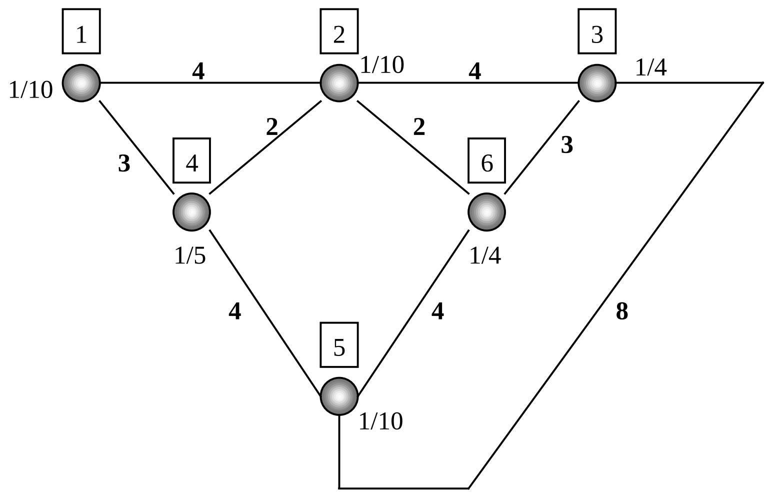
Massachusetts Institute of Technology
1.203J, 6.281J, 13.665J, 15.073J, 16.76J, ESD.216J

Logistical and Transportation Planning Methods

Quiz 2, Fall – 1999 Open Book

Briefly explain your reasoning. And good luck!

Consider the 6-node, 9-link bi-directional transportation network, G(N,A), shown below:



The integer adjacent to each arc is the length of that arc. The integer in a square adjacent to each node is the node number n, n=1,2,...,6. The fraction adjacent to each node k is the node weight w(k), i.e., the fraction of total customer demand originating from that node.

All parts of this quiz are based on this network. Each part is independent of other parts.

1. [10 points] Find a Hakimi two-median of G(N,A).

2. [10 points] Find all one-vertex-centers of G(N,A).

3. [10 points] Find all the absolute centers of G(N,A).

4. <u>This part deal with minimum spanning trees.</u>
    (a) [10 points] Find the length of a minimum-spanning tree, T, of G(N,A). Explain your method of constructing T.

    (b) [10 points] Find an absolute center on T. Briefly explain your work.

5. [15 points] Find an optimal solution to the Chinese postman problem. Explain briefly your methodology. What is the length of the optimal tour?

6. [15 points] Suppose that we want to locate a hospital on G(N,A) that operates as a FCFS M/G/1 queue. After serving a call located at one of the vertices, the only ambulance returns to the hospital. Moreover, our ambulance operates on G(N,A) following shortest paths.
We wish to find a Stochastic Queue Median (SQM) on G(N,A). The SQM is the hospital location that minimizes mean response time of the ambulance to a random customer, where response time is the sum of travel time and queueing delay. We assume a constant travel speed equal to one.

Let λ = Poisson arrival rate of customers from G(N,A), and let $\lambda w(k)$ = Rate of (independent) Poisson arrivals from node k (k = 1,2,..., 6).

(a) Find a SQM for $\lambda = 0+$.

(b) Define λ_{max} = maximum value for λ such that for any $\lambda \geq \lambda_{max}$ the M/G/1 queueing system is unstable for any facility location in G(N,A). Find a SQM for $\lambda = \lambda_{max} - \varepsilon$, with $\varepsilon > 0$ and $\varepsilon / \lambda_{max} << 1$.

(c) Suppose now that all calls that find the ambulance busy are lost (e.g., they use a taxi instead). Each lost call costs the Hospital $\gamma > 0$ dollars. Find a location for the Hospital that minimizes the expected cost of response to a random service request, where this cost is the sum of the expected travel time to answered requests plus the expected cost incurred by lost calls.

7. [20 points] Suppose two hospitals are located on G(N,A), one each at the midpoints of links (1,4) and (3,6) respectively. The system operates as the "2-server hypercube" loss system. All travel is along minimum distance paths. Assume that service times are i.i.d. negative exponential with mean equal to 1 (i.e., $1/\mu = 1.0$) and that the total arrival rate on the network is $\lambda = 1.0$. Moreover, assume a constant travel speed equal to c/2, where c is the speed of light; seriously, this is to make the travel time component of the total service time a negligibly small part. Virtually all of the negative exponential service time is on-scene time.

(a) Find the equal travel distance boundary between the two hospitals.

(b) Find the primary response areas (set of nodes) for each of the two facilities that minimize the mean response time to a random served customer, assuming steady state operation.