MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — May 8, 1998
Prof. Alan Guth

REVIEW PROBLEMS FOR QUIZ 4

QUIZ DATE: Thursday, May 14, 1998

COVERAGE: Lecture Notes 9, 10, and 11; Problem Set 5; Silk, Chapters 9 and 10. Since chemical equilibrium was not completed in time for the previous quiz, it will be a possible topic for this quiz — so be sure to review Problem 5 of Problem Set 4 and Problem 3 of 1996 Quiz 3. Note that Lecture Notes 12 will be discussed in class, but will NOT be covered on the quiz. **One of the problems on the quiz will be taken verbatim (or at least almost verbatim) from either the homework assignment or from this set of Review Problems, or from Quiz 4 of 1996, or from one of the chemical equilibrium problems mentioned above.**

PURPOSE: These review problems are not to be handed in, but are being made available to help you study. Except for a few that are marked otherwise, they are all problems that I would consider fair for the quiz. I have included here all problems from the 1994 quizzes that are relevant to this quiz, and a number of problems from earlier years as well. Quiz 4 from 1996 will be handed out separately. If a number of points is mentioned in this handout, it is based on 100 points for the full quiz.

INFORMATION TO BE GIVEN ON QUIZ:

The following material will be included on the quiz, so you need not memorize it. You should, however, make sure that you understand what these formulas mean, and how they can be applied.

COSMOLOGICAL EVOLUTION:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

$$\ddot{R} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right)R$$

EVOLUTION OF A FLAT ($\Omega \equiv \rho/\rho_c = 1$) UNIVERSE:

$$R(t) \propto t^{2/3} \qquad \text{(matter-dominated)}$$

$$R(t) \propto t^{1/2} \qquad \text{(radiation-dominated)}$$

EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2}$$

$$\ddot{R} = -\frac{4\pi}{3} G\rho R$$

$$\rho(t) = \frac{R^3(t_i)}{R^3(t)}\, \rho(t_i)$$

Closed ($\Omega > 1$):

$$ct = \alpha(\theta - \sin\theta) \; ,$$
$$\frac{R}{\sqrt{k}} = \alpha(1 - \cos\theta) \; ,$$
$$\text{where } \alpha \equiv \frac{4\pi}{3} \frac{G\rho R^3}{k^{3/2} c^2}$$

Open ($\Omega < 1$):

$$ct = \alpha\left(\sinh\theta - \theta\right)$$
$$\frac{R}{\sqrt{\kappa}} = \alpha\left(\cosh\theta - 1\right) \; ,$$
$$\text{where } \alpha \equiv \frac{4\pi}{3} \frac{G\rho R^3}{\kappa^{3/2} c^2} \; ,$$
$$\kappa \equiv -k \; .$$

COSMOLOGICAL REDSHIFT:

$$1 + Z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

ROBERTSON-WALKER METRIC:

$$ds^2 = -c^2\, d\tau^2 = -c^2\, dt^2 + R^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right\}$$

SCHWARZSCHILD METRIC:

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2$$
$$+ r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \; ,$$

GEODESIC EQUATION:

$$\frac{d}{d\lambda}\left\{g_{ij}\frac{dx^j}{d\lambda}\right\} = \frac{1}{2}\left(\partial_i g_{k\ell}\right)\frac{dx^k}{d\lambda}\,\frac{dx^\ell}{d\lambda}$$

$$\text{or:}\qquad \frac{d}{d\tau}\left\{g_{\mu\nu}\frac{dx^\nu}{d\tau}\right\} = \frac{1}{2}\left(\partial_\mu g_{\lambda\sigma}\right)\frac{dx^\lambda}{d\tau}\frac{dx^\sigma}{d\tau}$$

COSMOLOGICAL CONSTANT:

$$p_{\text{vac}} = -\rho_{\text{vac}}c^2 \qquad \rho_{\text{vac}} = \frac{\Lambda c^2}{8\pi G}$$

where Λ is the cosmological constant.

PHYSICAL CONSTANTS:

$$G = 6.672 \times 10^{-11}\ \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$$

$$k = \text{Boltzmann's constant} = 1.381 \times 10^{-16}\,\text{erg/K}$$

$$= 8.617 \times 10^{-5}\,\text{eV/K}\ ,$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-27}\ \text{erg-sec}$$

$$= 6.582 \times 10^{-16}\ \text{eV-sec}\ ,$$

$$c = 2.998 \times 10^{10}\ \text{cm/sec}$$

$$1\ \text{eV} = 1.602 \times 10^{-12}\ \text{erg}\ .$$

BLACK-BODY RADIATION:

$$u = g\frac{\pi^2}{30}\,\frac{(kT)^4}{(\hbar c)^3} \qquad p = \frac{1}{3}u \qquad \rho = u/c^2$$

$$n = g^*\frac{\zeta(3)}{\pi^2}\,\frac{(kT)^3}{(\hbar c)^3} \qquad s = g\frac{2\pi^2}{45}\,\frac{k^4T^3}{(\hbar c)^3}\ ,$$

where

$$g \equiv \begin{cases} 1 \text{ per spin state for bosons (integer spin)} \\ 7/8 \text{ per spin state for fermions (half-integer spin)} \end{cases}$$

$$g^* \equiv \begin{cases} 1 \text{ per spin state for bosons} \\ 3/4 \text{ per spin state for fermions ,} \end{cases}$$

and

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots \approx 1.202 \ .$$

EVOLUTION OF A FLAT RADIATION-DOMINATED UNIVERSE:

$$kT = \left(\frac{45\hbar^3 c^5}{16\pi^3 g G}\right)^{1/4} \frac{1}{\sqrt{t}}$$

For $m_\mu = 106$ MeV $\gg kT \gg m_e = 0.511$ MeV, $g = 10.75$ and then

$$kT = \frac{0.860 \text{ MeV}}{\sqrt{t} \text{ (in sec)}}$$

CHEMICAL EQUILIBRIUM:

$$n_i = g_i \frac{(2\pi m_i kT)^{3/2}}{(2\pi\hbar)^3} \, e^{(\mu_i - m_i c^2)/kT} \ .$$

where n_i = number density of particle

g_i = number of spin states of particle

m_i = mass of particle

μ_i = chemical potential

For any reaction, the sum of the μ_i on the left-hand side of the reaction equation must equal the sum of the μ_i on the right-hand side. Formula assumes gas is nonrelativistic ($kT \ll m_i c^2$) and dilute ($n_i \ll (2\pi m_i kT)^{3/2}/(2\pi\hbar)^3$).

PARTICLE PROPERTIES:

While working on this exam you may refer to any of the tables in Lecture Notes 10.

PROBLEM 1: SHORT ANSWERS:

The following questions were each worth 5 points on Quiz 3, 1994:

(a) Most cosmologists believe that the entire visible universe, like the planet Earth, is composed of matter rather than antimatter. Which of the following are valid reasons for this belief? List as many as apply.

- (A) The polarization of the photons from distant galaxies confirms that they are composed of matter and not antimatter.
- (B) The fact that we receive photons and not antiphotons from distant galaxies implies that they are composed of matter and not antimatter.
- (C) The fact that gamma rays associated with matter-antimatter annihilation are not detected suggests that nearby clusters of galaxies must be entirely matter or else entirely antimatter. If both matter and antimatter have significant abundances in the visible universe, it is difficult to imagine how they became segregated into distinct regions.
- (D) The fact that we receive neutrinos and not antineutrinos from stars in distant galaxies implies that they are composed of matter rather than antimatter.

(b) If the matter/antimatter asymmetry of the observed universe evolved from an initial state of matter/antimatter symmetry, then the underlying particle physics must possess which of the following properties? List as many as apply. For each property that you list, give a one or two sentence explanation of why the property is needed for the matter/antimatter asymmetry to arise.

- (A) The symmetry C, for charge conjugation, must be violated.
- (B) The symmetry P, for parity or reflection symmetry, must be violated.
- (C) The symmetry CP, for the combination of charge conjugation and parity, must be violated.
- (D) The conservation of baryon number must be violated.
- (E) The conservation of charge must be violated.

(c) Is the mass of a grand unified theory monopole expected to be about 1 GeV, 10^{10} GeV, 10^{16} GeV, or 10^{25} GeV?

(d) According to the unified electroweak theory, at the fundamental level an electron is identical to what other type of elementary particle? According to grand unified theories, it is identical to what two other types of particles?

(e) The Higgs fields that are introduced into the electroweak theory or grand unified theories to spontaneously break the internal symmetries are always (A) scalar fields, corresponding to spin 0 particles, (B) spinor fields, corresponding to spin $\frac{1}{2}$ particles, (C) vector fields, corresponding to spin 1 particles, or (D) tensor fields, corresponding to spin 2 particles?

PROBLEM 2: A FEW MORE SHORT ANSWERS

(a) *(8 points)* Hadrons are not thought to be elementary particles but instead are composed of more fundamental particles. What is the general name for these more fundamental constituents of the hadrons? Name a kind of lepton. Do leptons feel the strong interactions? Do hadrons? [Hint: "hadrons" refers to strongly interacting particles, the baryons and mesons.]

The following question was worth 5 points on Quiz 2, 1994. This would not be a fair question for 1998, since we didn't really talk about it.

(b) At a temperature of about $3 \times 10^{15}\,^{\circ}$K ($kT \approx 300$ GeV $= 3 \times 10^{11}$ eV), it is believed that the matter in the early universe underwent a phase transition. The phase transition was marked by the change in behavior of the interactions of physics. In particular, there are two interactions that are distinct at low temperatures, but behave in a unified way at temperatures above this phase transition. What are these two interactions?

PROBLEM 3: DID YOU DO THE READING?

The following questions appeared on the last quiz of 1992. Due to differences in coverage between this year and 1992, parts (d) and (e) of this problem would not be appropriate for this year's quiz. We will discuss the inflationary universe in Lecture Notes 12, but it will not be included on the quiz.

(a) *(5 points)* Suppose the universe was created with equal numbers of particles and antiparticles, and hence no net baryon number. To explain the net asymmetry that the universe appears to have today, one must provide a mechanism for going from a baryon symmetric universe to a baryon asymmetric universe. Three crucial ingredients are needed for a viable mechanism of creating such a baryon asymmetry. One condition is a departure from thermal equilibrium. The other two necessary ingredients are: (a) baryon number violation and nonconservation of electric charge (b) baryon number violation and CP (charge-parity) violation or (c) nonconservation of electric charge and lepton number violation.

(b) *(5 points)* Quarks carry a kind of charge known as color. The quarks come in three colors: red, blue, and yellow (or green—depending on who you talk to). However, all free particles that we know of today are colorless. There are two ways to build a colorless composite particle of quarks. What are these two ways? (Hint: One combination of quarks corresponds to the baryons and the other to the mesons.)

(c) *(5 points)* Grand unified theories (GUTs) predict the existence of magnetic monopoles, point-like topological defects. Is the mass of the monopole expected to be roughly (a) 10^{16} GeV (b) 1 GeV (c) 1/2 MeV or (d) zero? What is the most serious cosmological problem associated with the existence of GUT monopoles?

(d) *(5 points)* The cosmological evolution of the universe is described by the two Einstein equations given on the cover sheet of the exam. When the energy density of the universe is dominated by matter or by radiation, then the acceleration of the scale factor, described by one of the Einstein equations, is negative. Consequently the expansion of the universe is slowed. During an inflationary era, is the acceleration of the scale factor positive or negative? Why?

(e) *(5 points)* The standard calculations of big bang nucleosynthesis predict the abundances of light elements as functions of other parameters, such as the lifetime of the neutron and the mass density in ordinary baryonic matter. Let $\Omega_{\rm B} \equiv \rho_{\rm B}/\rho_c$, where $\rho_{\rm B}$ is the mass density in ordinary baryonic matter and ρ_c is the critical density (i.e., the density that is just barely sufficient to halt the expansion). For approximately what value of $\Omega_{\rm B}$ do the predictions of big bang nucleosynthesis agree with the current observations? Briefly identify one argument of modern cosmology which suggests that the total value of $\Omega = 1$.

PROBLEM 4: QUARK DIAGRAMS *(25 points)*

a) *(10 points)* The Ξ^{*-} (xi star minus, also called the cascade star minus) is not shown on the quark content diagram, because the particle is really an excited state of the Ξ^-, and has the same quark content. One likely decay for this particle is

$$\Xi^{*-} \rightarrow \Xi^0 \, \pi^- \ .$$

Draw a quark diagram for this decay. There might be more than one variation, but you are asked to draw only one.

b) *(10 points)* One common decay for the D^+ is

$$D^+ \rightarrow \bar{K}^0 \, \pi^+ \ .$$

The $\bar{K}^0$ has quark content $s\bar{d}$. (It is not shown on the quark content table because it does not remain a $\bar{K}^0$, but immediately undergoes a partial conversion into its own antiparticle, K^0, forming the superposition K zero short or K zero long. This complication, however, is not relevant to the question you are asked.) Draw a quark diagram for this decay ($D^+ \rightarrow \bar{K}^0 \, \pi^+$). Again there might be more than one variation, but you are asked to draw only one.

c) *(5 points)* Is the decay process described in (a) a strong or weak interaction? Is the decay process described in (b) a strong or weak interaction? Explain briefly.

PROBLEM 5: THE TOP QUARK *(30 points)*

The following problem was on the last quiz of 1994:

On Tuesday, April 26, 1994, the CDF (Collider Detector at Fermilab) Collaboration, consisting of 440 physicists from 34 institutions (including MIT), released a paper titled "Evidence for Top Quark Production in $\bar{p}p$ Collisions at $\sqrt{s} = 1.8$ TeV." $\bar{p}p$ refers to the colliding beams of antiprotons and protons in the Fermilab accelerator, and $\sqrt{s}$ refers to the total energy: 1.8 tera-electron volts, or 1.8×10^{12} eV. They found a total of 12 events that looked like top quark production, but they estimate that 6 of these events were most likely due to "background"— that is, events that did not involve a top quark, but which looked similar. Nonetheless, the group estimates that there is only 1 chance in 400 that all 12 events could be attributed to background.

(a) *(8 points)* One method by which top quarks are believed to have been produced in this experiment is the reaction

$$u\,\bar{u} \longrightarrow t\,\bar{t}\ ,$$

where the u and $\bar{u}$ quarks are part of the initial proton and antiproton, respectively. (Note that $\bar{u}$ denotes the antiparticle of the u quark.) Draw a quark diagram for this process. Is it a strong, electromagnetic, weak, or gravitational process?

(b) *(7 points)* In the same $p\bar{p}$ collisions, $t\bar{t}$ pairs can also be produced from a different type of quark-antiquark pair. What type is that?

(c) *(8 points)* One of the principal decay modes for the t quark is

$$t \longrightarrow b\,e^+\,\nu_e\ .$$

Draw a quark diagram for this decay. Is it a strong, electromagnetic, weak, or gravitational process?

(d) *(7 points)* Another principal decay mode for the t quark is

$$t \longrightarrow b\,u\,\bar{d}\ .$$

Draw a quark diagram for this decay. Is it a strong, electromagnetic, weak, or gravitational process?

PROBLEM 6: SHORT ANSWER QUESTIONS

Parts (c) and (d) are based on the **Scientific American** *article by Lawrence Krauss ("Dark Matter in the Universe", December 1986; reprinted in* **Particle Physics in the Cosmos**, *edited by Richard A. Carrigan, Jr. and W. Peter Trower). This source will not be covered in the coming quiz, but I include these questions anyway for the sake of general interest.*

(a) The V-A theory of the weak interactions, developed in 1958 by Feynman and Gell-Mann and independently by Marshak and Sudarshan, is listed on Table 3 of Lecture Notes 10 as a "flawed theory". In what way is this theory flawed. (Please answer in no more than 2-3 sentences.)

(b) The word "supersymmetry" refers to a symmetry that relates the behavior of one certain class of particles with the behavior of another class. What are these two classes?

(c) The best-documented evidence for dark matter is based on measurements by Vera Rubin of the velocities of rotation in spiral galaxies. Are these velocities larger or smaller than the velocities that would be expected in the absence of dark matter? Does the rotational velocity fall off rapidly with distance from the center of the galaxy, remain roughly constant with distance from the center, or does it grow rapidly with distance from the center?

(d) One candidate for the dark matter is the neutrino, which might have a nonzero mass. Since the mass would have to be quite small, however, one expects that the neutrinos would have moved at speeds comparable to the speed of light until almost the time of decoupling. In a few sentences, explain qualitatively what effect this would have on the evolution of structure in the universe.

(e) The prevailing belief that the universe is made primarily of matter, and not a 50-50 mix of matter and antimatter, is based on which of the following lines of reasoning:

   (i) Observations of the light emitted from distant galaxies indicate that they are probably composed of matter and not antimatter.

   (ii) If the universe consisted of patches of matter and patches of antimatter, then we would see radiation emitted at the boundaries where matter and antimatter would undergo annihilation. This radiation would be so powerful that we would detect it no matter where in the observable universe it originated.

   (iii) If the universe were composed partly of matter and partly of antimatter, then it is hard to imagine how the two components could have become so segregated into distinct regions.

(f) The hope for a dynamical explanation of the cosmic asymmetry between matter and antimatter depends crucially on the belief that total baryon number is not exactly conserved. It turns out that the properties of black holes provide an indication that this belief is correct. Give a short explanation.

(g) For the matter-antimatter asymmetry to be generated as the universe evolves, it is necessary that the laws of physics violate the symmetry known as CP. What does CP stand for?

PROBLEM 7: TIME SCALES IN COSMOLOGY

In this problem you are asked to give the approximate times at which various important events in the history of the universe are believed to have taken place. The times are measured from the instant of the big bang. To avoid ambiguities, you are asked to choose the best answer from the following list:

10^{-43} sec.
10^{-35} sec.
10^{-12} sec.
10^{-5} sec.
1 sec.
4 mins.
10,000 – 1,000,000 years.
2 billion years.
5 billion years.
10 billion years.
13 billion years.
20 billion years.

For this problem it will be sufficient to state an answer from memory, without explanation. The events which must be placed are the following:

(a) the beginning of the processes involved in big bang nucleosynthesis;

(b) the end of the processes involved in big bang nucleosynthesis;

(c) the time of the phase transition predicted by grand unified theories, which takes place when $kT \approx 10^{14}$ GeV;

(d) "recombination", the time at which the matter in the universe converted from a plasma to a gas of neutral atoms;

(e) the phase transition at which the quarks became confined, believed to occur when $kT \approx 300$ MeV.

Since cosmology is fraught with uncertainty, in some cases more than one answer will be acceptable. You are asked, however, to give **ONLY ONE** of the acceptable answers.

PROBLEM 8: QUARK DIAGRAMS

Draw quark diagrams for the following particle decays:

(a) $\Sigma^{*+} \rightarrow \Lambda\,\pi^+$

(b) $\Omega^- \rightarrow \Lambda\,K^-$

(c) $\Xi^0 \rightarrow \Lambda\,\pi^0$

PROBLEM 9: MORE QUARK DIAGRAMS

(a) Draw a quark diagram for the decay $\Xi^{*-} \rightarrow \Lambda\pi^-$. Is this a weak or a strong interaction? Although this decay is certainly allowed, it has nonetheless never been seen. Explain why.

(b) Through a strong interaction process it is possible to produce a particle called a K^0, with quark content $d\bar{s}$. This particle has not been included in the tables of Lecture Notes 10, however, because it does not remain a K^0. Instead, it partially converts to its own antiparticle, a $\bar{K}^0$, with quark content $\bar{d}s$. It then continues to exist as a superposition of the two. Draw a quark diagram for the conversion $K^0 \rightarrow \bar{K}^0$. Note that no other particles are given off. *[Hint: you will find it necessary to use two intermediate W lines in this diagram.]*

PROBLEM 10: BLACK-BODY RADIATION AT THE GUT SCALE

The simplest of the grand unified theories is the model that was first proposed by Howard Georgi and Sheldon L. Glashow in 1974. The particle content of this model includes of course the known quarks and leptons, as well as the as-yet-unobserved top quark and tau neutrino. In addition, the model includes a multiplet of 24 vector bosons, each of which behaves in a manner similar to photons. (This multiplet includes the particles that are later identified as the photon, the $W^{\pm}$, and the Z^0.) Finally, the model requires 34 spinless Higgs particles.

When kT is about 10^{15} GeV, all of these particles (including the quarks) would act like non-interacting massless particles, contributing to the black-body radiation.

(a) Find the value of g to be used in the calculation of the mass density of black-body radiation at $kT = 10^{15}$ GeV. Be sure to carefully indicate the various contributions to the answer, so that partial credit can be assigned.

(b) According to this grand unified theory, what is the mass density of black-body radiation at $kT = 10^{15}$ GeV? Please express your answer in gm/cm^3.

PROBLEM 11: GRAND UNIFIED THEORIES AND MAGNETIC MONOPOLE PRODUCTION

When grand unified theories are combined with standard (i.e., non-inflationary) cosmology, one is led to the conclusion that far too many magnetic monopoles are produced. This conclusion is based on an estimate of n_M/n_γ, the ratio of the number density of magnetic monopoles to the number density of photons. The estimated value of n_M/n_γ is proportional to a power of the critical temperature T_c of the grand unified theory phase transition. State the power, and explain why.

SOLUTIONS

PROBLEM 1: SHORT ANSWERS:

(a) This issue is discussed in *"The Cosmic Asymmetry Between Matter and Antimatter"*, by Frank Wilczek (**Scientific American**, December 1980). Statement (A) is false: polarization can provide evidence of magnetic fields, but it does not distinguish between matter and antimatter. Statement (B) is also false: a photon is its own antiparticle, so there is no such thing as an antiphoton. Statement (C) is true. It may not sound like a completely compelling reason, but it is the best that we have. Statement (D) is false. In principle, an anti-star can be distinguished from a star by the fact that it emits mainly antineutrinos rather than neutrinos. Unfortunately, however, we can detect neither neutrinos nor antineutrinos from any star other than our sun.

(b) This issue is also discussed in the **Scientific American** article by Frank Wilczek. Statement (A) is true. If C (charge conjugation symmetry) were not violated, then the symmetry would guarantee equal probabilities for the production of matter and antimatter. While the randomness of the production process would lead to a random excess of one type over the other, the laws of statistics imply that this imbalance would be far smaller than the observed excess of matter over antimatter. Statement (B) is false: parity symmetry relates a proton with spin along its momentum to a proton with spin opposite its momentum, but it says nothing about the relation between protons and antiprotons. Statement (C) is true. If CP (charge conjugation times parity symmetry) were not also violated, then this symmetry would guarantee equal probabilities for the production of matter and antimatter. That is, for every right-handed proton that is produced, there would be an equal probability that a left-handed anti-proton was produced. (Here "right-handed" and "left-handed" refer to the spin of the proton, which is either along the momentum or opposite it. Many accounts place more emphasis on CP violation then on C violation, but they are both essential. CP tends to be given more press coverage, apparently for historical reasons. C violation was known much earlier, so for many years CP symmetry seemed to be the obstacle to baryogenesis. So when CP violation was finally discovered, it was natural to think of it as the key to baryogenesis.) Statement (D) is true: if baryon number were exactly conserved, then matter and antimatter would always have to be produced in equal amounts, so the excess of matter could not be explained. Statement (E) is false: charge is believed to be exactly conserved, but this poses no obstacle to baryogenesis.

(c) 10^{16} GeV, as discussed in Lecture Notes 11.

(d) In the electroweak theory, the electron is fundamentally the same as the neutrino. In grand unified theories, the electron is fundamentally the same as both the neutrino and the quark. (Many students mentioned the symmetry between the electron, the muon, and the tau, which is a part of the symmetry between generations of fermions.

This symmetry remains a mystery, not explained by either the electroweak theory or grand unified theories.)

(e) Choice (A): the Higgs fields are scalars. This is essential for the consistency of the theory, since the Higgs fields have a nonzero value in the vacuum. If they were not scalars, then this nonzero value would be measured differently by observers who were rotated with respect to each other, so the rotational invariance of the vacuum would be violated. (The electron, by the way, is a spin-$\frac{1}{2}$ particle, the photon is an example of a spin 1 particle, and the graviton has spin 2.)

PROBLEM 2: A FEW MORE SHORT ANSWERS

(a) Hadrons are thought to be built out of quarks. Electrons, neutrinos, muons are all examples of leptons. Leptons do not feel the strong interactions. Hadrons do feel the strong interactions.

b) The weak and electromagnetic interactions. At temperatures above $kT \approx 300$ GeV (1 GeV $\equiv 10^9$ eV), these two interactions are believed to merge into what is often called the electroweak interaction. (It is also speculated, in a class of theories called grand unified theories, that the electroweak interactions merge with the strong interactions at $kT \approx 10^{16}$ GeV.)

PROBLEM 3: DID YOU DO THE READING?

a) The three necessary ingredients for a viable mechanism of creating the baryon asymmetry are a departure from equilibrium, (b) baryon number violation, and CP (charge-parity) violation.

b) A bound state of three quarks, one of each color, results in a colorless combination. All baryons are built out of three quarks. A bound state of a quark of some color and an antiquark carrying that anticolor is colorless. All mesons are built out of such quark-antiquark pairs.

c) GUT monopoles are expected to have a mass of (a) 10^{16} GeV. With the abundance predicted by GUTs, the incredibly massive monopoles would quickly come to dominate the energy density of the universe. The large energy density would speed up the cosmological evolution so that the universe was a mere few tens of thousands of years old today. Such a young and quickly evolving universe is in serious conflict with observations. Also, there are no confirmed observations of monopoles.

d) During an inflationary era the acceleration of the scale factor is positive. The pressure associated with the false vacuum which drives inflation is negative. The contribution to the acceleration from the negative pressure dominates over that from the positive energy density in the equation

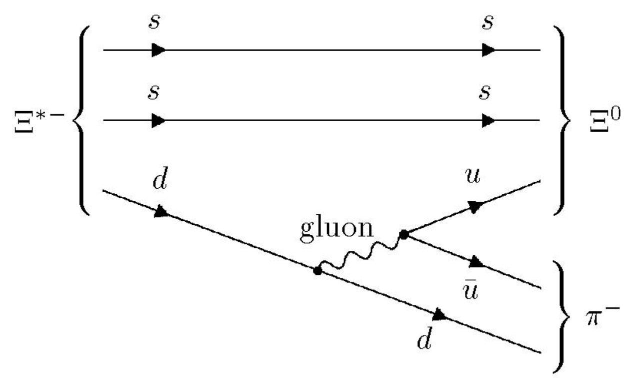
$$\ddot{R} = -\frac{4\pi}{3} G \left(\rho + \frac{3p}{c^2} \right) R \quad .$$

Consequently, the negative pressure powers a repulsive gravitational force.

e) In the *Scientific American* article by Lawrence Krauss (*"Dark Matter in the Universe"*, December 1986; reprinted in *Particle Physics in the Cosmos*, edited by Richard A. Carrigan, Jr. and W. Peter Trower), the value $\Omega_B < 0.2$ was given. However a more recent value is $\Omega_B \approx 0.012\ h_0^{-2} \pm 0.003$, where $h_0 = H_0/(100\ \text{km/sec/Mpc})$ and H_0 is the Hubble constant today. h_0 can vary from 1 to about 0.4. Inflation predicts that Ω will be driven to 1, regardless of its initial value.

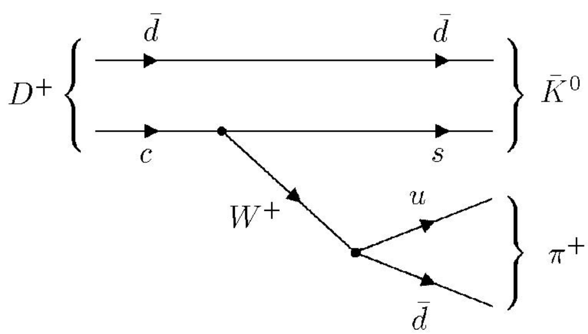
PROBLEM 4: QUARK DIAGRAMS

a)

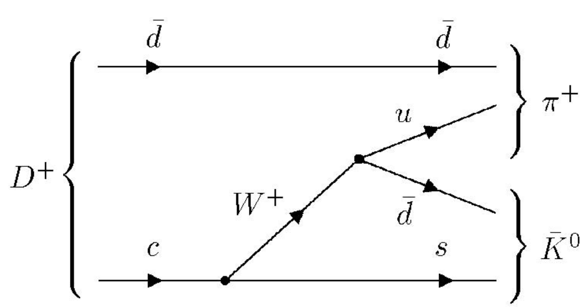


Other diagrams are also possible. In particular, the gluon that creates the u-$\bar{u}$ pair can be emitted from any one of the three quark lines.

b)



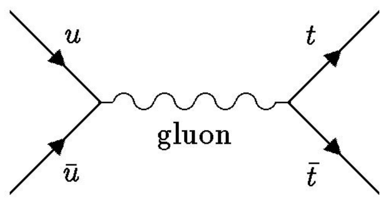
or



c) The diagram for part (b) contains a W particle, while the diagram in (a) contains only gluons. Thus the reaction in (a) is a strong interaction, while the reaction in (b) is a weak interaction.

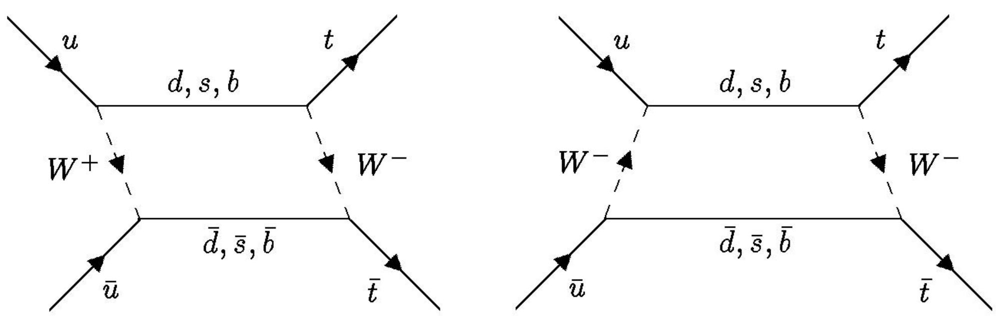
PROBLEM 5: THE TOP QUARK

a) The primary mechanism for $u\,\bar{u} \longrightarrow t\,\bar{t}$ is illustrated by the diagram:

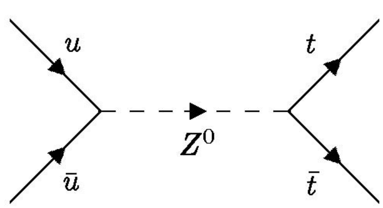


Since the process involves only gluons as intermediate particles, it is a strong interaction.

A number of students drew weak interaction diagrams, such as



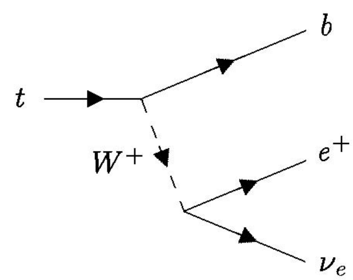
or



Here the label "d, s, b" indicates a single particle which might be a d, s, ***OR*** b; more explicitly, I could have drawn three diagrams and labeled one with a d, one with an s, and one with a b. These diagrams are valid, but the presence of the intermediate vector bosons W^+, W^-, and Z^0 implies that they are weak interactions. These processes, therefore, are much rarer than the dominant strong interaction that was sketched above. (More precisely, in the language of quantum theory, these processes make only a negligible contribution to the probability amplitude for the process $u\,\bar{u} \longrightarrow t\,\bar{t}$.) I gave partial credit for diagrams of this sort, but only the strong interaction diagram received full credit.

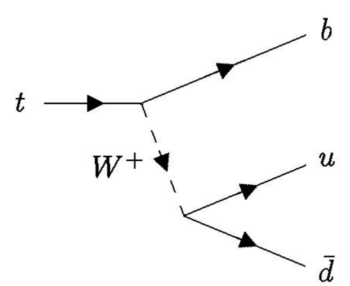
b) The proton contains u and d quarks, while the antiproton contains only $\bar{u}$ and $\bar{d}$ quarks. Thus, the combination $d\,\bar{d}$ can produce $t\,\bar{t}$ pairs in the same way that $u\,\bar{u}$ pairs do. (Some students gave $c\,\bar{c}$ as an answer to this question. It is true that $c\,\bar{c}$ can produce a $t\,\bar{t}$ pair by an analogous process, but the c and $\bar{c}$ quarks are not contained in the protons or antiprotons.)

c) The diagram for $t \longrightarrow b\,e^+\,\nu_e$ is:



It is a weak interaction, due to the presence of the W^+ in the intermediate state. Note that the emission or absorption of a gluon never changes the identity of a quark, so there is no way that this reaction can happen by gluons alone.

d) The diagram for $t \longrightarrow b\,u\,\bar{d}$ is given by:



As in (c), this is a weak interaction process.

PROBLEM 6: SHORT ANSWER QUESTIONS

(a) It was flawed in that it was not "renormalizable". That is, calculations based on quantum mechanical perturbation theory lead to infinities in the sums over intermediate states, which is similar to the situation with QED (quantum electrodynamics). Unlike QED, however, the infinities here cannot be absorbed into a redefinition of the fundamental constants of the theory.

(b) Fermions and bosons.

(c) As discussed in the **Scientific American** article by Lawrence Krauss, the velocities are larger than expected, indicating that if the orbits are stable there must be more mass in the galaxies than can be accounted for in visible stars. The rotational velocity is observed to be roughly constant with distance. (This, by the way, refers to linear velocity, not angular velocity.)

(d) Again from Krauss' article, one learns that the relativistic speeds of the neutrinos prior to decoupling lead to the classification of neutrinos as "hot dark matter". The effect of this motion is to smooth out density perturbations on all length scales up to about the horizon distance at the time of decoupling, which corresponds roughly to the scale of superclusters today. Thus the only perturbations that survive are at longer wavelengths, so in this model it has to be the largest structures that form first. In fact, detailed numerical simulations of a neutrino-dominated universe show that ordinary galaxies form too late for the model to be acceptable.

(e) As discussed in the **Scientific American** article by Frank Wilczek, the correct answer is (iii). *[Note: statement (i) is blatantly false, since the photons that would be given off by a galaxy of antimatter would be indistinguishable from those given off by a galaxy of matter. Neutrinos, on the other hand, would indicate the difference, if they could be detected. Statement (ii) is true for patches of matter and antimatter within our own supercluster, but we would not be able to detect the annihilation radiation that would result if some of the other superclusters were composed of antimatter.]*

(f) Again as discussed in Wilczek's article, general relativity implies that the only properties of a black hole that can be measured from the outside are its mass, its charge, and its angular momentum. *[Actually, if the black hole had a net magnetic charge, which is allowed for example in grand unified theories, the magnetic charge would also be measurable.]* Since baryon number is not included on this list, the baryon number of a system can change if some of the baryons disappear by falling into a black hole.

(g) Wilczek's article explains that CP is the combined symmetry of charge conjugation and parity. *[Note: To understand what this means, imagine that a motion picture of a physical experiment is shown through a mirror so that left and right are reversed (parity symmetry). An exact CP symmetry would imply that if the identification of the particles were modified to substitute each particle by its antiparticle (charge conjugation), then the resulting image on the screen would show an experiment completely* *consistent with the laws of physics. CP violation is necessary for the dynamical generation of the matter-antimatter asymmetry, because an exact CP symmetry would imply that for each baryon that is produced, on average an antibaryon would also be produced, in a state that would be the mirror image of the state of the baryon.]*

PROBLEM 7: TIME SCALES IN COSMOLOGY

(a) 1 sec. *[This is the time at which the weak interactions "freeze out", so that free neutron decay becomes the only mechanism that can interchange protons and neutrons. From this time onward, the relative number of protons and neutrons is no longer controlled by thermal equilibrium considerations.]*

(b) 4 mins. *[By this time the universe has become so cool that nuclear reactions are no longer initiated.]*

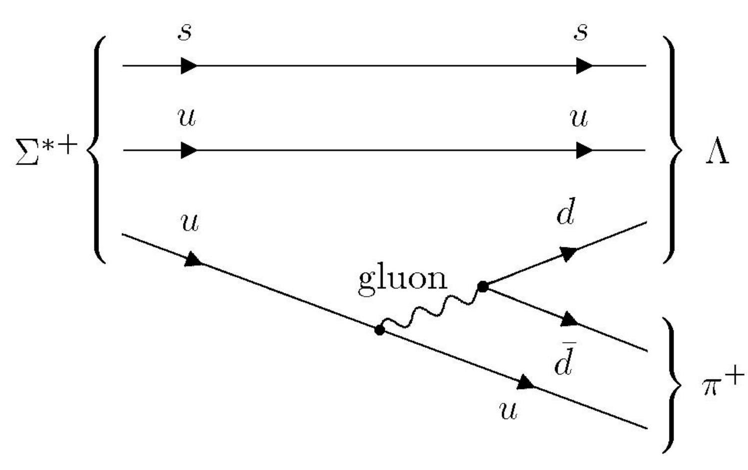
(c) 10^{-35} sec. *[We learned in Lecture Notes 7 that kT was about 1 MeV at $t = 1$ sec. Since 1 GeV = 1000 MeV, the value of kT that we want is 10^{17} times higher. In the radiation-dominated era $T \propto R^{-1} \propto t^{-1/2}$, so we get 10^{-34} sec.]*

(d) 10,000 – 1,000,000 years. *[This number was estimated in Lecture Notes 7 as 200,000 years.]*

(e) 10^{-5} sec. *[As in (c), we can use $t \propto T^{-2}$, with $kT \approx 1$ MeV at $t = 1$ sec.]*

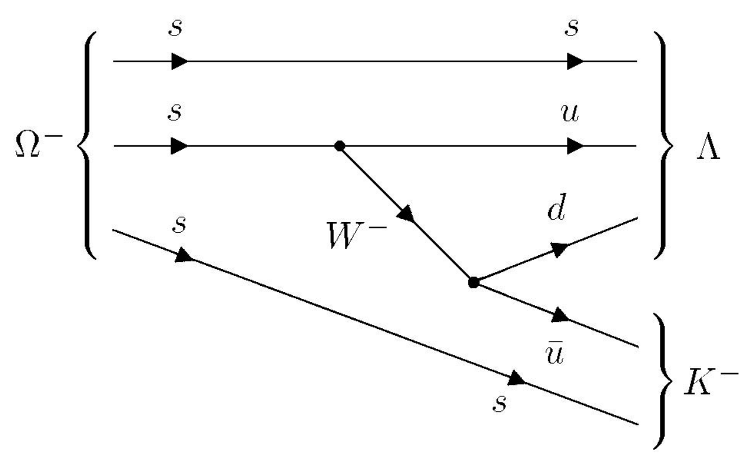
PROBLEM 8: QUARK DIAGRAMS

(a) $\Sigma^{*+} \rightarrow \Lambda\,\pi^+$



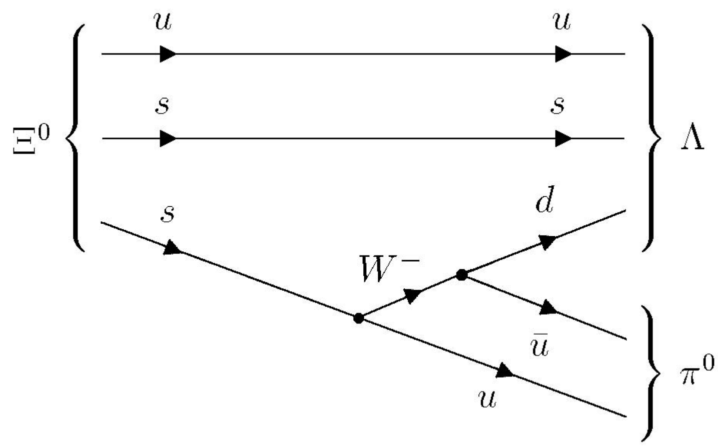
Note that by comparing the quark content of the initial and final states, one can see that the reaction requires the creation of a d-$\bar{d}$ pair. This extra d-$\bar{d}$ pair can be created by a gluon. The general rule is that a gluon can be emitted or absorbed by any quark without changing its flavor, it can create a quark-antiquark pair of the same flavor, or a quark-antiquark pair of the same flavor can annihilate to create a gluon. The gluon that creates the d-$\bar{d}$ pair can be emitted from any of the quark lines, and an arbitrary number of gluon exchanges can be added to the diagram. The same reaction could take place with either a photon or a Z^0 taking the place of the gluon, but such diagrams would have the strength of the electromagnetic or weak interactions, respectively. The strong interaction diagram involving the gluon will therefore dominate over the other possibilities.

(b) $\Omega^- \to \Lambda K^-$



This time the counting of quarks shows that the net effect of the reaction is to turn an s quark into a u, d, and $\bar{u}$. Thus quark flavors must be changed, which implies that W particles must be involved. The rules for the couplings of W particles are explained in Problem 4 of Problem Set 7. The only way an s quark can disappear is to emit a W^- and then become a charge $\frac{2}{3}$ quark. The simplest possibility is then for that charge $\frac{2}{3}$ quark to be the needed u, and then the W^- can decay into the d-$\bar{u}$ pair. Since this reaction requires a W, it is a weak interaction. (Interactions which require Z^0 intermediate particles are also weak interactions.)

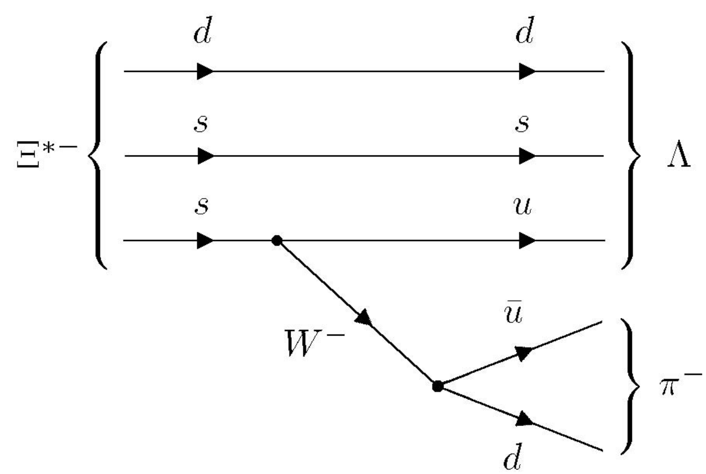
(c) $\Xi^0 \rightarrow \Lambda\, \pi^0$



Quark counting here shows that the net reaction is the same as the previous case, with one s quark turning into a u, d, and $\bar{u}$. The simplest way to do this is again for an s quark to emit a W^-, becoming a u quark. The W^- can then decay into a d-$\bar{u}$ pair.

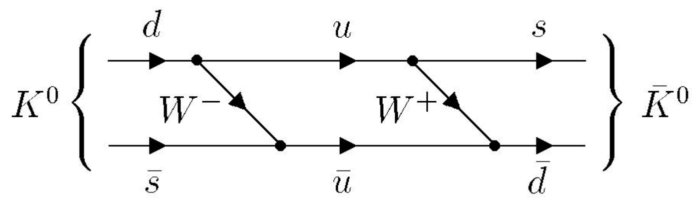
PROBLEM 9: MORE QUARK DIAGRAMS

(a) By comparing quark contents of initial and final states, one can see that a d and an s quark continue through the reaction as "spectators", while an s-quark is converted into a $u\bar{u}d$. The conversion can occur by the s-quark emitting a W^-, thereby becoming a u. The W^- can then decay into a $\bar{u}d$:



Since the reaction requires a W-meson, it is a weak interaction. The Ξ^{*-} can also decay to $\Xi^0\pi^-$ or $\Xi^-\pi^0$, with a lifetime of about 10^{-22} sec. This is clearly a strong interaction, so its rapid rate will make it impossible to see the much rarer weak decays.

(b) This reaction must convert a d-quark to an s-quark, and similarly a $\bar{s}$-quark to a $\bar{d}$-quark. (It must be this way and not $d \to \bar{d}$, since baryon number is conserved by the weak or strong interactions.) The conversion of a d to an s changes the identity of the quark but conserves charge, and this cannot be done in a single step. To see this, note that the only conceivable way to do it in one step would be to emit a neutral vector boson— a Z^0 or a gluon. But none of the neutral vector mesons can change the identity (flavor) of a quark, so it has to be a two step process. For example, a d can emit a W^- to become a u-quark, which in turn can emit a W^+ to become an s-quark. Putting the pieces together:



Note that the intermediate u and $\bar{u}$ could just as well have been a c and $\bar{c}$.

[Note: this reaction, in fact, was a significant part of the motivation for inventing the c quark in the first place. During the 1960s it was noticed that the rate for this reaction is much **smaller** *than one would expect from the diagram shown above, but no one understood why. In the early 1970s Glashow, Iliopoulos, and Maiani pointed out that the low probability for this and several similar reactions could be explained by assuming the existence of a fourth quark, which was a radical notion at the time. The fourth quark was assumed to interact in such a way that the diagrams with intermediate c-quarks would destructively interfere, in the quantum mechanical sense, with the diagram above, so that the net rate would be very small. It has now been verified that the charmed quark interacts exactly as predicted.]*

PROBLEM 10: BLACK-BODY RADIATION AT THE GUT SCALE

(a) The u-quark has 3 colors with 2 spin states for each, for a total of 6 states. The $\bar{u}$-quark contributes the same, bringing the total to 12. Including also the d and $\bar{d}$ quarks, the total is up to 24. The first generation of spin-$\frac{1}{2}$ particles is completed by adding a contribution of 4 for e^+e^- (2 spin states each), and a contribution of 2 from ν_e and $\bar{\nu}_e$. The total is then 30 for the quarks and leptons of the first generation only. Adding the next two generations brings the number of spin states up to 90. These particles are all fermions, and therefore g contains a factor of 7/8. Thus, the contribution to g from all the quarks and leptons is $\frac{7}{8} \times 90 = 78\frac{3}{4}$. Each of the "photon-like" vector bosons contributes 2 to g, for a total of 48. Finally, each spinless

Higgs particle contributes 1 to g, for a total of 34. Adding these contributions gives $\boxed{g = 160\frac{3}{4}.}$

(b) Using $\rho = u/c^2$ and the formula for u on the exam,

$$\rho = (160.75)\frac{\pi^2}{30}\frac{\left(10^{15}\ \text{GeV}\right)^4}{\left(6.582 \times 10^{-16}\ \text{eV-sec}\right)^3 \left(2.998 \times 10^{10}\ \text{cm-sec}^{-1}\right)^5}$$
$$\times \left(\frac{10^9\ \text{eV}}{1\ \text{GeV}}\right)^4 \frac{1.602 \times 10^{-12}\ \text{erg}}{1\ \text{eV}} .$$

Using 1 erg = 1 gm-cm^2-sec^{-2}, the units become

$$\frac{\text{GeV}^4}{\text{eV}^3\text{-sec}^3\text{-cm}^5\text{-sec}^{-5}} \times \frac{\text{eV}^4\text{-gm-cm}^2\text{-sec}^{-2}}{\text{GeV}^4\text{-eV}} = \text{gm/cm}^3 .$$

So

$$\boxed{\rho = 1.23 \times 10^{79}\ \text{gm/cm}^3 .}$$

PROBLEM 11: GRAND UNIFIED THEORIES AND MAGNETIC MONOPOLE PRODUCTION

Then number density of monopoles produced in the grand unified theory phase transition is roughly one per horizon volume. The horizon distance is proportional to t, which in turn is proportional to $1/T_c^2$. Thus, the number density of magnetic monopoles is proportional to $1/t^3 \propto T_c^6$. The number density of photons at temperature T_c is proportional to T_c^3, So the ratio

$$\boxed{n_M/n_\gamma \propto T_c^3 .}$$