QUIZ 2 SOLUTIONS
Logistical and Transportation Planning Methods
Massachusetts Institute of Technology
December 5, 1999

(1) Nodes 4 & 6 as well as nodes 3 &4 comprise Hakimi 2-median
(2) Nodes 2, 4 or 6 are all one-vertex centers

The maximum distance vector is:

Node, k	Max, m(k)
1	8
2	6
3	8
4	6
5	7
6	6

(3) The absolute center is easy to compute if one remarks the symmetry of the problem. Only, arc (2,3), (2,6), (3,6), (5,6) have to be inspected. Moreover, if one starts looking at arc (2,6) the local center is at 1 distance unit from node 2 (middle of the arc) and the maximum distance is 5. Looking at the maximum distance vector above, we conclude:

$\frac{m(2)+m(3)-d(2,3)}{2} = \frac{6+8-4}{2} = 5$. Therefore, we can discard the candidacy of arc (2,3).

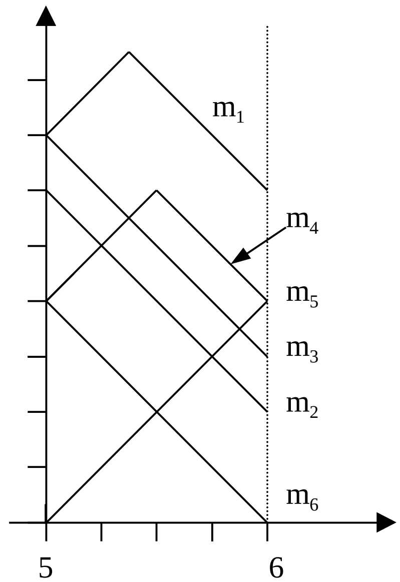
Similarly for arc (3,6) because:

$$\frac{m(3)+m(6)-d(3,6)}{2} = \frac{8+6-3}{2} = 5.5$$ We can discard arc (3,6).

For arc (5,6):

$$\frac{m(5)+m(6)-d(5,6)}{2} = \frac{7+6-4}{2} = 4.5$$ We have to look at (5,6).

Searching for the local center on arc (5,6) we find:

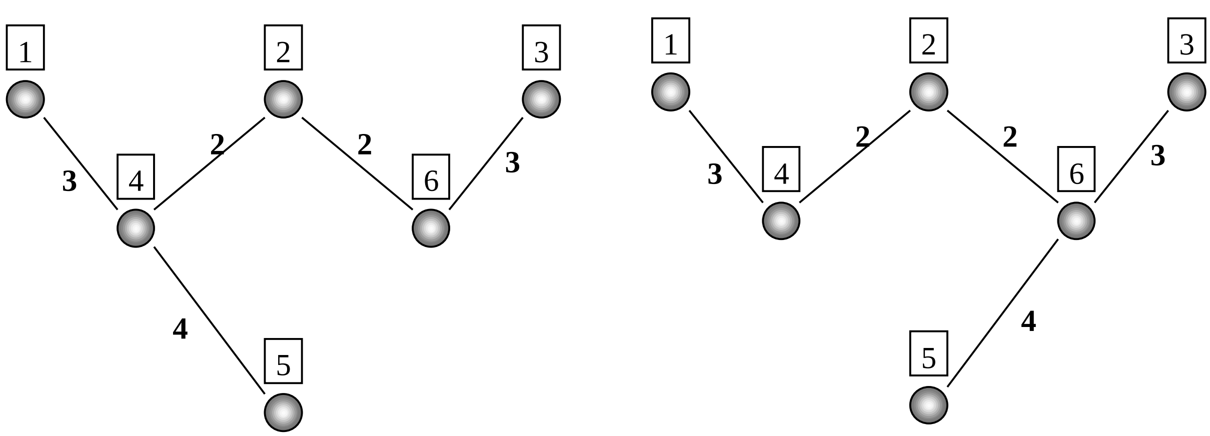


We find that the local center of arc (5,6) is in node 6, with a maximum distance of 6.

Conclusion: By symmetry the only two absolute centers are located on the center of arc (2,6) and arc (2,4).

(4) We build the MST using the greedy MST algorithm, we find 2 solutions:

The length of a minimum spanning tree is: 14.



(5) Center on T is located on arc (2,4) at 1/2 from 2.

(6) Chinese Postman:

Set of odd-degree nodes: {3,4,6,5}

Best Pair wise matching: {3,6} and {4,5}

Length of the Chinese Postman Tour: 34+ 3 + 4 =41.

(7)

(a) The SQM for $\lambda = 0+$ is the 1-median:
We have to find the 1-median. According to Hakimi's theorem, we can find one at a vertex.
The distance matrix is:

F/T	1	2	3	4	5	6
1	0	4	8	3	7	6
2	4	0	4	2	6	2
3	8	4	0	6	7	3
4	3	2	6	0	4	4
5	7	6	7	4	0	4
6	6	2	3	4	4	0

Multiplying row k by 10.w(k) one finds a 1-median at: 6

F/T	1	2	3	4	5	6
1	0	4	8	3	7	6
2	4	0	4	2	6	2
3	20	10	0	15	17.5	7.5
4	6	4	12	0	8	8
5	7	6	7	4	0	4
6	15	5	7.5	10	10	0
Sum	*52*	*29*	*38.5*	*34*	*48.5*	***27.5***

(b) The SQM for $\lambda = \lambda_{max}-\varepsilon$ is also the 1-median because it is the last feasible position before system blows up according to Pollaczek-Khintchine (M/G/1).

(c) The position of the Loss median is the same as the Hakimi median. Therefore, using Lemma in class + web notes, an optimal location is at the Hakimi 1-median: node 6.

(8)

(a) The equal travel distance boundary between the 2 hospitals passes through nodes 2 and 5.

(b) We define Hospital #1 to be the center of (1,4) and Hospital #2 to be the center of (3,6). We define $\eta=\lambda/2\mu=0.5$
Intuitively one can speculate that the optimum primary response areas of 2 will be smaller than the primary response area of 1 because there is a greater chance of getting a call closer to 2 than to 1. We use Carter, Chaiken and Ignall formula to find s_0:

$$s_0 = \frac{2\eta}{2\eta+1} \times [T_1(B) - T_2(B)]$$

$10. T_1(B) =$ 1.5+3.5+7.5*2.5+1.5*2+5.5+5.5*2.5=46
$10. T_2(B) =$ 7.5+3.5+1.5*2.5+5.5*2+5.5+1.5*2.5=35

Therefore, s_0=0.45 s_0/2=0.225
The optimal boundary line is slightly "East" of nodes 2 and 5.

Conclusion:

Node 1,2,4, and 5 belongs to the "optimal" (or "primary") response area of Hospita1 1

Node 3 and 6 belongs to the "optimal" (or "primary") response area of Hospita1 2.