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Density Profile Measurements in LDX using Microwave Reflectometry

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Abstract: The levitated dipole experiment (LDX) has a sharply peaked, stationary plasma density profile as shown recently by measurements with a four-channel microwave interferometer. More precision in profile measurement is needed to fully explore the stability of high beta LDX plasmas. A 4-8 GHz O-mode scanning reflectometer is being implemented that will probe the entire LDX density profile (the density in LDX varies in the range of 2-8x10^{17} m^{-3}). This will add to information obtained from the four channels of the interferometer array and will be the first time a continuous density profile measurement is achieved in a levitating dipole plasma. This is important because it is needed to confirm predictions of a $1/r^4$ density profile obtained by Abel inversion of the interferometer data and to obtain the exact location of the peak density in LDX for the first time.

1. Introduction

The levitating dipole experiment (LDX) is an alternative scheme for confinement of high beta plasma inspired by the simple dipole magnetic field of planets. A dipole field is obtained by levitating a superconducting coil (F-coil) inside a large vacuum vessel so that a significant fraction of the field lines are closed and particle loss along field lines is reduced. The plasma is heated and sustained by 28 kW of electron cyclotron resonance heating (ECRH) at 2.45, 6.4, 10.5, and 28 GHz. In a dipole field inward turbulence induced transport causes a centrally peaked plasma density profile in what is called a turbulent pinch.[1] This is consistent with an equal number of particles per magnetic flux tube, $N = \langle n \rangle \delta V$ where n is the density and δV is the differential volume of a tube of magnetic flux. This condition on particles per flux tube is a stability condition for the centrifugal interchange stability. Because the magnetic field is higher going inward toward the F-coil, the plasma groups near the inner closed flux surface of the plasma. A four-channel microwave interferometer currently installed on LDX has shown that the line integrated density decreases radially across the four interferometer chords. By doing an Abel inversion on this data, it has been estimated that the plasma has a $1/r^4$ density profile. In comparison, the line density seems to be uniform across the four interferometer chords when the F-coil is supported because there are no longer any closed field lines and the plasma escapes to the vacuum chamber wall. This comparison is shown in Figure 1. In order to get more detail of the density profile shape a 4-8 GHz sweep O-mode reflectometer is being developed to measure densities between 2x10^{17} m^{-3} and 8x10^{17} m^{-3}.

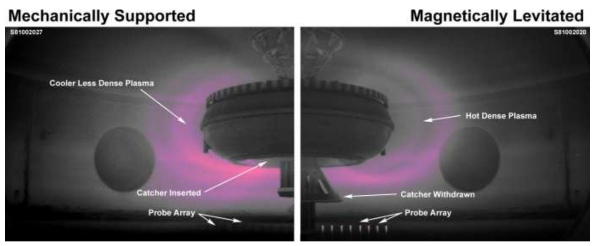


Figure 1: Comparison of plasma between mechanically supported and magnetically levitated runs.

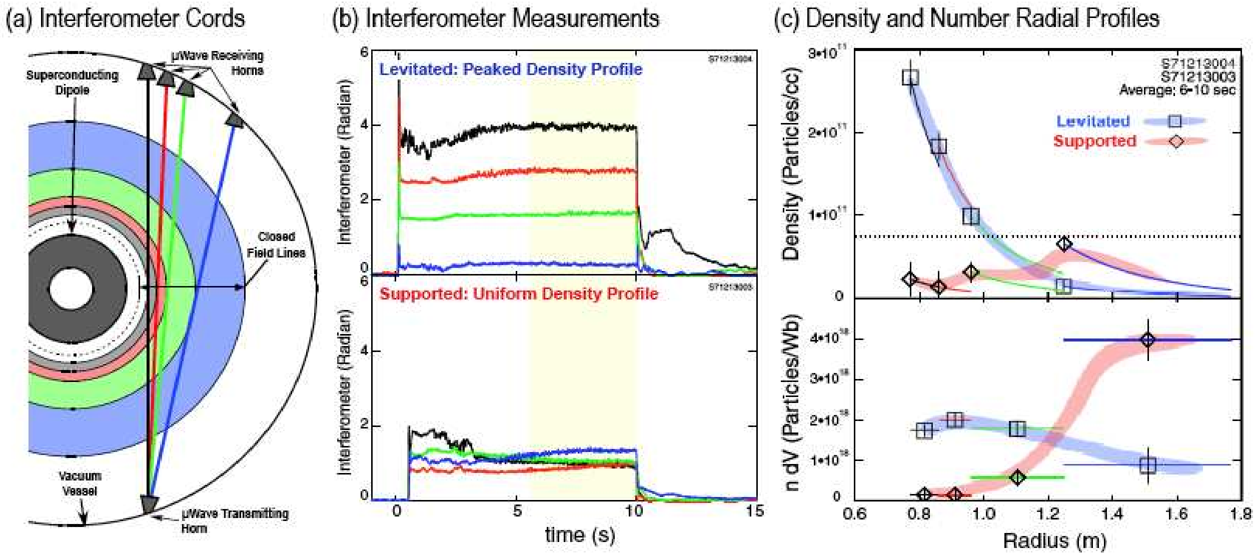


Figure 2: Interferometer measurements show peaked density profile. a) The four interferometer chords pass through different radial regions of the plasma **b)** When the dipole is levitated the line density measurements increase for the central chords and decrease in the outer region **c)** Reconstructed plasma density increases by nearly an order of magnitude near R~0.8m. The particle number per unit magnetic flux, $\langle n \rangle \delta V$, is hollow with a supported dipole and uniform when the dipole is levitated.

Ref: Boxer et al.,Nature Physics, (2010).

Frequency swept reflectometery would be a practical density diagnostic tool because it is easy to implement, especially when the low densities of LDX allow frequencies in the GHz range to be used. Reflectometry is non pertabative because it uses powers on the order of 1 W. It provides easy access to the interior of the plasma because of the simple dispersion relation for O-mode. This diagnostic would also allow implementation of a similar fluctuation reflectometer that could use most of the same hardware and measure with high accuracy the fluctuations in density caused by turbulence. One challenge in implementing the reflectometer is that it must send its signal into the plasma in O-mode. In LDX this is fairly simple because of the dipole field geometry. Another complication comes from the overlap in bandwidth with ECRH heating sources. This paper will discuss the methods used and challenges faced in implementing the reflectometer.

2. Four Channel Interferometer

The four channel interferometer has four receiving horns that pick up the microwave signal transmitted across four chords of the plasma at different radial positions. The phase shift along each chord is determined by the line integrated density along the chord. The implication of this is that if the phase shift along one interferometer chord exceeds that of the next chord out radially, then the inner density must be higher.[2]

Figure 2a shows the orientation of the interferometer chords within LDX. Figure 2b shows that the phase shift is clearly greater for chords that are farther into the plasma when the F-coil is levitated. On the other hand, when the F-coil is not levitated the interferometers do not show such a sharply peaked density profile. The density profile can be approximated by Abel inversion as shown in Figure 2c, but this approximation is only based on the density at four points. A new type of measurement is needed to get a continuous density profile. The reflectometer will be used for this purpose.

3. Reflectometer Theory

Reflectometry is based on the reflection of an electromagnetic wave at a cutoff layer where the index of refraction goes to zero. Using the cold-plasma approximation the dispersion relation of an electromagnetic wave propagating perpendicular to the magnetic field can be derived as[3]

$$N_{\perp}^{2} - N_{\perp}^{2}\left(\frac{\kappa_R \kappa_L}{\kappa_{\perp}} + \kappa_{||}\right) + \frac{\kappa_R \kappa_L \kappa_{||}}{\kappa_{\perp}} = 0 \qquad (1)$$

where $N_{\perp}$ is the index of refraction in the mode propagating perpendicular to the field and κ_L, κ_R, $\kappa_{||}$, and $\kappa_{\perp}$ are the wavenumbers for the left, right, parallel, and perpendicular modes respectively.

This dispersion relation can be solved for the index of refraction which will be different for different for different polarizations of the incident wave. For the O-mode (polarization parallel to the magnetic field) the index of refraction is

$$N_{\perp}^{2} = 1 - \frac{\omega_p^2}{\omega^2} \qquad (2)$$

where ω_p is the plasma frequency. In the O-mode the index of refraction will go to zero and the wave will be reflected when the frequency of the wave is equal to the local plasma frequency. For the X-mode (polarization perpendicular to the magnetic field) the index of refraction is

$$N_{\perp}^{2} = 1 - \frac{\omega_p^2(\omega^2 - \omega_p^2)}{\omega^2(\omega^2 - \omega_h^2)} \qquad (3)$$

where ω_h is the upper hybrid frequency. This mode is more complicated and has a resonance at ω_h and two cutoffs at the frequencies

$$\omega_R = \frac{1}{2}\left[\omega_C + ({\omega_C}^2 + 4{\omega_P}^2)^{1/2}\right] \qquad (4)$$

$$\omega_L = \frac{1}{2}\left[-\omega_C + ({\omega_C}^2 + 4{\omega_P}^2)^{1/2}\right] \qquad (5)$$

The reflectometer for LDX is meant to measure the density which can be accomplished most easily by measuring the plasma frequency by operating the reflectometer in O-mode.

The distance that an O-mode EM wave travels into the plasma, r, can be found from the time of flight, τ, and the scanning frequency, f, by the integral[3]

$$r(f) = \frac{c}{\pi}\int_0^f \tau(f)\frac{d\eta}{\sqrt{f^2 - \eta^2}} \qquad (6)$$

The time of flight will cause a difference in phase between the signal that is reflected off the plasma and a reference signal. These signals are mixed and get a beat frequency, f_B, which can easily be related to the time of flight.

$$\tau(f) = f_B(f)\frac{1}{df/dt} \qquad (7)$$

The scanning frequency must be changing in time with a rate df/dt so that the signal returning from the plasma has a different frequency than the reference signal and a beat frequency can be created. By comparing equation 6 and equation 7 it is possible to relate the observed beat frequency to the distance the wave travelled into the plasma

$$f_B(f) = \frac{2r(f)}{c}\frac{df}{dt} \qquad (8)$$

The function r(f) can be inverted to get the data of interest, the plasma density profile, $\omega_p(r)$.

4. Sweep Mode Reflectometer Design and Operation

4.1 Design

A reflectometer operating in the O-mode is now installed on LDX. It uses a YIG oscillator controlled by a function generator to sweep linearly in frequency from 4-8 GHz. The signal is split so that some of the signal can go into the LO port of a mixer as a reference signal. The rest is sent into the plasma by a log periodic antenna. Since this is an O-mode reflectometer, the wave is reflected when it reaches the plasma density at which the index of refraction $N_\perp$ goes to zero. This frequency range is capable of probing plasma densities between $2x10^{17}$ m^{-3} and $8x10^{17}$ m^{-3}. The reflected wave is received by a separate receiving antenna and this signal is filtered by both a high and low pass filter so that only frequencies between 4 and 8 GHz get through. This prevents the reflectometer from picking up signal from the ECRH sources. There is also a notch filter at 6.4 GHz to filter out the ECRH at that frequency. After this filtering the signal goes into the RF port of the mixer. This layout is shown in Figure 3. The wave coming out of the mixer has a beat frequency, f_B, equal to the difference in frequency between the signal received from the plasma and the reference signal.

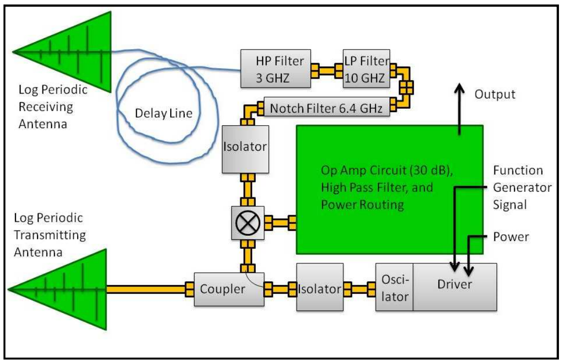
By looking at this beat frequency as a function of the frequency scanned by the YIG oscillator the reflection at the plasma density can be mapped out for the whole range of frequencies using equation 8. This will give a density profile within the full range of the reflectometer ($2\text{-}8x10^{17}$ m^{-3}).

The signal that went into the plasma is much weaker than the reference signal because it decays over the distance it travels according to the radar equation

$$P_R \propto \frac{P_T}{R^4} \tag{9}$$

where P_T and P_R are the transmitted and received powers and R is the distance the beam travels before reflection. For this reason the received signal must be amplified by more than 30 dB before it is digitized and recorded. One problem with this is that the noise is of a similar intensity to the signal. Luckily most of the noise comes from reflections within the radio frequency components and has a slightly lower frequency than the signal received from the plasma. By adding a delay line the signal of interest goes to an even higher frequency. Then most of the noise from reflections can be filtered out with a high pass filter in the op-amp circuit.

(a) Reflectometer Schematic



(b) Internal electronics

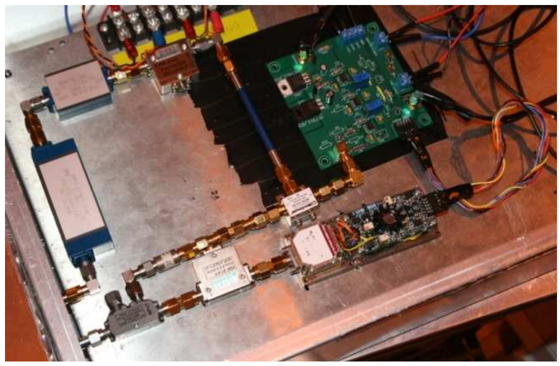


Figure 3: a) Schematic of the reflectometer design described in this section **b)** Photo of the actual layout within the device

These reflections within the microwave components are also cut down by a couple of isolators. One isolator protects the YIG oscillator from these reflections because they could potentially destabilize it. The other prevents signals from leaving the mixer through the RF port and going the wrong direction towards the receiving antenna. This leakthrough current is inherent to mixers when the LO power is much higher than the RF power. In this case the LO power is approximately 40 dB higher because the received power from the plasma is so low. This isolator cuts down most of this signal so that reflections off the filters and receiving antenna do not mask the actual received signal.

Another possible signal that can mask the received signal is the direct coupling of the two antennas. In this case a delay line is of no use, because the delay line could only be placed between antenna and the connection to the reflectometer and this would increase the frequency of the direct communication signal too. A piece of microwave frequency foam absorber cut down the coupling significantly but it was still necessary to take a reference signal without any reflective surface and subtract that background measurement from every shot. This was effective at isolating the beat frequency of interest from those caused by the coupling of the two antennas. The antenna casing including both antennas and the foam absorber between them is shown in figure 4.

4.2 Data Analysis

The amplified signal can be analyzed with a short time FFT to obtain a peak beat frequency as a function of time. With a peaked density profile the beat frequency increases with increasing scanning frequency. When the scanning frequency is high enough such that none of the plasma has a high enough density to reflect it, the beam is reflected from the F-Coil which is made of stainless steel and will reflect any wave in this frequency range.

Each sweep through the full frequency range takes 80 milliseconds as set by the specifications of the YIG oscillator, but each data shot is more than 20 seconds long. This gives over 250 sweeps

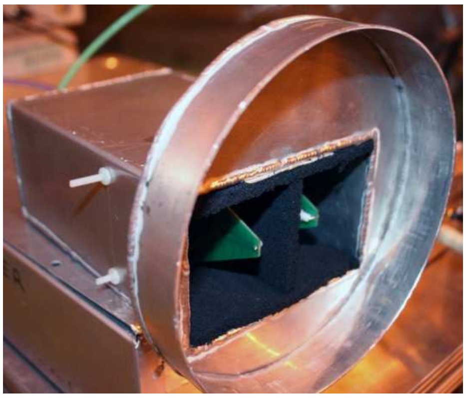


Figure 4: Photo of the antenna casing with both log periodic antennas and the Echosorb RF absorbing foam between them.

per data shot to work with in the frequency analysis.

In order to get the beat frequency as it changes in time, a short time Fourier transform is needed. But since the shot is 20 seconds long it would take too long to analyze the whole shot. Instead, individual sweeps are analyzed with Matlab's Spectrogram function. This gives a plot of intensity as a function of frequency and time as shown in Figure 5. This plot can be analyzed for a single sweep or averaged over several sweeps. At this stage the spectrogram plot of the background signal can easily be subtracted to get a spectrogram plot that is a difference in intensity from the background.

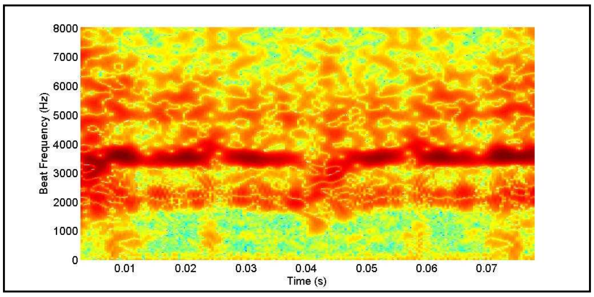


Figure 5: Spectrogram of a shot with a reflecting plate 2 meters from the antennas. The plate shows up as a red line (highest intensity) at 3.5 kHz.

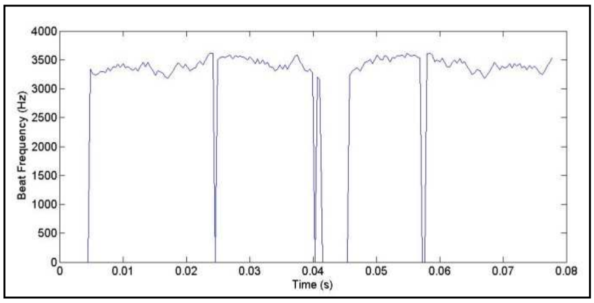
One challenge of using a short time FFT is picking the right size FFT window to use. An FFT window is a short section of the signal that is repeated in time in order to do an ordinary FFT. The intensity has to be multiplied by a window function in order to reduce the signal at the edge of the window and make the signal continuous at the discontinuities between repeated segments. In this case a Hamming window was used which has the functional form of a cosine. In splitting the original signal up into windows many parameters can be controlled. The size of the window determines how much of the signal will be averaged over to get the frequency spectrum in that window. This must be made large enough to include a significant amount of the wave to obtain the frequencies but small enough to avoid averaging over discontinuities such as the one that occurs when the sweep goes through 6.4 GHz where the notch filter cuts out the signal. Another important parameter is the overlap of the windows. If the overlap is made to be a large fraction of the window size then only a small amount of data is unique to each window. This allows many windows to fit into one sweep and shortens time resolution.

The data needed to get the distance to the reflecting layer can be obtained by finding the frequency with the maximum intensity for each point in time. There are often stray signals in the spectrogram but the correct signal can be picked out by visual inspection. In order for the code to find the correct signal a start frequency is entered and the code follows that signal continuously in time. This method gets a maximum beat frequency of the signal for each time as shown in Figure 6a.

Figure 6a has regions where the signal is lost for a short period of time. These periods of weak signal are caused by discontinuities in the sweep. For example the RF port of the mixer receives no signal when the frequency passes through 6.4 GHz and is filtered out by the notch filter. The control signal for the sweep is a triangle wave to prevent any frequency range from being lost by discontinuities. The discontinuity at the bottom of the sweep causes some low frequencies to be lost in the up sweep. This can be seen in the first 5 ms of the sweep. Similarly, the discontinuity at the top of the sweep causes some high frequencies to be lost in the down sweep. This can be seen 4-4.5 ms into the sweep. By averaging the up sweep with the down sweep a maximum beat frequency can be obtained for the whole scanning frequency range as shown in Figure 6b.

This plot can be directly related to the changing reflection distance as higher scanning frequencies travel farther into the plasma before cutoff.

(a)



(b)

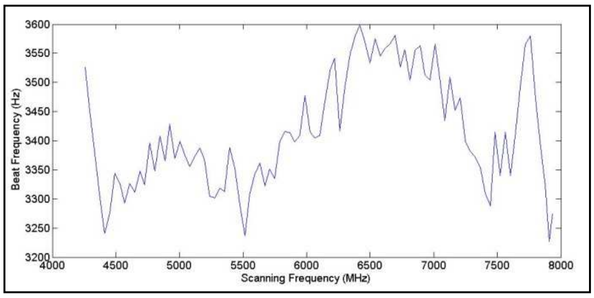


Figure 6: a) Frequency with maximum intensity of the spectrogram versus time **b)** Maximum frequency after the up sweep and down sweep have been averaged to plot versus scanning frequency

4.3 Calibration

The YIG oscillator had to be calibrated to make sure the frequency sweep was actually linear with sweep voltage. This was done with a frequency counter which measured the frequency output for voltages applied with a variable voltage power supply. The results of this calibration show that frequency output from the oscillator is to high accuracy completely linear.

The beat frequency of the reflectometer needed to be calibrated to a predetermined reflection distance to make sure that the linearity of equation 8 held true and to find the constant offset added into the equation by the presence of delay lines. The delay lines add an additional distance for the waves to travel that is internal to the reflectometer.

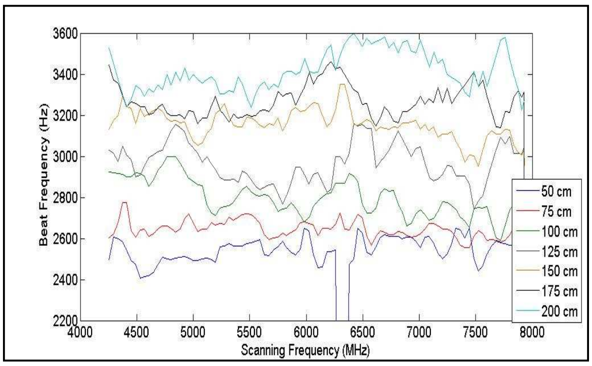
This calibration was done by pointing the antennas at a large steel reflecting sheet. After all the averaging described in the data analysis section a maximum beat frequency versus scanning frequency plot was obtained for each distance of the reflecting plate. Figure 7a shows all of these maximum frequency plots superimposed. This shows that the signals are noisy and even with plates separated by 25 cm the signals overlap for certain frequencies. Though this noise has not been tracked down it is probably due to the response of the YIG to the control signal. The YIG specifications say there will be residual noise of ±10 kHz but the additional noise could be from the discontinuities in the sweep caused by the triangle wave shape

Even with this noisy data linear fits of the data are obviously distinguishable from one another. The linear fits are approximately flat at the average beat frequency for each reflection distance. Figure 7b shows a plot of this average beat frequency against reflection distance. This data provides an empirical approximation for equation 8.

$$f_B(r) = 2220 \text{ Hz} + \left(600 \text{ Hz / m}\right) r \qquad (10)$$

The constant offset of 2220 Hz implies there is 3.7 m of extra distance travelled within the delay lines and the reflectometer itself. This distance is the extra difference in path length between the LO signal and the RF signal besides the path difference caused by going into the plasma. This is also in correspondence with the observation of a strong signal from the coupling of the antennas at around 2200 Hz. The slope differs from the expected value by 10%. This could be explained by inaccuracy of the YIG oscillator which will be discussed in the conclusions sections.

(a)



(b)

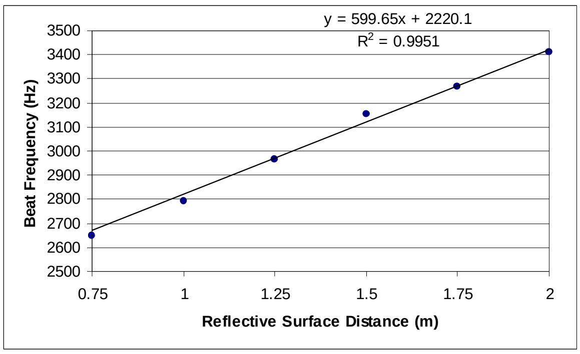


Figure 7: a) Maximum beat frequency versus scanning frequency for different distances of the reflecting plate away from the antenna. **b)** Average

5. Fluctuation Reflectometry

Reflectometry is also useful in making fluctuation measurements to understand the turbulent modes present in the plasma. Turbulence will cause density fluctuations. By reflecting off a constant density fluctuations will be detected as a fluctuating distance to the reflecting layer.

The reflectometer was set up to have a second Op-Amp circuit that could be switched in to make density fluctuation measurements. Instead of sweeping through a range of frequencies a set voltage is applied to the oscillator which outputs a constant frequency. This frequency will reflect when the density increases to the point that the plasma frequency is equal to the frequency of the oscillator. The reflected wave will have a phase difference based on the additional distance travelled than the reference signal. When the phase shifted wave is mixed with the reference signal of the same frequency, a constant DC offset is output from the IF port.

If there are fluctuations in the distance to the reflecting layer caused by fluctuations in the density profile this will cause a fluctuation in the phase shift which will show up as a fluctuation in the DC offset. This fluctuation can be recorded using the same data processing system and the frequency shows the frequency of density perturbations.

The only difference in the circuit is that the fluctuation circuit does not have any high pass filters. The fluctuations are predicted to be between 0 and 4000 Hz with a richer spectrum in the low frequency range. One fluctuation expected is a quasi coherent mode caused by the rotation of the plasma at around 1 kHz. The high pass filtering needed for the swept reflectometer would eliminate a lot of the low frequency part of the fluctuation spectrum.

The reflectometer was calibrated for fluctuation measurements by pointing it at a large steel bladed fan. The short time FFT in Figure 8 shows a frequency peak at the frequency of the fan blade along with peaks at its harmonics caused by the non-sinusoidal shape of the oscillation. All of these peaks begin to decay in frequency when the fan is turned off.

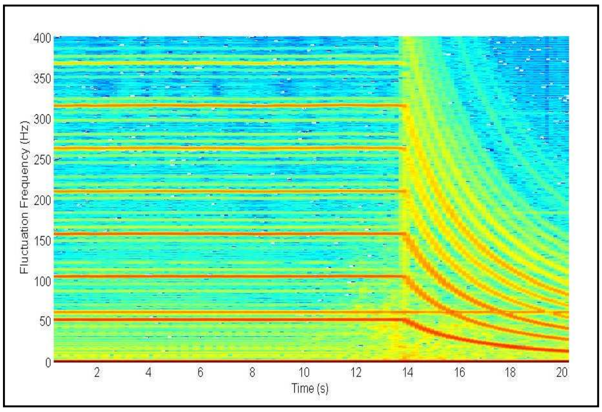


Figure 8: Fluctuation frequency versus time throughout at shot. The 3200 rpm (53 Hz) fan is shutoff and slowly decelerates at 14 seconds.

6. Conclusions

LDX was run for 7 lifts of 2-3 hours each on the week of August 2, 2010. Each lift had time for about 15 data shots. During this plasma campaign the reflectometer was installed and collecting data. The reflectometer was unable to see reflection from the plasma or even from the F-coil so the run was used to track down problems with the device for future modification.

The signal was already fairly weak in the calibration shots so when the antennas were attached to the device the extra attenuation from the window and the port opening contributed to the lack of a signal from the plasma. This was apparent from a magnetics shot with no plasma in which the reflectometer was still unable to see the F-coil. It should be noted that this problem could also be caused by the negative curvature of the F-coil which will tend to disperse the radar beam.

These problems should be addressed together. A possible solution would be to place the antennas inside the vacuum chamber. The additional room inside the vacuum chamber would allow more directional but bigger horn antennas. This directionality would be helpful in increasing the size of the reflected signal from a negative curvature object such as the F-coil and the plasma. It would also reduce the signal from cross coupling because there would be less antenna pattern overlap.

Placing the antennas inside the vacuum chamber would also correct the problem of attenuation of the radar wave by the antenna casing and port window. It could also be used to correct the slightly off center position of the antennas relative to the F-coil midplane caused by the port being off center. In addition, internal antennas would be closer to the dense region of the plasma so less power would be lost as the signal decays over a distance. Therefore, moving the antennas inside the vacuum chamber would fix numerous problems associated with the antenna geometry.

One of the other problems with the device was discovered in the calibrations. The observed noise in the calibration shots will decrease the spatial resolution and accuracy of the reflectometer because a reflection at a single distance will cause beat frequencies to be present over a small range of frequency. This noise has similar structure from sweep to sweep for a large range of reflection distances. It actually stays almost exactly the same for sweep after sweep in a single shot. This suggests that it may be an inherent system problem more than just random noise. Because the structure of this noise repeats on the scale of one sweep, it is likely that it is associated with the sweep.

The YIG oscillator frequency/voltage calibration was only calibrated for one frequency at a time. At each control voltage it was given sufficient time to adjust to that voltage. Even in this ideal scenario it was seen to drift slightly over time with the changing temperature of the device. It is within the specifications of the YIG oscillator to scan through 4 GHz in 40 ms but the high accuracy needed of < 100 Hz at this scan rate may be outside the accuracy of the oscillator. The noise was observed to be on the order of 100 Hz. At this sweep rate the oscillator has 10 kHz residual FM. This remaining modulation may be responsible for the inaccuracy of the beat frequency measurements. The solution to this problem is most likely to abandon the use of the YIG oscillator and find a more precise frequency swept oscillator. It would be desirable to find an oscillator with a faster maximum df/dt which would increase all the beat frequencies and get them above all the noise inherent to low frequencies.

The only response of the reflectometer during the run was to ambient noise when systems were activated for a plasma shot. It correlated to whether the ECRH sources were on or not. Instead of indicating that the reflectometer may be seeing plasma this probably indicates that the ECRH heating was affecting the electronics of the reflectometer. The electronics were all shielded by an aluminum case in which they were separated from by dielectric supports, but all of the external wiring for communicating with the antennas and the digitizers was exposed to the noise from the ECRH sources. The pathways that are introducing this noise into the refectometer system must be found and shielded if the reflectometer is to be successful in the future.

The reflectometer was proven usable with calibration shots but problems with the antenna geometry prevented it from detecting the reflection from the plasma. The calibration also revealed problems with the frequency swept oscillator which should be fixed in order to improve the accuracy of the diagnostic. This paper has shown the potential application of a reflectometer density diagnostic to LDX and highlighted several problems to need to be fixed before it can become operational.

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