

Urban OR Quiz 1  
October 20, 2003

“We blew it.”

--Henry Fonda (Easy Rider)

Don't blow it: Just answer all questions, showing all work. Good luck.

1. Suppose that  $x_1$  and  $x_2$  are independent and both  $U(0,a)$ , and let  $G(a)$  be the expected value of the *cube* of the larger of the two variables. Use Crofton's method to determine  $G(a)$ .
2. Suppose that vehicles are distributed over a large city under a spatial Poisson process with parameter  $\lambda$ , and let  $v_i$  be the straight-line distance to a given point from the  $i^{\text{th}}$  nearest vehicle to that point. Find the probability that  $v_4 > 4v_1$  *and* also that  $v_4 > 2v_2$ . (We are referring to the joint event involving  $v_1$  and  $v_2$ , not two separate events.)
3. Mendel arrives at a random time to ride the Lemon Line, on which the intervals between trains are equally like to be 4, 5, and 6 minutes.
  - (i) What is the probability that he waits between 4 and 5 minutes for the next train? (HINT: What is the conditional probability that he waits between 4 and 5 minutes if he arrives in an interval of length  $X$ ?)
  - (ii) Would the answer to (i) increase or decrease if the Lemon Line became less variable and all intervals between trains were exactly 5 minutes? Intuitively, why does the answer move in that direction?
4. Consider a small factory that has 3 machines that are all subject to breaking down. Machines break down independently of each other. Whenever a machine breaks down, it is sent to the factory's repair shop for service. The repair shop has two parallel and identical repair stations. Machines at the shop are repaired in a first come, first served order. It has been observed that the time needed to repair a machine at a repair station has an exponential pdf with an expected value of 2 hours. The time until a repaired machine breaks down again (and has to go back to the repair shop) also has an exponential pdf with an expected value of 9 hours.

Find the expected number of machines that are operating at this factory in steady state. A numerical answer is expected. *Please explain your work clearly.*