1.203J/6.281J/15.073J/16.76J
Logistical and Transportation Planning Methods
Massachusetts Institute of Technology

Quiz 2

December 3, 2003

---

1. Total time allotted is 90 minutes.
2. Please print your name clearly on each page you turn in.
3. Please answer each problem on a separate page

Problem 1 (32 points)
At a facility with a single server, customers arrive in a Poisson manner at the rate of 24 per hour. Exactly 50% of the customers require a service that takes exactly 1 minute, while the other 50% require service that takes exactly 3 minutes. "1-minute customers" and "3-minute customers" arrive randomly intermingled. There is infinite queue capacity.

(a) (20 points) Suppose that "1-minute customers" are assigned non-preemptive priority over "3-minute customers". Find the expected waiting time (i.e., not including time spent in service) for (i) "1-minute customers", (ii) "3-minute customers" and (iii) a randomly selected customer. Furthermore, (iv) compare your answer to (iii) with the expected waiting time of a random customer, if all customers were served in a First-Come, First-Served (FCFS) order at the facility, irrespective of the length of their service time. Finally, (v) explain in a couple of sentences why the relationship (greater than, less than, equal) between your answers to (iii) and (iv) is what it is. [NUMERICAL ANSWERS ARE EXPECTED FOR PARTS (i) – (iv).]

(b) (9 points) Suppose now that the demand rate increases to 40 per hour (with 50% "1-minute customers" and 50% "3-minute customers"). Suppose, as well, that "1-minute customers" are assigned non-preemptive priority over "3-minute customers". Compute the expected waiting time for (i) "1-minute customers" and (ii) "3-minute customers".

(c) (3 points) With the situation as in (b), i.e., a demand rate of 40 per hour, suppose now that "3-minute customers" are assigned non-preemptive priority over "1-minute customers". In a few (at most) sentences, please explain what is going to happen at this facility. No math please!

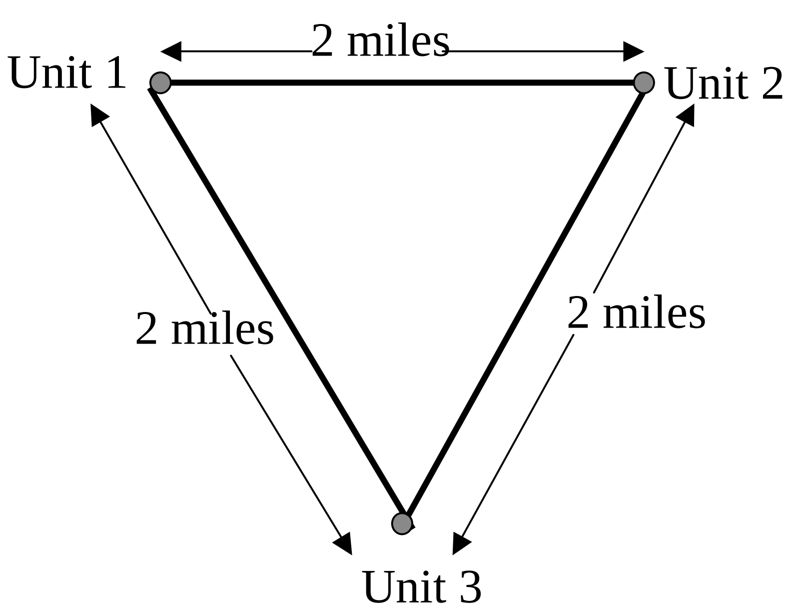
Problem 2 (32 points)
Three ambulances are positioned symmetrically at the three respective corners of an equilateral triangle, as shown in the figure. Each triangle leg is two miles in length. Calls for ambulance service are uniformly and independently distributed over the edges of the triangle. These calls arrive in time as a homogeneous Poisson process with a total rate of λ calls per hour from the entire triangle. Mean on scene time at an ambulance call for service, including the time it takes the ambulance to travel back to its home location at one of the corners of the triangle, is one hour. The pdf for this on-scene time is negative exponential. Response speed is 60 miles per hour, so -– as the usual engineering approximation --- we can forget travel time as part of service time. The dispatcher always assigns a closest available ambulance to a call for service and the ambulance always takes the shortest route to the call it is dispatched to. There is an infinite capacity queue allowed. Queued calls are handled in the first come, first served manner.

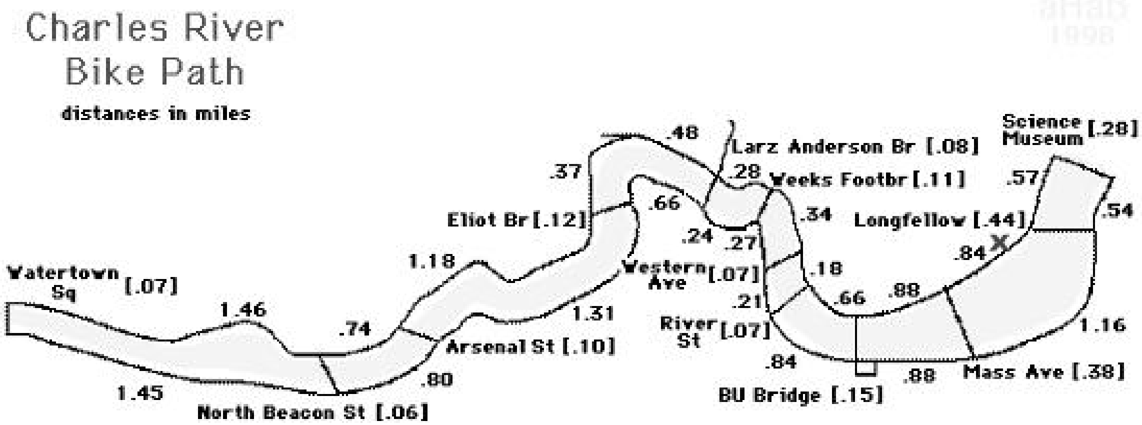
(a) What is the mean response distance (i.e., travel distance for the ambulance to travel to the scene of the call for service) when $\lambda = 10^{-6}$?

(b) What is the maximum allowable value of λ, such that for any value greater than this value, no steady state exists and the queue will grow without bound? Call this value
λ_{max} .

(c) Suppose now we have a very high value of λ, say $\lambda = \lambda_{max}$ – *epsilon*, where *epsilon* is very small but positive. What is the mean response distance in this case? Explain how you derive it!

(d) For an arbitrary value of λ determine the fraction of dispatch assignments that assign an ambulance to a call farther than one mile from its home location.





The Twelve Bridges of Cambridge Problem

( )

Problem 3 (36 points)

According to a famous Boston tourist web site, the "...Charles River Bike Path is a 16.7 mile loop along the banks of the Charles, from the Museum of Science in downtown Boston to Watertown Square and back. The dozen bridges allow for a loop walk/bike of almost any length (see map above with distances over and between bridges)." We at MIT have tabulated all of the land segment lengths and the bridge lengths, and we arrive at a total mileage of 18.27. The stated figure of 16.7 miles is in fact the length of the outer loop or cycle, including all the land paths plus the two end bridges, the Watertown Square Bridge and the Science Museum (land) bridge. The remaining 1.57 miles is the total length of the ten bridges between the two end ones. The figure shows the distances between consecutive bridges on each side of the river. The names of the twelve bridges are also shown along with the length of each bridge (in brackets, [ ]). The details are in the spreadsheet on the next page.

(a) Mr. Mike Jogger wants to run a route that covers every inch of the network of paths (on both sides of the river) and bridges at least once, but he wants to do it in minimum total distance. Create such a shortest-distance jogging path for Mike. What is the total distance he will have to run?

(b) (True Story!) Jon, a former Ph.D. student at the MIT Operations Research Center jogged every day at lunchtime. The ORC is shown with a red 'X' on the map. He numbered the 12 bridges from 1 to 12, as shown on the spreadsheet. Each day, to determine that day's jogging route, he would pick two sample values of random variables that were uniformly independently distributed on the integers 1 to 12. Those two experimental values would imply a jogging route. For instance, if he obtained the numbers 3 and 7, he would leave MIT and jog to bridge 3 and cross it, jog from bridge 3 to bridge 7 on the "Boston side" of the river, cross bridge 7, and then return back to MIT on the Cambridge side. If he picked the same two numbers, say 3 and 3, he would go to bridge 3, cross it and then immediately make a U-turn and cross it again, and then return to MIT. Carefully explain how you would determine the probability law for the random variable, "the number of miles Jon jogs each day." Please make sure to define carefully any variables or quantities you use. DO NOT work out the numerical details.

(c) Suppose Jon could move his office to any land-based (i.e., not on a bridge) location on the network, thereby freeing himself from his MIT home location.

(i) Are there other locations on the network that would result in a lower mean distance jogged each day, assuming he still selects his jogging routes randomly as described above? Can you identify one and explain why it is better than the original MIT location?

(ii) How would you think about finding an optimal location for Jon, where optimal means minimizing mean mileage jogged per day? Can you precisely formulate this problem? Will an optimal solution exist solely on the set of nodes, i.e., the juncture points between the bridges and the banks of the river?

Br. #	Bridge	length	Segments on north bank (Cambridge side)			Segments on south bank (Boston Side)		
1	Science Museum	0.28	from	to	length	from	to	length
2	Longfellow	0.44	Science Museum	Longfellow	0.57	Science Museum	Longfellow	0.54
3	Mass Ave	0.38	Longfellow	Mass Ave	0.84	Longfellow	Mass Ave	1.16
4	BU Bridge	0.15	Mass Ave	BU Bridge	0.88	Mass Ave	BU Bridge	0.88
5	River St.	0.07	BU Bridge	River st	0.66	BU Bridge	River St.	0.84
6	Western Ave	0.07	River St.	Western Ave	0.18	River St.	Western Ave	0.21
7	Weeks Footbr	0.11	Western Ave	Weeks Footbr	0.34	Western Ave	Weeks Footbr	0.27
8	Larz Anderson Br	0.08	Weeks Footbr	Larz Anderson Br	0.28	Weeks Footbr	Larz Anderson Br	0.24
9	Eliot Bridge	0.12	Larz Anderson Br	Eliot Bridge	0.85	Larz Anderson Br	Eliot Bridge	0.66
10	Arsenal St.	0.10	Eliot Bridge	Arsenal st.	1.18	Eliot Bridge	Arsenal St.	1.31
11	North Beacon St	0.06	Arsenal St.	North Beacon st	0.74	Arsenal St.	North Beacon St.	0.8
12	Watertown Sq	0.07	North Beacon St	Watertown Sq	1.46	North Beacon st	Watertown Sq	1.45
	total	1.93		total	7.98		total	8.36

Grand total = 18.27 miles