14.384 Time Series Analysis
Fall 2008

14.384 Time Series Analysis, Fall 2007
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Recitation 5

Variance Decomposition

Suppose we have a VAR and we have some way to identify orthonormal shocks, so that

$$y_t = \tilde{C}(L)u_t \text{ and } Eu_t u_t' = I_k$$

Impulse response functions let us report how y responds to changes in u. Another question we might ask is: how important is variation in u for explaining variation in y? This question is addressed by reporting variance-decompositions. Then the error of the forecast of y_{t+s} given all information up to time t is

$$\hat{y}_{t+s|t} - y_{t+s} = \sum_{k=1}^{s} \tilde{C}_{s-k} u_{t+k}$$

So the MSE of the forecast is

$$MSE(\hat{y}_{t+s|t} - y_{t+s}) = \sum_{k=1}^{s} \tilde{C}_{s-k} I_k \tilde{C}'_{s-k}$$

Then we can decompose the variance of y_t at different horizons into components due to each of the shocks by looking at

$$V(s,j) = \frac{\sum_{k=1}^{s} \tilde{C}_{s-k} 1_{jj} \tilde{C}'_{s-k}}{MSE(\hat{y}_{t+s|t} - y_{t+s})}$$

where 1_{jj} is a matrix of all zeros except for the jth diagonal element, which is one. $V(s,j)$ is the portion of the variation in y at horizon s due to shock j.

Table 1 shows a variance decomposition from Blanchard-Quah (1989).

Kilian (1998) – Bootstrap after Bootstrap

In Lecture 8, we saw that the bootstrap can be used to construct confidence intervals and to bias correct nonlinear functions of consistent estimates. Our motivation for introducing the bootstrap was to compute confidence intervals for impulse-response functions. We said that

Percentage of variance due to demand

Horizon (Quarters)	Output	Unemployment
1	99.0 (76.9, 99.7)	51.9 (35.8, 77.6)
2	99.6 (78.4, 99.9)	63.9 (41.8, 80.3)
3	99.0 (76.0, 99.6)	73.8 (46.2, 85.6)
4	97.9 (71.0, 98.9)	80.2 (49.7, 89.5)
8	81.7 (46.3, 87.0)	87.3 (53.6, 92.9)
12	67.6 (30.9, 73.9)	86.2 (52.9, 92.1)
40	39.3 (7.5, 39.3)	85.6 (52.6, 91.6)

Variance decomposition of output and unemployment (Change in output growth at 1973/1974: unemployment detrended)

the asymptotic distribution of impulse-response functions might be a poor approximation because the impulse-response function is nonlinear function of our AR coefficients. One might also wonder whether the impulse-response function has bias that bootstrap can correct. Kilian (1998) addresses this question.

Let the model be:

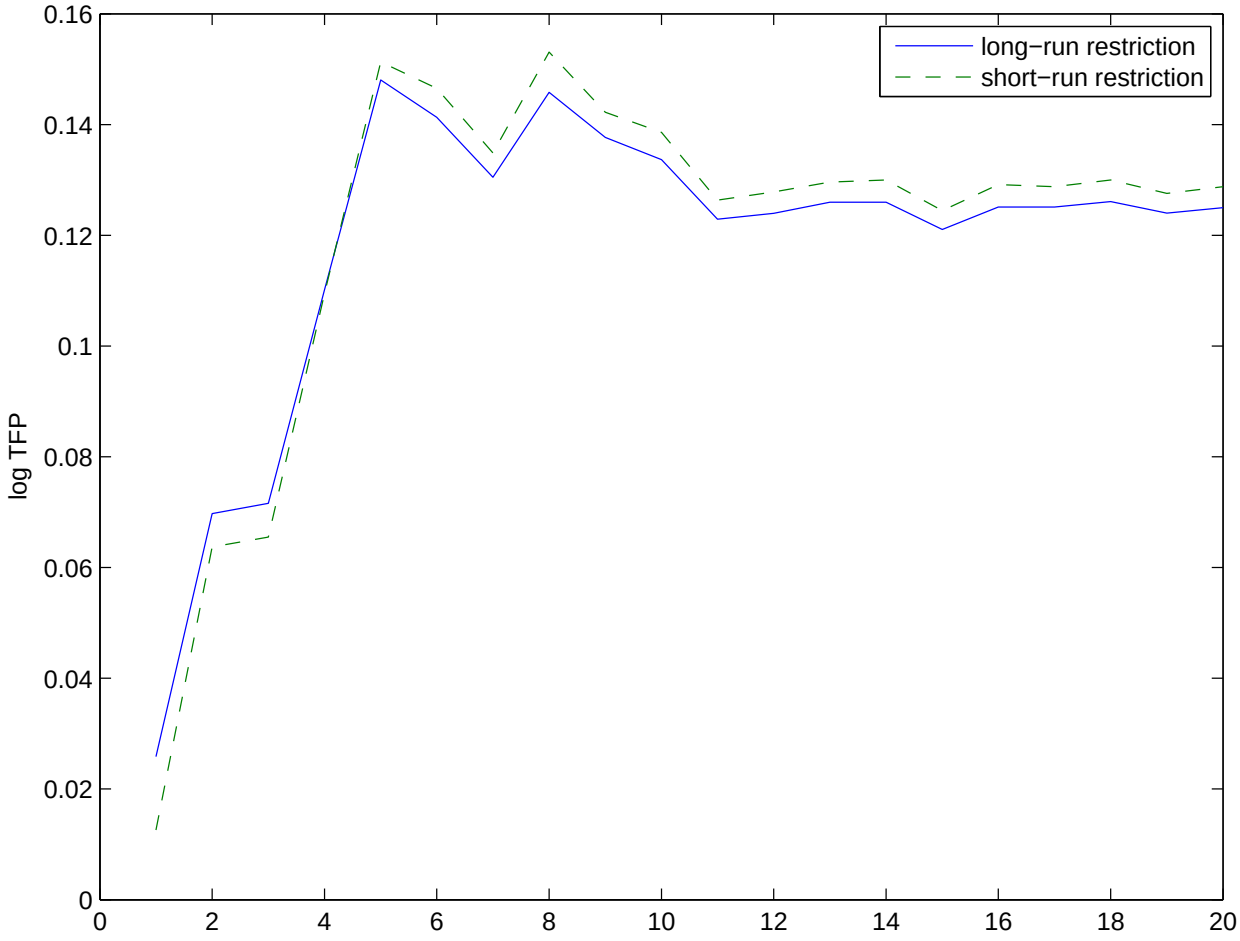
$$A(L)y_t = e_t \text{ where } A(L) \text{ is order } p$$

Let $\theta(A(L), \Sigma)$ denote the impulse response function. Kilian proposes the following bootstrap after bootstrap algorithm:

1. Estimate $\hat{A}(L)$, $\hat{\Sigma}$, $\hat{e}_t$ by OLS
2. Compute a bias correction for $\hat{A}(L)$, i.e.
   (a) For $b = 1..B$, sample $e^*_{t,b}$ from $\hat{e}_t$, form $y^*_{t,b} = \hat{A}(L)^{-1}e^*_{t,b}$.
   (b) Estimate $\hat{A}^*_b(L)$ by OLS on $y^*_{t,b}$.
   (c) Estimate the bias $\hat{\Psi} = \frac{1}{B}\sum_{b=1}^{B} \hat{A}^*_b(L) - \hat{A}(L)$
3. Apply the bias correction to form $\tilde{A}(L)$, but preserve stationarity if $\hat{A}(L)y_t$ is stationary
   (a) If a root of $det(\hat{A}(z))$ is inside the unit circle (nonstationary), let $\tilde{A}(L) = \hat{A}(L)$
   (b) If all roots of $\det(\hat{A}(z))$ are outside the unit circle (stationary), let $\tilde{A}(L) = \hat{A}(L) - \hat{\Psi}(1-\delta)^j$, where j is the minimal non-negative integer such that all the roots of $\det(\tilde{A}(L))$ are outside the unit circle
4. Bootstrap $\tilde{A}(L)$ and $\theta(\tilde{A}(L), \hat{\Sigma})$
   (a) For $b = 1..B$, sample $e^*_{t,b}$ from $\tilde{e}_t$, form $y^*_{t,b} = \tilde{A}(L)^{-1}e^*_{t,b}$.
   (b) Estimate $\hat{\tilde{A}}^*_b(L)$ and $\hat{\Sigma}^*_b$
   (c) Bias correct as above to form $\tilde{\tilde{A}}^*_b$. This would lead to a nested, bootstrap within a bootstrap. If that is too burdensome, you can reuse the original bias estimate, $\hat{\Psi}$.
   (d) Compute $\theta^*_b(\tilde{\tilde{A}}^*_b, \Sigma^*_b)$
   (e) Use quantiles of $\theta^*_b(\tilde{\tilde{A}}^*_b, \Sigma^*_b)$ to form a confidence region.

Kilian proves that this procedure is asymptotically valid. He also gives some simulation evidence of its finite sample performance. In his simulations, the bootstrap after bootstrap calculates confidence intervals of impulse responses better than the traditional bootstrap and the asymptotic distribution.

Figure 1: Impulse-response of TFP



Beaudry and Portier (2006)

Beaudry and Portier study the relationship between stock prices and TFP. They are interested in the ability of stock prices to forecast future TFP. They run a VAR on log TFP growth and log stock index growth. They consider two different identification schemes.

1. **short-run**: one of the shocks cannot have a contemporaneous effect on TFP
2. **long-run**: one of the shocks cannot have a long-run effect on TFP

One could think of approach 1 as identifying a shock to stock prices that is orthogonal to current and past TFP. Approach 2 identifies a shock that is responsible for long-run TFP changes. Their main finding is that these two shocks are nearly identical.

Beaudry and Portier's dataset can be downloaded from the AER website. On the class website, there is the Matlab code used to create figures 1 and 2. I wrote this code in an hour or two, so it is not very well-documented, it probably is not robust, and may not even be correct. Nonetheless, it could be useful for problem set 2.

Figure 2: Impulse-response of SI