14.05: Practice Exercise
Social Security in Diamond's OLG Models (Solutions)

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Exercise (Social Security in Diamond Overlapping Generations Model - Exam #2 Fall 2004)**.**

Consider the Diamond overlapping generations model. L_t individuals are born in period t and live for two periods, working and saving in the first and living off capital in the second period. Assume population is growing at a constant rate, n, and there is no technological progress, $g = 0$, so that we can normalize $A = 1$. Markets are competitive and labor and capital are paid their marginal products. There is no capital depreciation ($\delta = 0$). Utility is logarithmic and, for simplicity, we assume that the individual discount rate is zero, ie, $\rho = 0$.

$$U = \log(c_t) + \log(c_{t+1}).$$

The production function in per capita terms is

$$y_t = f(k_t) = k_t^{\alpha}.$$

The government taxes each young individual an amount T. It allocates a fraction γ of this amount to an Individual Retirement Account. This fraction is used to purchase capital and the individual receives $(1 + r_{t+1})\gamma T$ when old. The remaining fraction $(1 - \gamma)$ is used to pay retirement benefits to the current old. Therefore, when he retires, the individual gets a benefit $(1 + n)(1 - \gamma)T$.

(a) What kind of social security system is this? Is it funded, unfunded or a mixture of the two? Explain. Also explain why $(1 + n)$ appears in the benefit formula.

Answer. This social security system is partially funded and partially unfunded. There is a fraction γ of taxes that is invested and used to pay retirement benefits to the current young when they are old in period $t + 1$, but the remaining fraction is used to pay benefits to the current old. An unfunded (or pay as you go) system would have $\gamma = 0$ and a fully funded system would have $\gamma = 1$. $(1+n)$ appears in the benefit formula because population grows at rate n. Therefore, in period $t + 1$ there are $L_t(1 + n)$ young people paying taxes, which are partially used to pay benefits to L_t people born in period t.

(b) Write down the budget constraint faced by an individual born at time t.

Answer.

$$c_{t+1} = (1 + r_{t+1})(w_t - c_t - T) + (1 + r_{t+1})\gamma T + (1 + n)(1 - \gamma)T.$$

We can simplify this to get

$$c_{t+1} = (1 + r_{t+1})(w_t - c_t) + (1 - \gamma)(n - r_{t+1})T.$$

(c) Use the budget constraint and the individual's utility function to solve for first period consumption c_t, and first period savings s_t. [*Hint: You should get a function for savings that depends on the wages and the social security tax in the following way:* $s_t = Bw_t - Z_t T$, *where* Z_t *is a function of* n, γ *and the interest rate at* $t+1$ $(1+r_{t+1})$.]

Answer. The Euler equation is given by

$$\frac{c_{t+1}}{c_t} = 1 + r_{t+1}$$

Substituting in the budget constraint gives first period consumption

$$c_t = \frac{1}{2}w_t + \frac{(1-\gamma)(n-r_{t+1})}{2(1+r_{t+1})}T$$

First period savings is given by

$$s_t = w_t - c_t - T$$

Substituting for c_t gives

$$s_t = \frac{1}{2}w_t - \left[1 + \frac{(1-\gamma)(n-r_{t+1})}{2(1+r_{t+1})}\right]T$$

So, $B = 1/2$ and $Z_t = 1 + \frac{(1-\gamma)(n-r_{t+1})}{2(1+r_{t+1})}$.

(d) Use the savings function derived in part (c) and the production function to determine the relationship between k_{t+1} and k_t. Assume T is not too large and draw a graph of the relationship between k_{t+1} and k_t such that there are two steady states for k. Which of these two steady states is stable? Explain your reasoning.

Answer. Total capital stock is equal to savings by the consumers in the first period plus the amount allocated to the Individual Retirement Account (which is used to purchase capital):

$$K_{t+1} = s_t L_t + \gamma T\ L_t$$

It follows that

$$k_{t+1} = \frac{1}{1+n}\left(\frac{1}{2}w_t - Z_t T + \gamma T\right)$$

Using the production function, we know that

$$w_t = (1-\alpha)k_t^{\alpha}$$

Substituting above, we get, after some algebra

$$\begin{aligned} k_{t+1} &= \frac{1}{1+n}\left[\frac{1}{2}(1-\alpha)k_t^{\alpha} - (1-\gamma)\left(\frac{r_{t+1}+n+2}{2(1+r_{t+1})}\right)T\right] \qquad (1) \\ k_{t+1} &= m(k_t, k_{t+1}, \gamma, T) \end{aligned}$$

There are two steady state levels of k, see figure 1. The one to the right is stable because it cuts the 45 degree line from above.

(e) For this question, assume we are at the stable steady state found in part (d). Now assume that the government decides to allocate a higher fraction of taxes to the Individual Retirement Account, ie, γ increases. How does this affect the steady state value of k? What is the effect on welfare of current old and future generations if the economy is initially in a steady state that is dynamically efficient? What happens if the initial steady state is dynamically inefficient?

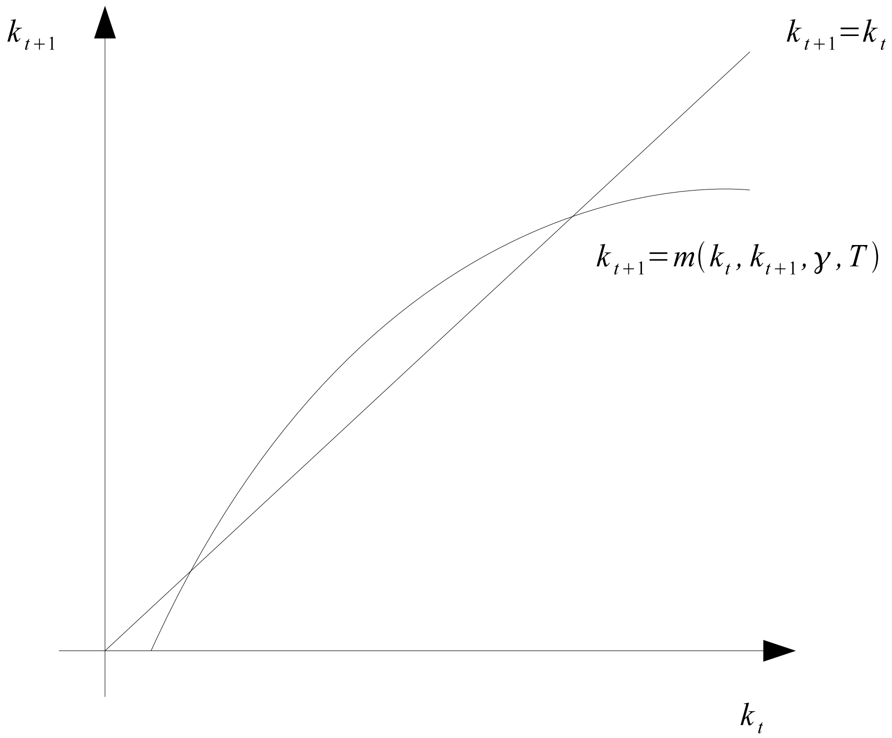

Figure 1: The Dynamic System, Equation (1).

Answer. When γ increases, the curve describing the relationship between k_{t+1} and k_t shifts up. The steady state value of k increases. If the economy is initially in a dynamically efficient steady state, welfare of future generations increases because the economy will get closer to the golden rule level of k. if the initial steady state is dynamically inefficient, the increase in k reduces welfare of future generations because k is above the golden rule. The current old lose in all cases, because they receive smaller benefits. Note that $(1+n)(1-\gamma)T$ goes down.