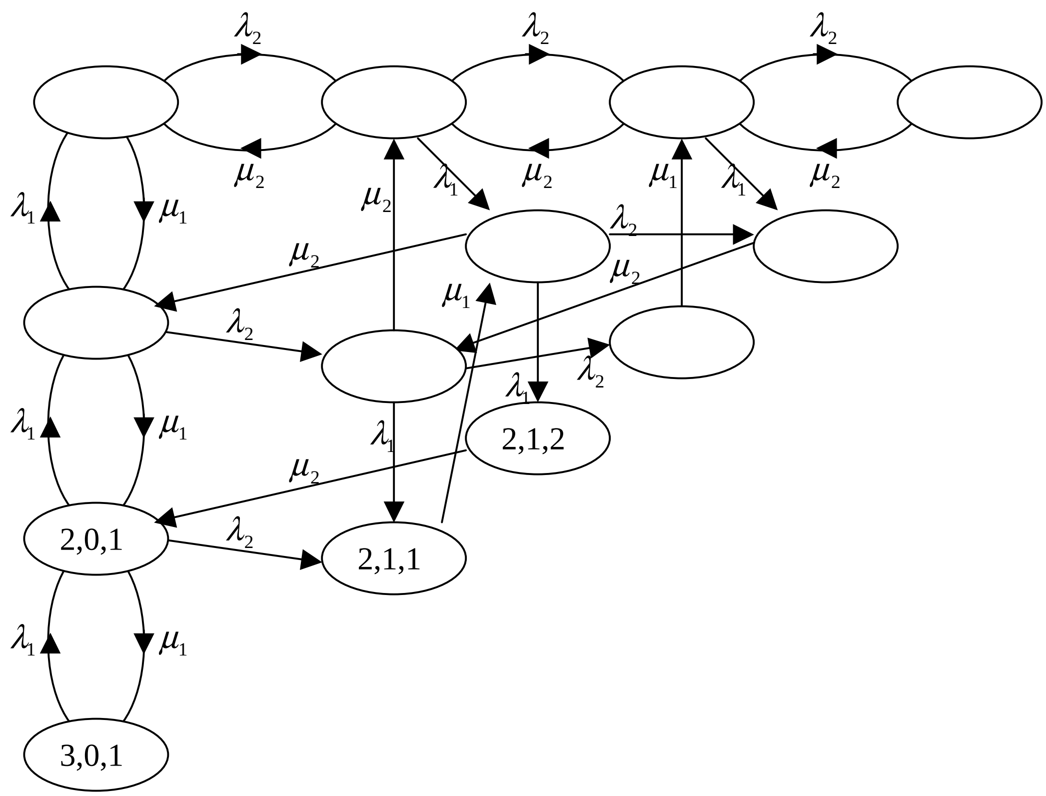
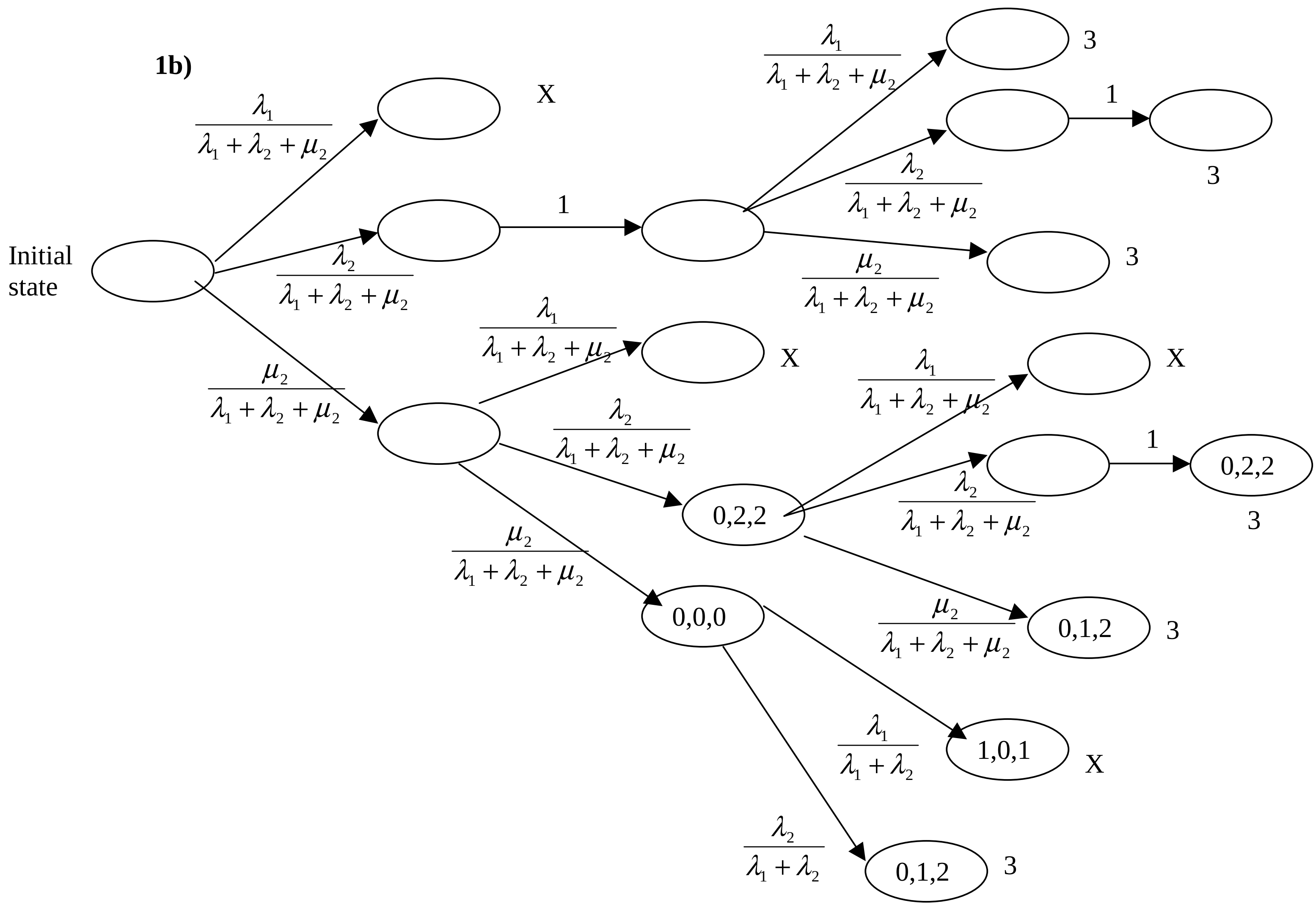
Urban OR 2000 Quiz #2 Solutions
12/12/00
Prepared by JTH

1a)



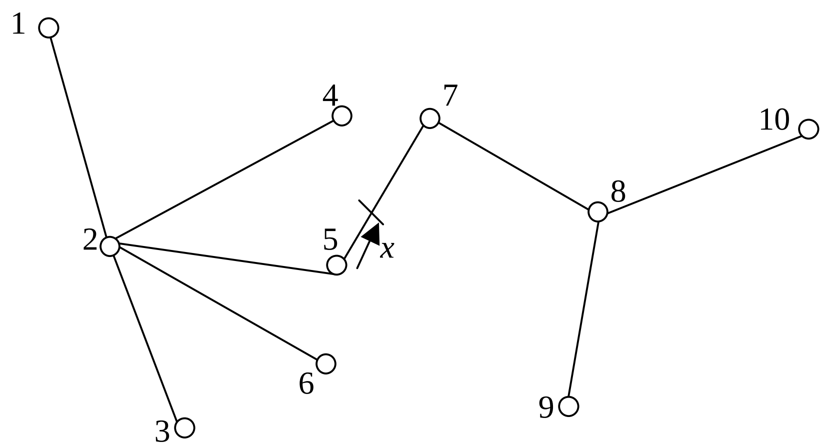
1b)



Answer:

$$\frac{\lambda_2^2}{(\lambda_1+\lambda_2+\mu_2)^2}+\frac{\lambda_2\mu_2}{(\lambda_1+\lambda_2+\mu_2)^2}+\frac{\lambda_2^2\mu_2}{(\lambda_1+\lambda_2+\mu_2)^3}+\frac{\lambda_2\mu_2^2}{(\lambda_1+\lambda_2+\mu_2)^3}+\frac{\lambda_2\mu_2^2}{(\lambda_1+\lambda_2+\mu_2)^2(\lambda_1+\lambda_2)}$$

2)



a) 1-medians. Any point (including nodal end points) on link (2,5).

b) A longest path is (1,2,5,7,8,10). Nodes 5 and 7 are vertex centers, with max distance equal to 3 km.

c) Absolute center is the half way point on link (5,7). Max distance to a node in this case is 2.5 km.

d) For $\alpha = 1$, we have any 1-medians, as in (a) above.
For $\alpha = 0$, we have the absolute center as in (c) above.

For $\alpha = 1/2$, suppose we have a point *x* measured from node 5 on link(5,7), where $(0 \le x \le \frac{1}{2})$.

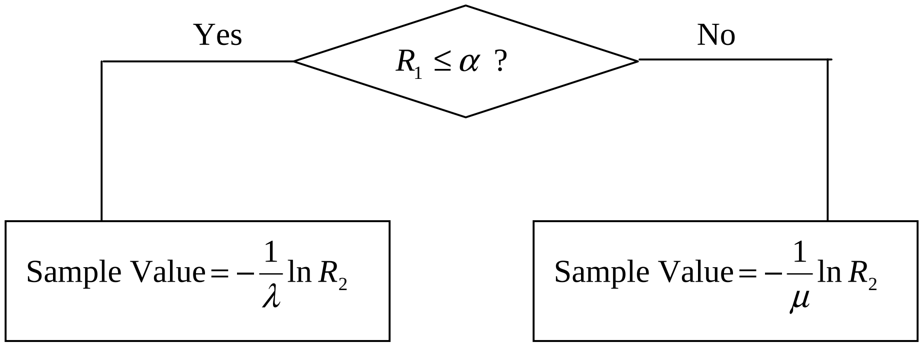
Then the objective function can be written:
$\frac{1}{2}\left[\frac{1}{10}\{x + (x+1) + 4(x+2) + (1+x) + (2-x) + 2(3-x)\}\right] + \frac{1}{2}\left[2 + (1-x)\right]$

which can be simplified to:
$-\frac{2x}{5} + \frac{24}{10}$, which is minimized at x = ½

Note that for any value of α the objective function is a linear function of *x*, which is minimized at an extreme point, *x* = 0 or *x* = ½

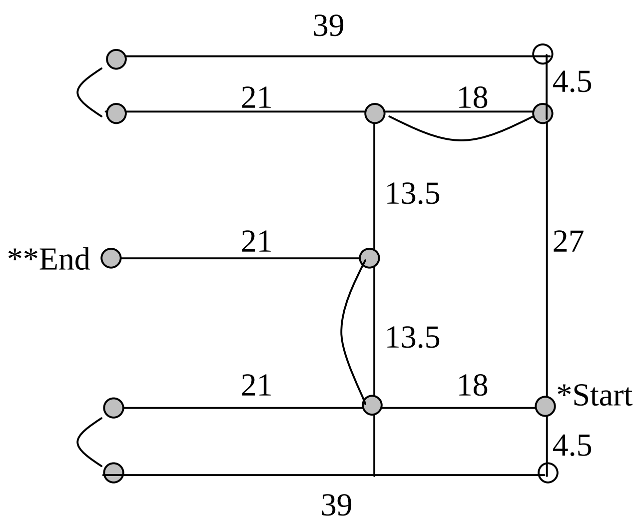
So, we must solve for a value of α, call it $\alpha *$, which is a type of "break point" in the linear function. At the break point, we must have:
Obj function(*x*=0) = Obj function(*x*=½)
$\alpha\left[\frac{1}{10}\{0+1+8+1+2+6\}\right] + (1-\alpha)\left[2+1-0\right] = \alpha\left[\frac{1}{10}\{2 \cdot \frac{1}{2} + 18\}\right] + (1-\alpha)(3 - \frac{1}{2})$
Solve for $\alpha = \alpha *$, we get $\alpha * = \frac{5}{6}$
For any $\alpha \le \alpha *$, $X* =$ absolute center; otherwise $X* =$ node 5.

3)



4)

- **a)** No, the graph is not a tree. (There is a cycle)
- **b)** # of Nodes of odd degree = 10 (gray nodes below)
- **c)**



We need to match 8 of the 10 odd degree nodes. Then we can find an Euler Path (not a Tour) That minimizes the total distance traversed. By inspection, we added 2 links of 4.5 ft, one of 13.5 and one of 18, in a sense using the greedy matching algorithm. There is no other matching that can use shorter links. So we have added (9 + 13.5 + 18) ft = 40.5 ft. Sweeper can start at Node(*) and end at Node(**).