MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — February 23, 1994
Prof. Alan Guth

QUIZ 1

USEFUL INFORMATION:

DOPPLER SHIFT:

$$Z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$Z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$Z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(relativistic)}$$

SPECIAL RELATIVITY:

Time Dilation Factor:

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \ , \qquad \beta \equiv v/c$$

Lorentz-Fitzgerald Contraction Factor: γ

Relativity of Simultaneity:

Trailing clock reads later by an amount $\beta \ell_0 / c$.

Lorentz Transformation:

$$x'^0 = \gamma \left(x^0 - \beta x^1 \right)$$

$$x'^1 = \gamma \left(x^1 - \beta x^0 \right)$$

$$x'^2 = x^2$$

$$x'^3 = x^3 \ ,$$

where $x^0 \equiv ct$, $x^1 \equiv x$, $x^2 \equiv y$, and $x^3 \equiv z$.

Lorentz-Invariant Dot Product:

$$x \cdot y \equiv x^1 y^1 + x^2 y^2 + x^3 y^3 - x^0 y^0 \ .$$

Four-Velocity and Four-Momentum:

$$u^\mu \equiv \frac{dx^\mu}{d\tau} \qquad u^i = \gamma v^i \quad (i = 1, 2, 3) \qquad u^0 = \gamma c \ .$$

$$u^2 \equiv -c^2 \ .$$

$$p^\mu \equiv m u^\mu \ , \qquad p^2 \equiv -m^2 c^2 \ .$$

$$E = \sqrt{\vec{p}^{\,2} c^2 + m^2 c^4} \ .$$

where m is the rest mass.

PROBLEM 1: DID YOU DO THE READING? *(25 points)*

The following questions are worth 5 points each:

a) In 1750 the English instrument maker Thomas Wright published *Original Theory or New Hypothesis of the Universe*. In this book Wright described an astronomical object that is known today as the Crab Nebula, the solar system, the Milky Way, or the local supercluster?

b) In 1755 Immanuel Kant published his *Universal Natural History and Theory of the Heavens*. What new hypothesis was put forward in this book?

c) Estimate the diameter and thickness of the disk of the Milky Way galaxy. Any numbers within a factor of 2 of those given in Weinberg's book will be accepted.

d) The mathematical theory of an expanding universe was first published in 1922 by the Russian mathematician Alexandre Friedmann, the Dutch Astronomer Willem de Sitter, the American astronomer Edwin Hubble, or the Belgian cleric Georges Lemaître?

e) After discovering an inexplicable hiss coming from their radio telescope, Arno Penzias and Robert Wilson of Bell Laboratories learned that P.J.E. Peebles, a Princeton theorist, had calculated that the big bang would produce a background of cosmic radiation with a temperature today of 10° K. What MIT radio astronomer informed them of Peebles' work? [Announcement: Peebles will be visiting MIT to give the Herman Feshbach Lectures on March 10, 11, and 15.]

PROBLEM 2: THRESHOLD FOR PARTICLE PRODUCTION *(25 points)*

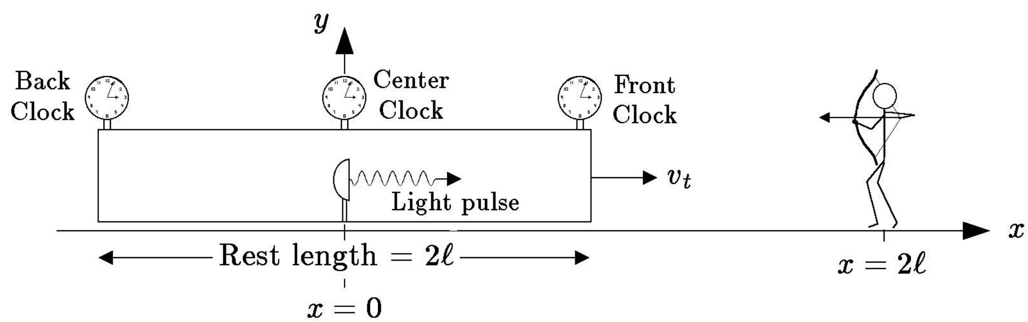
An electron of energy E (in the laboratory frame) collides with a proton at rest. Note that E denotes the ***total*** energy of the electron, which means kinetic energy plus rest energy. Let m_e denote the rest mass of the electron and let m_p denote the rest mass of the proton. In terms of m_e, m_p, and c, what is the minimum value of E that would allow the reaction

$$e^- + p \rightarrow e^- + p + p + \bar{p}$$

to take place? Here $\bar{p}$ denotes an antiproton, which also has a rest mass m_p. *(Hint: when E is at its minimum, the four particles of the final state will all move together, as if they were one particle of rest mass $m_e + 3m_p$. Any motion of these particles relative to each other would require a larger value of E.)*

PROBLEM 3: TRAINS, LIGHT PULSES, AND ARROWS *(25 points)*

A fast train of rest length 2ℓ moves along a straight track at speed v_t (subscript t for "train"). Clocks aboard the train are synchronized in the frame of the train, while clocks on the ground are synchronized in the frame of the ground. Along the track is a coordinate axis x, calibrated in meters as measured by an observer at rest with respect to the ground. The clocks on the ground read $t = 0$ at the instant when the center of the train passes $x = 0$. At this instant the clock on board the train, at its center, reads $t' = t_0$.



a) *(5 points)* At the instant the center of the train passes $x = 0$, a light pulse is sent out from this point in the positive x-direction. The event when the pulse reaches the front of the train will be called A. What time t'_A is read on the clock at the front of the train when the light pulse reaches it?

b) *(5 points)* In the ground-based coordinate system, at what place x_A and time t_A does the light pulse reach the front of the train?

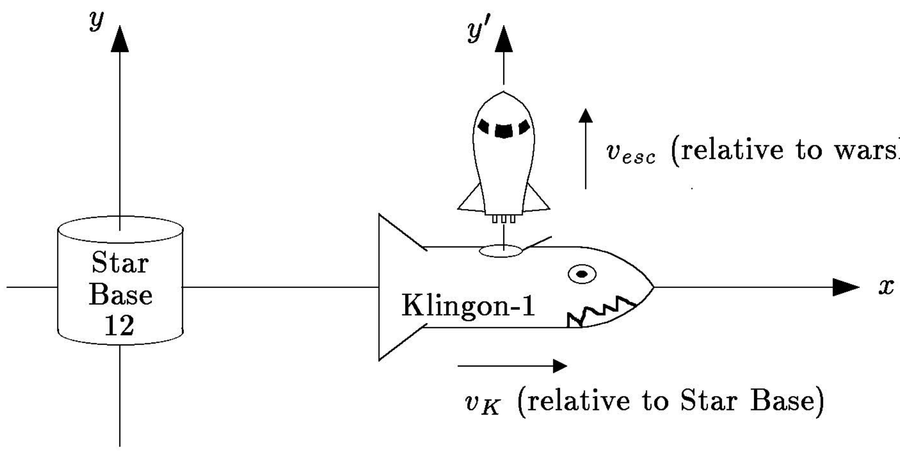
c) *(10 points)* The light pulse emitted in part (a) is also received by an archer standing on the ground at $x = 2\ell$. He receives the pulse, and immediately shoots an arrow in the negative x-direction, at speed v_a (subscript a for "arrow"), alongside the train. The arrow hits a clock at the back of the train. (You may ignore any motion in the vertical direction.) The collision of the arrow and clock is an event that we label B. In the ground-based coordinate system, at what point x_B and time t_B does the arrow hit the clock?

d) *(5 points)* What time t'_B does the clock at the back of the train read when it is hit by the arrow? For this part it will be sufficient to express your answer in terms of given variables and/or the answers to previous parts; it is not necessary to substitute the answer(s) from previous parts, or to simplify the expression. Be sure, however, that your notation is well-defined.

PROBLEM 4: TRANSFORMATION OF TRANSVERSE VELOCITIES *(25 points)*

A Klingon warship strafes Star Base 12 and then escapes along the x-axis at uniform speed v_K. For convenience, we will assume that the clocks in the Star Base coordinate system (x, y, t) are set to $t = 0$ at the moment the Klingon ship departs. The clocks in the warship coordinate system (x', y', t') are also set to zero (*i.e.*, $t' = 0$) when the ship departs the star base, but these clocks are synchronized in the warship frame of reference. The y' axis in the warship coordinate system goes through the middle of the warship.

Afraid that the warship might be destroyed by the photon torpedoes fired from the star base, the Klingons bail out in an escape vessel, launched from the side of the warship. The escape vessel is launched at time $t' = \Delta t_1$, in the time system of the warship.



a) *(4 points)* In the coordinate system of Star Base 12, where and when is the escape vessel launched? Call the coordinates $(x_{esc}, y_{esc}, t_{esc})$.

b) *(4 points)* In the frame of reference of the Klingon warship, the escape vessel is launched at speed v_{esc}, along the y' axis (*i.e.*, perpendicular to the line of sight to the star base). In the warship frame of reference, what is the four-velocity of the escape vehicle?

c) *(4 points)* Using the Lorentz-transformation properties of four-vectors, what is the four-velocity of the escape vehicle as seen in the frame of reference of Star Base 12?

d) *(5 points)* After its launch, the escape vehicle waits a time Δt_2 as measured on its own clocks, and then emits a telemetry signal. In the coordinate system of the Klingon warship, what are the coordinates (x'_s, y'_s, t'_s) at which this signal is emitted?

e) *(4 points)* In the coordinate system of the star base, what are the coordinates (x_s, y_s, t_s) at which the telemetry signal is emitted?

f) *(4 points)* In the coordinate system of Star Base 12, what angle does the escape vessel velocity make with the positive x-axis? *[Hint: you can calculate this answer either by using the answer from (c), or by combining the answers from (e) and (a). To be really safe, you might want to calculate it both ways and compare.]*