

## Urban Operations Research 1999 Quiz 1

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This possible problem notwithstanding, please answer all questions, showing all work. Good luck.

1. At a particular ATM, customers arrive singly in Poisson manner with parameter  $\lambda = 0.5$ . There is one server, the queue discipline is first-come first-served, and each service takes exactly one time unit to complete.

(i) Suppose that Mendel arrives at a random time. How long on average will he spent in the system (until his service completion)?

(ii) Minerva arrives at a random time to find six customers in the system. What is the probability distribution of her time in the system?

(iii) If a busy period has just begun for the server, what is the minimum time it could last, and what is the probability it attains that minimum?

(iv) If a busy period has just begun, what is its mean duration given that it is longer than the minimum value in (iii)? (HINT: What is the overall mean busy period?)

2. Suppose that  $x_1$  and  $x_2$  are  $U(0,a)$ , and let  $J(a) = E(x_1^2 + x_2^2)$ . Use Crofton's method to determine  $J(a)$ .

3. Suppose that a city has the shape of a square with side  $L$  with the lower left corner at  $(0,0)$ , and that it has a diagonal barrier along  $y=x$  up to the corner at  $(L, L)$ . There is one break in the barrier at  $(.8L, .8L)$ , through which traffic can pass.

Suppose that emergencies occur uniformly over the city, and the response vehicle starts out from a location that is also uniform over the city. (The locations of the emergency and of the starting response vehicle are independent.) The streets follow the Manhattan-metric grid scheme. What is the average distance the vehicle travels to the scene of an emergency?