14.05: Section Handout #2
New Growth Theory

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Exercise 1 *Human Capital and Growth*

It is frequently argued that education is beneficial for growth. The following simple model attempts to formalize this intuition. Production is determined by the function $Y = K^{\alpha}(L_Y h)^{\beta}$, where K is capital, L_Y is the amount of labor allocated to production, and h is the level of human capital of each worker (assumed to be identical for all workers). Capital evolves according to $\dot{K} = sY$. Each worker is endowed with one unit of labor (1 "hour") of which she devotes a fraction $(1-e)$ to production activities, and a fraction e to education activities. The total number of workers evolves according to $\dot{L} = nL$. Finally, human capital per worker (h) depends on the amount of time each worker devotes to education, and on the existing level of human capital of each individual (teacher's quality), according to $\dot{h} = \phi(e)h$, where $\phi(e) > 0,\ \phi' > 0,\ \phi'' < 0$.

(a) Write an expression for L_Y in terms of the other variables in the model.

(b) Write expressions for the rate of growth of K (in terms of K and Y), L, Y, and h.

(c) Under what conditions does this economy have a balanced growth path (BGP)? Determine the growth rate of capital K, output Y, and output per worker in the BGP? What needs to be true for output per worker to grow at a positive rate? Interpret your expression.

(d) Assuming the the production function has constant returns to scale, what is the principal determinant of cross country differences in the growth of output per capita? Would Alwyn Young agree with this assesment when explaining the fast growth of East Asian countries? Explain in 1-2 sentences.

(e) Continue to assume constant returns to scale, and now write an expression for the logarithm of output per capita (Y/L) as a function of (K/L), h, and e. Discuss the effects of an increase in e in the level of output per capita in the short run ad long run.

Solution 1

(a) It is straightforward that $L_Y = L(1-e)$.

(b) From the equations of the model, we can obtain the following expressions

$$
\begin{aligned}
g_K &= s\frac{Y}{K}, && (1)\\
g_L &= n, && (2)\\
g_h &= \phi(e), && (3)\\
g_Y &= \alpha g_K + \beta(n+\phi(e)). && (4)
\end{aligned}
$$

Where the last expression follows from

$$
\begin{aligned}
L_Y &= (1-e)L\\
\ln(L_Y) &= \ln(1-e) + \ln L\\
\frac{\dot{L}_Y}{L_Y} &= \frac{\dot{L}}{L}
\end{aligned}
$$

(c) A BGP requires that all variables grow at a constant rate. In particular, it requires in this case that g_K is constant. This will only be true if $g_Y = g_K$ (see equation 1). Plugging this condition into equation (4) we obtain that:

$$g_Y = \frac{\beta(n+\phi(e))}{1-\alpha}$$

As previously stated, the growth rate of capital is equal to the growth rate of output in the BGP in this model. The rate of growth of output per worker is

$$
\begin{aligned}
g_y &= g_Y - n\\
&= \frac{\beta(n+\phi(e))}{1-\alpha} - n.
\end{aligned}
$$

For output per capita to grow at a positive rate we need the latter expression to be non-negative (output per worker cannot fall below zero), that is

$$\beta\phi(e) > n(1-\alpha-\beta)$$

Therefore, if $\alpha+\beta<1$ the economy will have a positive BGP only if $\phi(e)$ is large enough.

(d) Constant returns to scale requires $\beta = 1-\alpha$. In this case the expression for the growth rate of out out per worker will be

$$g_y = \phi(e),$$

so, only differences in education determine the differences in GDP per capita growth across countries. Alwyn Young would not agree with this since he argues that a substantial amount of faster NIC growth can be explained by the accumulation of factor inputs.

(e) Under constant returns to scale we obtain that

$$\ln \frac{Y}{L} = \alpha \ln \frac{K}{L} + (1-\alpha)\ln((1-e)h).$$

From this expression it is clear that an increase in e has two opposite effects on output per capita. The short run effect is that as people devote more time to study, time used to produce falls. For a given level of human capital per worker this reduces output per worker. However, an increase in e increases the growth rate of human capital per worker, therefore, for any given initial level, it increases the level of human capital per worker at any point in time. The former effect operates immediately, while the latter operates on a longer horizon. Therefore, an increase in e should reduce output per worker in the short run, while increasing it in the long run.