14.05: Practice Exercise
Social Security in Diamond's OLG Models

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Exercise 1 Social Security in Diamond Overlapping Generations Model - Exam #2 Fall 2004

Consider the Diamond overlapping generations model. L_t individuals are born in period t and live for two periods, working and saving in the first and living off capital in the second period. Assume population is growing at a constant rate, n, and there is no technological progress, $g = 0$, so that we can normalize $A = 1$. Markets are competitive and labor and capital are paid their marginal products. There is no capital depreciation ($\delta = 0$). Utility is logarithmic and, for simplicity, we assume that the individual discount rate is zero, ie, $\rho = 0$.

$$U = \log(c_t) + \log(c_{t+1}).$$

The production function in per capita terms is

$$y_t = f(k_t) = k_t^{\alpha}.$$

The government taxes each young individual an amount T. It allocates a fraction γ of this amount to an Individual Retirement Account. This fraction is used to purchase capital and the individual receives $(1 + r_{t+1})\gamma T$ when old. The remaining fraction $(1 - \gamma)$ is used to pay retirement benefits to the current old. Therefore, when he retires, the individual gets a benefit $(1 + n)(1 - \gamma)T$.

(a) What kind of social security system is this? Is it funded, unfunded or a mixture of the two? Explain. Also explain why $(1 + n)$ appears in the benefit formula.

(b) Write down the budget constraint faced by an individual born at time t.

(c) Use the budget constraint and the individual's utility function to solve for first period consumption c_t, and first period savings s_t. [*Hint: You should get a function for savings that depends on the wages and the social security tax in the following way:* $s_t = Bw_t - Z_tT$, *where* Z_t *is a function of* n, γ *and the interest rate at* $t+1$ $(1+r_{t+1})$.]

(d) Use the savings function derived in part (c) and the production function to determine the relationship between k_{t+1} and k_t. Assume T is not too large and draw a graph of the relationship between k_{t+1} and k_t such that there are two steady states for k. Which of these two steady states is stable? Explain your reasoning.

(e) For this question, assume we are at the stable steady state found in part (d). Now assume that the government decides to allocate a higher fraction of taxes to the Individual Retirement Account, ie, γ increases. How does this affect the steady state value of k? What is the effect on welfare of current old and future generations if the economy is initially in a steady state that is dynamically efficient? What happens if the initial steady state is dynamically inefficient?