Massachusetts Institute of Technology
Logistical and Transportation Planning Methods
Aero & Astro (16.76J), Civ and Env (1.203J), EE & CS (6.281J)
Ocean (13.665J), Urban Stud & Plan (11.526J), and Sloan (15.073J)

Quiz #2 Fall 2000 December 11, 2000

Open Book

There is no more beautiful life than that of a student.

~ F. Albrecht ~

Learn as though you would never be able to master it; hold it as though you would be in fear of losing it.

~ Confucius ~

Problem 1. Queueing (35%)

Consider a single-server queuing system that operates as follows:

- There are two types of customers, Type 1 and Type 2, each type arriving at the system in a Poisson manner at rates λ_1 and λ_2, respectively.

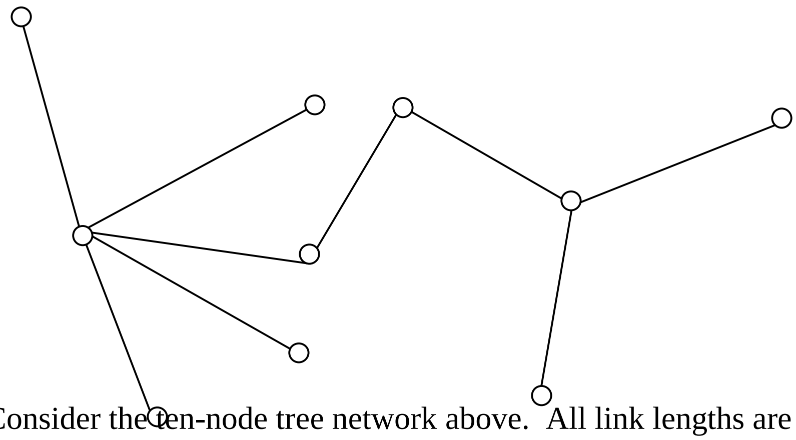
- Customers of Type 1 have negative exponential service times with expected value $1/\mu_1$ per service time. Similarly, customers of Type 2 have negative exponential service times with expected value of $1/\mu_2$ per service time. Successive service times are mutually independent.

- The queuing system has capacity of three customers, including the one receiving service. In other words, whenever three customers of any type are in the system (one receiving service and two waiting) the system is full and any additional arriving customers are rejected and go elsewhere.

- The next customer to be processed by the server is selected in the following way: If, after the completion of a service, there is no other customer waiting, then the next customer to arrive at the system, no matter of what type, obtains access to the server. If, after the completion of a service, there is at least one customer waiting, then customers (if any) of the OPPOSITE type than that of the customer who just completed service receive priority for access to the server. (For example, if the last customer was of Type 1, then Type 2 customers, if any are present, receive priority.) If no customers of the same type are waiting, then a customer of the other type, if any is waiting, is admitted for service. Customers within each type are served in a first-come, first-served (FCFS) way.

(a) Draw a neat state transition diagram for this queuing system. Make sure to define clearly the states and to indicate the transition rates for every possible transition. (Omit the "dt" to keep the picture uncluttered.)
(b) Suppose an observer arrives at the queueing system and finds a total of 2 customers of Type 2 in the system (including the one receiving service) and no customers of Type 1. What is the probability that the next 3 departures from the system (of serviced customers) will all be Type 2 customers? [HINT: Note that the first departure is guaranteed (with probability 1) to be a Type 2 customer.]

Problem 2. A Ten-Node Network (25%)



Consider the ten-node tree network above. All link lengths are 1 km and all nodal weights are 0.100.

a) Find all 1-medians of the network.
b) Find all vertex centers of the network.
c) Find all absolute centers of the network.
d) Suppose we wished to find a location $X^* \in G$ that minimizes the following quantity:

$$\alpha \sum_{i=1}^{n} h_i d(X^*, i) + (1-\alpha) \underset{i \in N}{MAX} \{d(X^*, i)\}, \quad 0 \le \alpha \le 1,$$

where n = 10 nodes and N = the set of ten nodes.
Find X^*.

Problem 3. Monte Carlo Simulation (20%)

Consider a hyperexponential probability density function, defined as follows:

$$f_X(x) = \alpha\lambda e^{-\lambda x} + (1-\alpha)\mu e^{-\mu x}, \quad 0 < \alpha < 1,\ x \geq 0,\ \lambda > 0,\ \mu > 0.$$

Using pseudo random numbers $r_1, r_2, \ldots, r_n, \ldots$, draw a simple flow chart showing how you would simulate a random sample from this probability density function. Each sample should use no more than two random numbers.

Problem 4. Cleaning the Lines of a Tennis Court (20%)

An "official diagram" of a tennis court is shown in the diagram on the next page. Just below the "official diagram" we have shown a simplified *network* or *graph depiction* of the *boundary and service lines* of the right hand side of the tennis court. The dimensions of the network depiction (i.e., lengths of the respective network links in feet) are the same as shown in the official diagram of the tennis court. For a tennis court that is a clay court, after the players have completed play, each player must "sweep" the *boundary and service lines* of his/her side of the court. This sweeping is done with a special mechanical sweeper, hand-directed by the tennis player, which is used to sweep over every inch of the *boundary and service lines*.

a) Treating the *boundary and service lines* as a graph, is this graph a tree?
b) How many nodes of odd degree are there in the graph?
c) Assume that the player who is directing the sweeper is permitted to walk (Euclidean metric) anywhere on the tennis court. That is, his/her path need not be restricted to the *boundary and service lines.* Derive an optimal path for sweeping the *boundary and service lines*, where optimal means a minimum distance sweeping path. What is the length of the optimal path? How does that compare with the summed lengths of the individual links of the graph? [The optimal path may or may not be a tour. The sweeper need not complete the sweeping at the same point that he started. But the entire graph must be swept.]

Official Diagram of a Tennis Court (above) and Simplified Graph Depiction (below)