Massachusetts Institute of Technology
1.203J/6.281J/13.665J/15.073J/16.76J/ESD.216J
Logistical and Transportation Planning Methods
Fall 2002

Quiz 2 Solutions

1

(Larson 2002)

(a) With probability 0.3, the emergency occurs on one of the two links incident to the garage location of ambulance 2. In this case, the travel distance is $U(0,1)$. With probability 0.7, the emergency occurs on one of the two links not incident to the garage location of ambulance 2. In this case, the travel distance is $U(1,2)$. Accordingly, as shown in Figure 1, the conditional pdf of the travel distance for ambulance 2 to travel to the scene of the emergency incident is

$$f_D(d) = \begin{cases} 0.3, & d \in [0,1) \\ 0.7, & d \in [1,2] \end{cases}$$

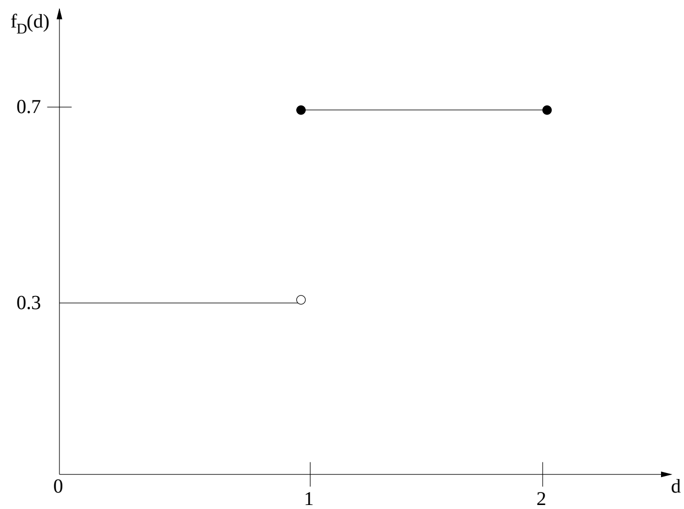

Figure 1: Conditional pdf of Travel Distance

(b)

The state transition diagram for this system is shown in Figure 2, where first component of the state indicates whether ambulance 2 is busy and the second component indicates whether ambulance 1 is busy. We can thus write the following balance equations.

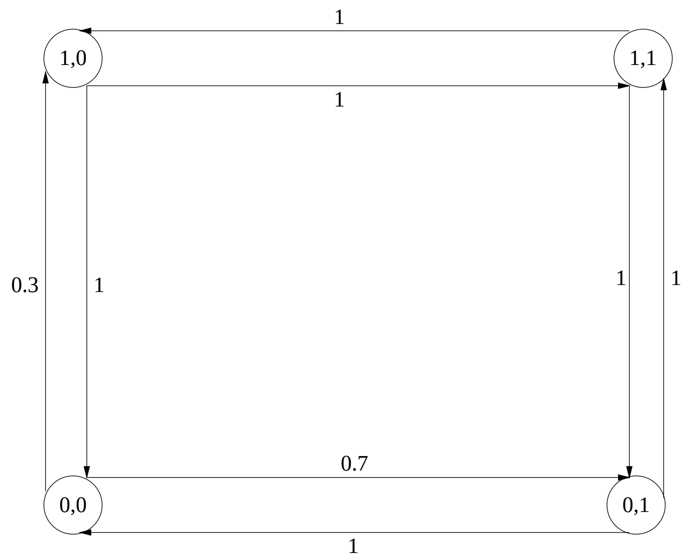

Figure 2: State Transition Diagram

$$
\begin{aligned}
P_{0,0}(0.3+0.7) &= P_{0,1}+P_{1,0} \\
2P_{0,1} &= 0.7P_{0,0}+P_{1,1} \\
2P_{1,1} &= P_{1,0}+P_{0,1} \\
P_{0,0}+P_{0,1}+P_{1,0}+P_{1,1} &= 1
\end{aligned}
$$

Solving this system, we obtain

$$
\begin{aligned}
P_{0,0} &= \frac{2}{5} \\
P_{0,1} &= \frac{6}{25} \\
P_{1,0} &= \frac{4}{25} \\
P_{1,1} &= \frac{1}{5}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\rho_1 &= P_{0,1}+P_{1,1} = \frac{11}{25} \\
\rho_2 &= P_{1,0}+P_{1,1} = \frac{9}{25}
\end{aligned}
$$

(c) This is a straightforward application of Equation 5.18 from the textbook.

$$
s_0 = \frac{2\eta}{2\eta+1}\left(T_2(B)-T_1(B)\right)
$$

where $T_n(B)$ gives the average travel time for unit n to travel to a random service request from anywhere in the entire service region. Note that s_0 is therefore given in time units. Let us multiply the RHS through by the travel speed and let $D_n(B)$ denote the average travel distance for unit n to travel to a random service request from anywhere in the entire service region. We can then write s_0 as follows, in units of distance rather than time.

$$
\begin{aligned}
s_0 &= \frac{2\eta}{2\eta+1}\left(D_2(B) - D_1(B)\right) \\
D_1(B) &= 0.1 \cdot 1.5 + 0.2 \cdot 1.5 + 0.3 \cdot 0.5 + 0.4 \cdot 0.5 \ = \ 0.8 \\
D_2(B) &= 0.1 \cdot 0.5 + 0.2 \cdot 0.5 + 0.3 \cdot 1.5 + 0.4 \cdot 1.5 \ = \ 1.2 \\
\eta &= \frac{\lambda}{2\mu} \ = \ \frac{1}{2} \\
s_0 &= \frac{0.4}{2} \ = \ 0.2 \text{ km}
\end{aligned}
$$

This means we shift the equal-travel-time boundary line away from the northwest and southeast corners of the square and toward the northeast corner of the square, but moving only 0.2 km in those directions.

2

(Odoni 2002)

(a) The length of the optimal CPP tour will be the length of the original network, plus the length of the optimal pairwise matching of odd-degree nodes. The odd degree nodes are C,F,G,H,J,I,P, and L. By inspection, the optimal matching is C-F (8), G-H (9), J-I (6), and P-L (16), where the numbers in parentheses give the cost of this matching. So, the optimal CPP tour has length $130 + 8 + 9 + 6 + 16 \ = \ 169$.

(b) Let us first use the majority theorem. Consider isthmus (G,H), which separates the network into two distinct subnetworks with node sets

$$
\begin{aligned}
N_1 &= \{A, B, C, D, E, F, G\} \\
\text{and } N_1' &= \{H, I, J, K, L, M, N, O, P, Q, R, S\}
\end{aligned}
$$

The total weights of these node sets are given by $W(N_1) = 7$ and $W(N_1') = 12$, respectively (we'll use W instead of the usual H notation to avoid confusion with node H). As a result, we can discard the portion of the original graph involving node set N_1 and consider the subnetwork with node set N_1' and all node weights 1 except for node H, which now has weight 8.

Now let us consider isthmus edge (L,M). Since the total weight of nodes M,N,O,P,Q,R,S is 7, we can disregard this part of the graph as well. We know that the absolute 1-median lies in the subnetwork given by nodes H,I,J,K,L, where nodes I,J, and K have weight 1 and nodes H and L each have weight 8. The minimum distance matrix for this subnetwork is

	H	I	J	K	L
H	0	4	3	7	8
I	4	0	6	3	7
J	3	6	0	9	5
K	7	3	9	0	4
L	8	7	5	4	0

The weighted minimum distance matrix is

	H	I	J	K	L
8 x H	0	32	24	56	64
I	4	0	6	3	7
J	3	6	0	9	5
K	7	3	9	0	4
8 x L	64	56	40	32	0
	78		79		80

Therefore, the absolute 1-median is H.

3

(Odoni 2002)

(a)

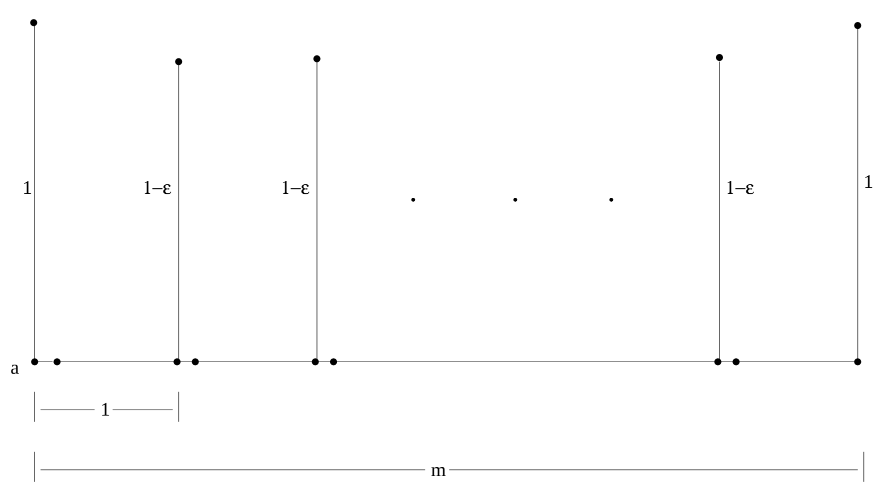

Figure 3: MST

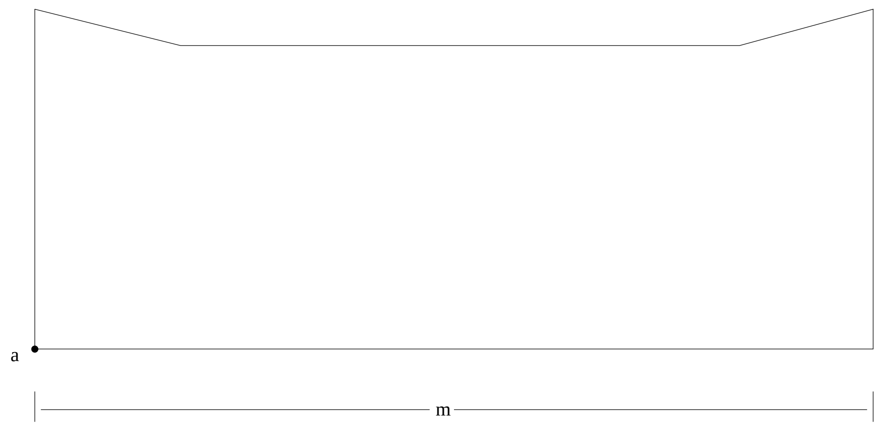

Figure 4: Optimal TSP Tour

(b)

(c) The curved lines in the below diagram indicate that upon reaching the upper-rightmost point, you travel a distance of 1 straight down to the lower-rightmost point and then travel a distance of m straight left to point a.

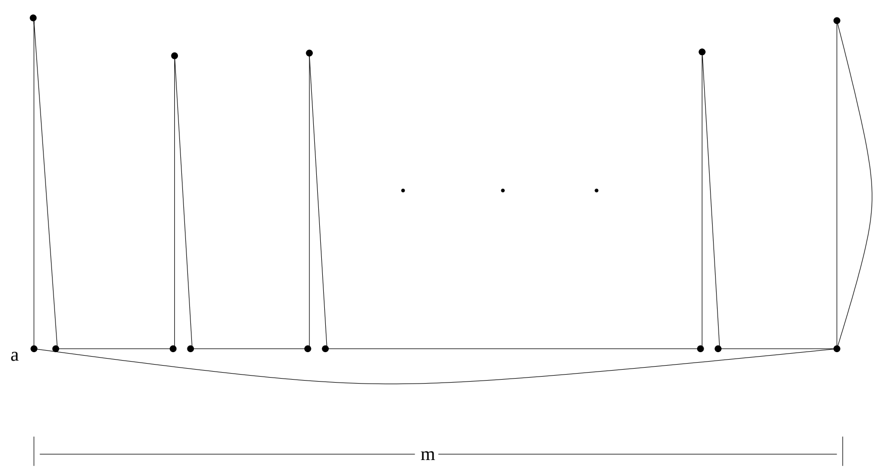

Figure 5: 2-MST HEURISTIC Tour

$$\begin{aligned}
L(2-\text{MST HEURISTIC}) &= m + m(1-\epsilon) + 2 + 1 + (1+\beta) \\
&\quad + (m-1)(1-\epsilon) + (m-1)(1-\delta) \\
&= 2m + 4 + 2m - 2 + O(\epsilon) + O(\delta) + O(\beta) \\
&= 4m + 2 + O(\epsilon) + O(\delta) + O(\beta) \\
L(\text{TSP}) &= m + m + 2\gamma + 2 \;=\; 2m + 2 + 2\gamma \\
\lim_{\beta,\delta,\epsilon,\gamma\to 0} \frac{L(2-\text{MST HEURISTIC})}{L(\text{TSP})} &= \frac{4m+2}{2m+2} \\
\lim_{m\to\infty}\left(\lim_{\beta,\delta,\epsilon,\gamma\to 0} \frac{L(2-\text{MST HEURISTIC})}{L(\text{TSP})}\right) &= 2
\end{aligned}$$

where $O(h)$ denotes a function s.t. $\lim_{h\to 0} O(h) = 0$. Therefore, this is an example of the worst-case performance of the 2-MST HEURISTIC.