particle slowing down using two energy groups in each radial zone. The eigenvalues of the resulting set of linear equations are obtained. The number of eigenvalues is equal to four times the number of zones. If the real part of the largest eigenvalue, $Re(s_{max})$, is positive then the system is unstable. The runaway time, $\tau_{runaway}$, is defined by

$$\tau_{runaway} = \frac{1}{Re(s_{max})} \qquad (5)$$

As the number of zones is increased the number of eigenvalues increases. The additional eigenvalues correspond to modes that are highly oscillatory in radius and are strongly damped. The largest eigenvalues, however, remain approximately constant as the number of zones is increased. Therefore the use of a relatively small number of zones (~ 3) is sufficient to calculate the stability properties of the equilibirium.

Figure 1 shows contours of constant runaway time as a function of the peak ion temperature and $Q$ for steady state heating. A parabolic ion auxiliary heating profile is assumed. Since the runaway time depends upon $\tau_i/\tau_e$, the ratio of the ion energy confinement time to the electron energy confinement time, it is necessary to specify device parameters[8]; the device parameters are $a \sim 0.52$ m, $R \sim 1.35$ m, $B_T \sim 9.5$ T and $I_p \sim 3.5\ 10^6$ A, corresponding to a recent design of an Ignition Test Reactor.[10] As seen in the figure, low values of $Q$ are necessary for the plasma to be stable (corresponding to $\tau_{runaway} \sim \infty$) even at high values of the plasma temperature.[2] Only one unstable mode is found. All eigenvalues are real.

III. Thermal Stabilization by Actively Controlled Auxiliary Heating

Actively controlled auxiliary heating is employed for thermal stabilization as follows: as the plasma heats up due to a positive temperature perturbation, the auxiliary heating power is reduced; the maximum reduction is complete shut-off of the auxiliary heating; it is assumed that the time response of the auxiliary heating control is much more rapid than the characteristic time for thermal runaway. If the temperature excursion is sufficiently small, the reduction in heating power would force the plasma back towards the equilibrium. For given power multiplication $Q$ and ion temperature there is a limited range of temperature excursions where the control system is effective. For fixed $Q$, larger temperature deviations are allowed at higher temperatures where the fusion power production increases more gradually with temperature.

The control of positive temperature perturbations is more demanding than that of negative temperature perturbations. The control of negative temperature perturbations depends only on the availability of adequate heating capability; it may be possible to use the auxiliary heating system provided for the initial plasma heating.

The average fusion power multiplication factor, $Q_{ave}$ is defined by

$$Q_{ave} = \frac{average\ fusion\ energy\ released}{average\ input\ energy\ to\ plasma} = \frac{\frac{1}{4}\ \overline{n^2 <\sigma\ v>_{ave}}\ (E_\alpha + E_n)}{P_{ext,i\ ave} + P_{ext,e\ ave}} \qquad (6)$$

If the system is symmetric (i.e. the maximum allowable negative perturbation is equal to the maximum allowable postive perturbation), then

$$Q_{ave} \sim Q$$

where $Q$ is the equilibrium power multiplication factor from last section. The peak auxiliary heating power for a symmetrical system, which occurs for a negative temperature perturbation, is equal to twice the average heating power.

The maximum controllable perturbation is calculated by solving the time dependent linearized system described in the previuos section. The linearized model is diagonalyzed and the perturbation that results in $\frac{\partial}{\partial t} = 0$ for the unstable mode is found when the auxiliary heating is removed. This is the maximum allowable perturbation, as a slightly larger perturbation would result in a runaway solution. The solution has been checked by using a nonlinear 1-D code.

The initial perturbation has been taken to be a multiple of the eigenvector corresponding to the largest eigenvalue obtained from the linear system; if the system is close to equilibrium and allowed to evolve, the asymptotic behavior corresponds to that of the mode with the largest growth rate. As pointed out in the previous section, there is only one growing mode. Furthermore, the fastest growing mode is such that the profiles remain approximately constant. Therefore, the relative size of the perturbation can be defined as $\delta T/T$, with $T$ being either the electron or the ion temperature. For consistency, the relative size of the perturbation has been defined as $\delta T_{io}/T_{io}$, where $T_{io}$ is the central ion temperature.

Figures 2 and 3 show contours of constant value of the allowable perturbation as a function of the plasma $Q$ and the central ion temperature for auxiliary heating of the ions. Figure 2 has been drawn for centrally peaked ion heating while Figure 3 is for edge ion heating. Central heating refers to a parabolic heating profile while edge heating refers to a heating profile with a radial dependence given by

$$P_{ext} \sim r^2 \left(1 - \frac{r^2}{a^2}\right)$$

The allowable $Q$ at fixed ion temperature for comparable dynamic ranges of the control system is about a factor of 1.5 larger for central heating than it is for edge heating. For a 10% dynamic range, central heating results in $Q \sim 17$ at $T_{io} \sim 15$ keV, while edge heating results in $Q \sim 12$. The scaling of the allowable perturbation with $Q$ approximately goes as

$$\left( \frac{\delta T_{io}}{T_{io}} \right)_{max} \sim \frac{1}{Q}$$

Figures 4 and 5 show contours of constant value of the maximum allowable value of $\delta T_{io}/T_{io}$ as a function of $Q$ and the peak ion temperature for auxiliary heating of the electrons. Figure 4 and 5 are for centrally peaked electron heating and edge electron heating, respectively.

Peaked electron heating and edge ion heating result in similar dependence of $\delta T_{io}/T_{io}$ upon $Q$.

Figures 2-5 have been drawn for the same device parameters as were used for Figure 1. However, the results are approximately independent of these parameters and are generally applicable to tokamak reactors with $\tau_i/\tau_e > 1$;[8] for example, for $a \sim 1.2$ m, $R \sim 5$ m and $B_T \sim 6$ T, for fixed maximum allowable perturbation, the value of $Q$ in Figures 2-5 change by < 5% relative to those calculated using the parameters of the Ignition Test Reactor[10] ($a \sim 0.52$ m, $R \sim 1.35$ m and $B_T \sim 9.5$ T).

The desired operating point can be reached by a startup heating scenario which progresses as jumps between thermally stabilized equilibrium points. This type of startup could increase the accuracy of reaching the plasma parameters of the operating point. The trajectory of the startup scenario can be such that the control requirements of the intermediate points are less than the requirements of the final state.

As the ion temperature increases and/or as the $Q$ decreases, not only does the allowable perturbation for a controllable system increase, but the time response requirements are also reduced. This can be seen from Figure 1, as $\tau_{runaway}$ of the equilibrium obtained in the previous section does not depend on whether the system is driven by steady-state or actively-controlled heating.

The achievement of sufficiently small characteristic times (< $\tau_{runaway}$) for change of the auxiliary heating power should be possible with both neutral beams and RF heating. The neutral beam slowing down time is in general much shorter than the runaway time. The most demanding requirement for the effective use of variable auxiliary heating for burn control is the development of sufficiently sensitive diagnostics.

## IV. Conclusions

In contrast to steady auxiliary heating, the use of variable auxiliary heating for thermal stability control of nearly ignited tokamak plasmas allows the possibility of high $Q$ ($Q > 20$) operation over a wide temperature range. Tradeoffs between $Q$ and the maximum allowable temperature fluctuation have been determined by means of a one-dimensional calculation. The use of centrally peaked ion heating offers significant advantages over edge heating and central electron heating. For a centrally heated tokamak plasma described by neoclassical transport scaling for the ions and $\tau \sim na^2$ empirical scaling for the electrons, $Q \sim 25$ can be obtained at $T_{io} \sim 25$ keV if $\delta T_{io}/T_{io} < 15\%$.

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## List of Figures

Figure 1. $\tau_{runaway}$ as a function of the central ion temperature and the power multiplication factor $Q$ for the Ignition Test Reactor. $\tau_{runaway} = \infty$ is the boundary of the stable region

Figure 2. Maximum allowable deviation from equilibrium as a function of the central ion temperature and the power multiplication factor $Q$ for centrally peaked auxiliary heating of the ions

Figure 3. Same as Figure 2 but for edge auxiliary heating of the ions

Figure 4. Maximum allowable deviation from equilibrium as a function of the central ion temperature and the power multiplication factor $Q$ for centrally peaked auxiliary heating of the electrons

Figure 5. Same as Figure 4 but for edge auxiliary heating of the electrons

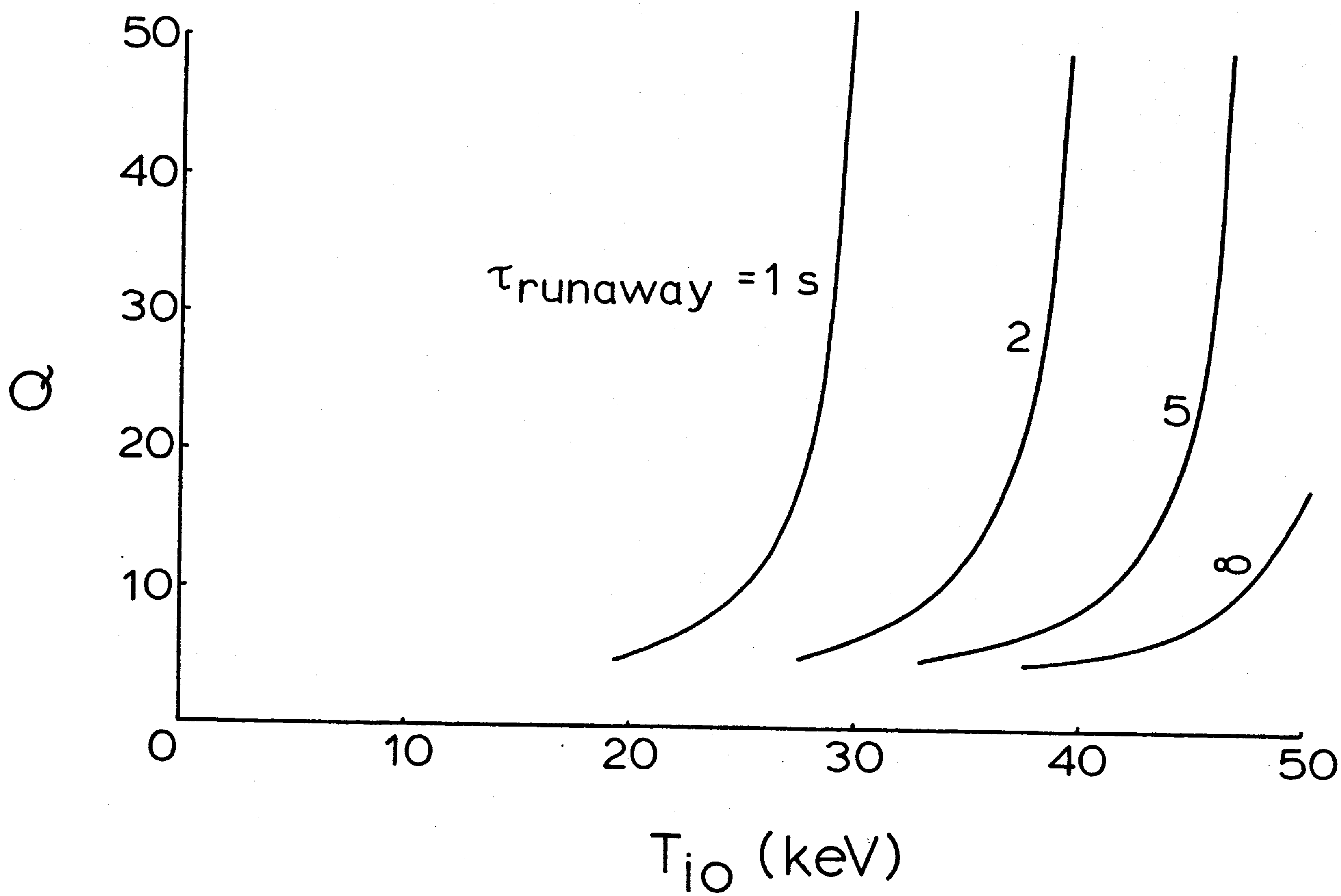


Figure 1

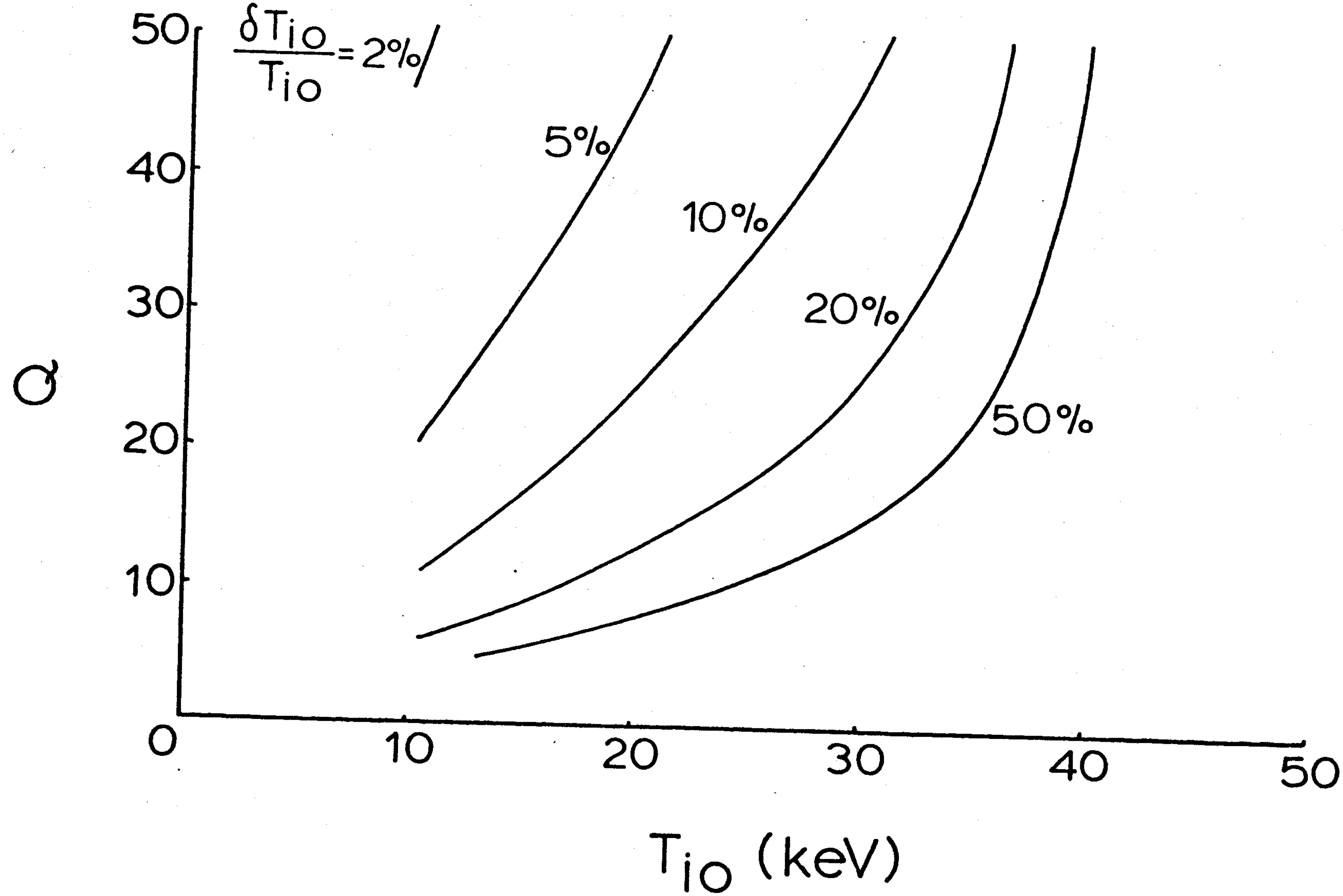


Figure 2

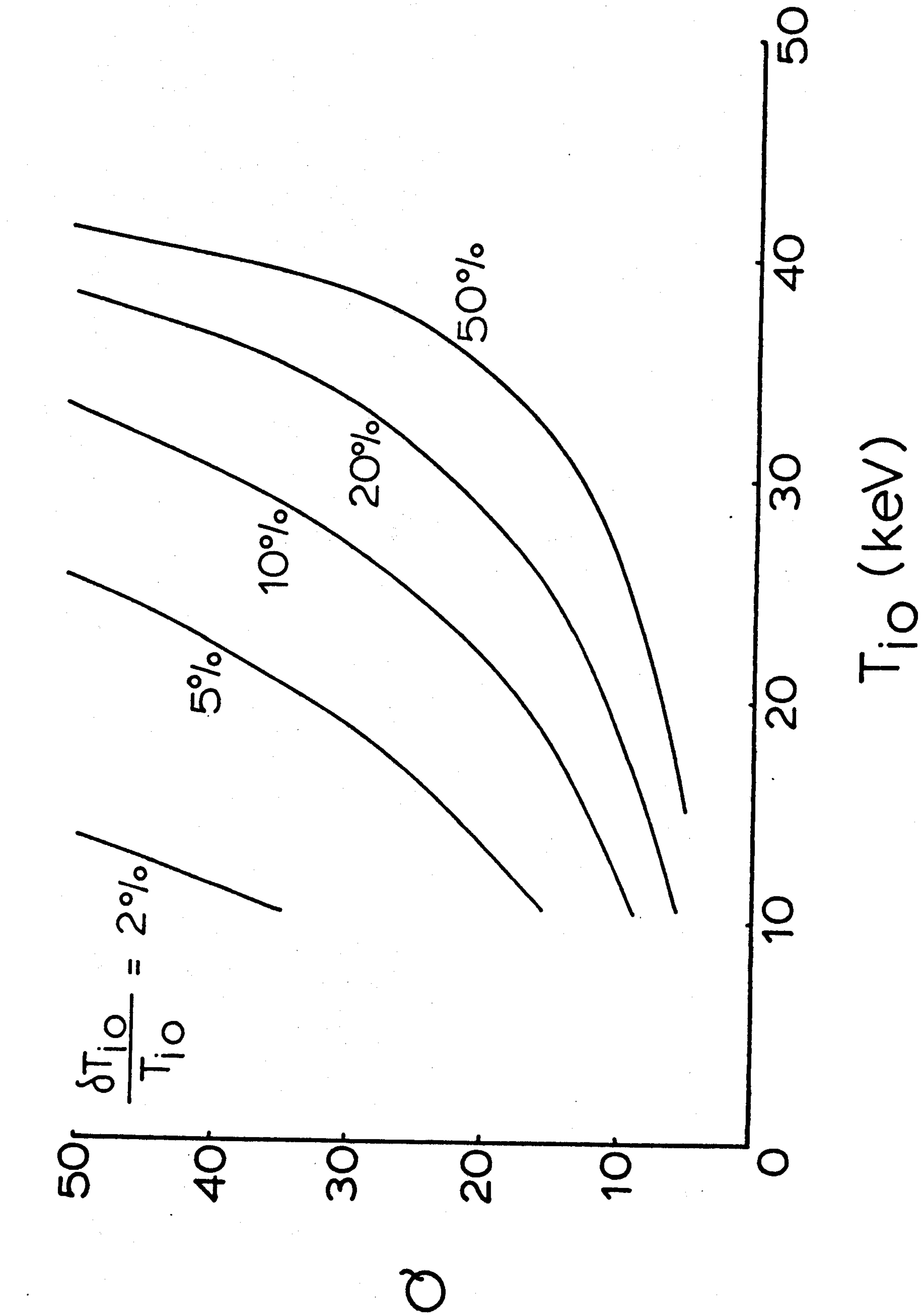
Q
50
40
30
20
10
0
10
20
30
40
50
$\frac{\delta T_{io}}{T_{io}}$ = 2%
5%
10%
20%
50%
$T_{io}$ (keV)

Figure 3

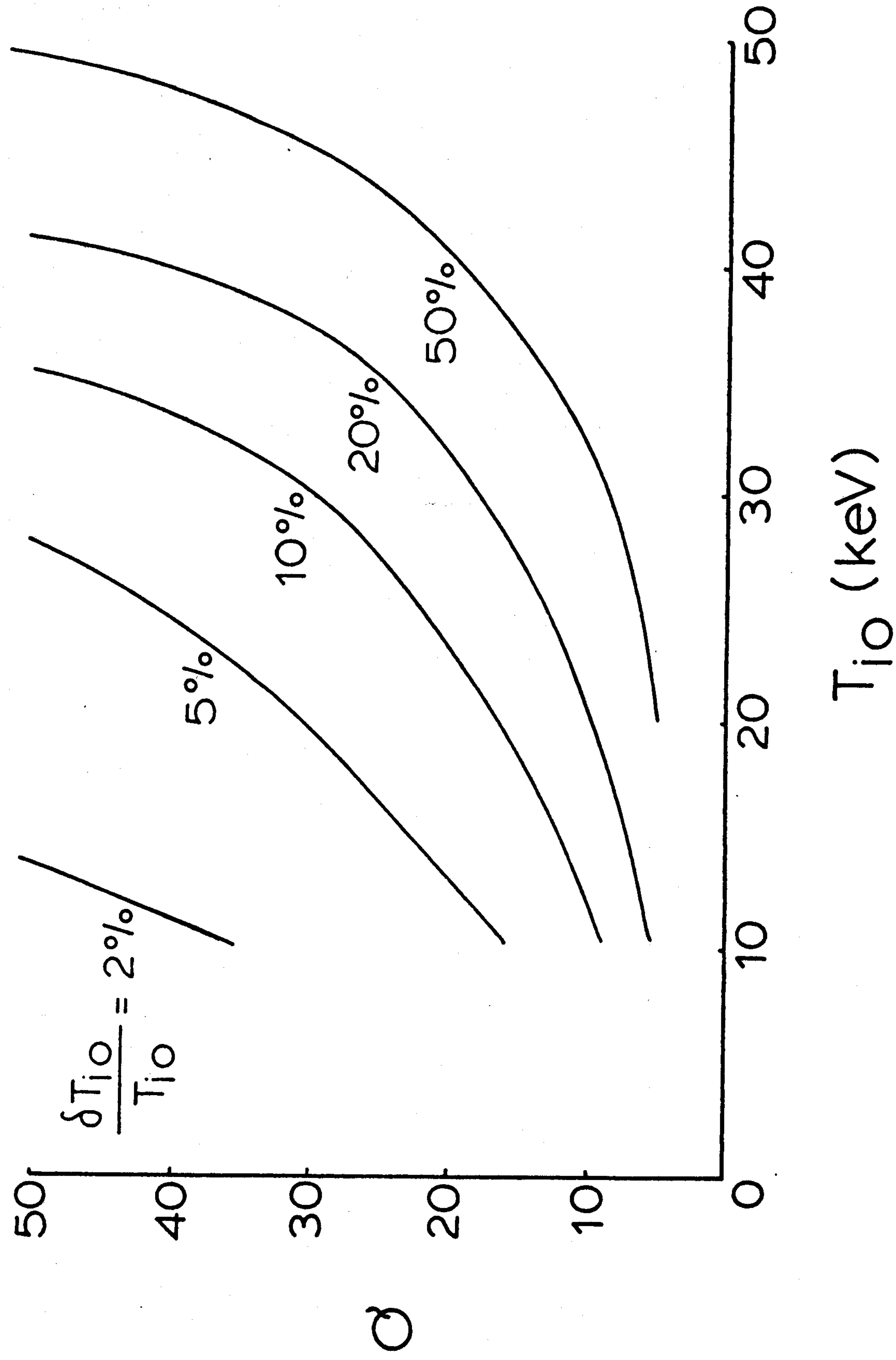


Figure 4

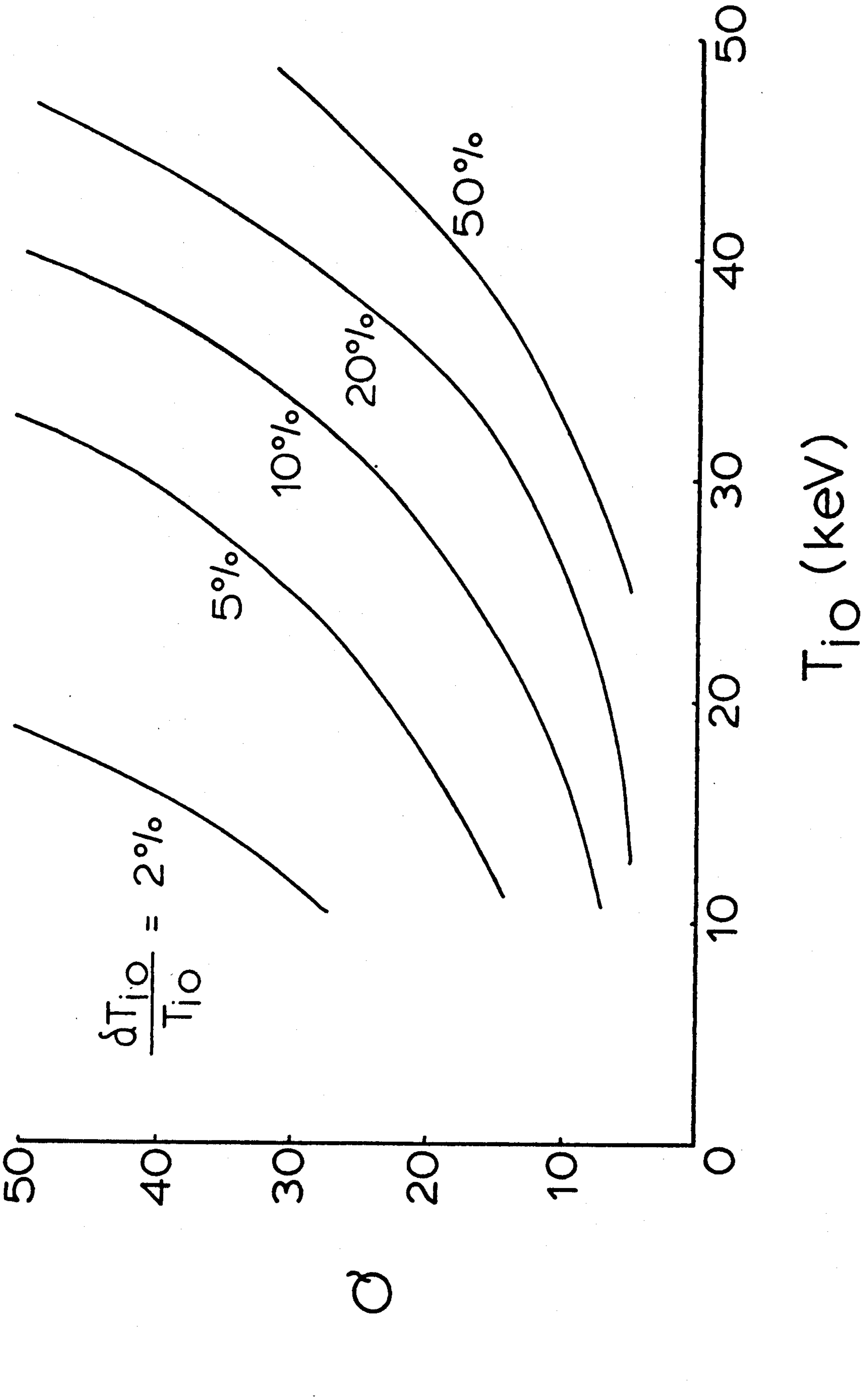


Figure 5