Exam 2
Econ 14.05

Instructions: Answer the three following questions. Each one of them counts for 1/3. Each subsection counts 1/5.

1. **Solow model with endogenous population growth**

   (a) Consider a standard Solow model described by the following equations: $Y = K^{\alpha}(AL)^{1-\alpha}$, $0 < \alpha < 1$, $\dot{K} = sY$, $\dot{L} = nL$. Define capital per effective worker as $k = K/AL$, and derive an expression for its evolution over time: dk/dt.

   (b) Consider now the case in which population growth depends on the level of output *per worker* Y/L, denoted by $\hat{y}$. In particular, assume that

   $$n = n_0 - n_1 \hat{y}^{\frac{\alpha-1}{\alpha}}.$$

   With n_0, $n_1 > 0$. How does population growth changes with an increase in $\hat{y}$? At which value of $\hat{y}$ is population constant? What happens for levels of income per worker below and above that threshold? Given your findings, provide an economic interpretation for this threshold value.

   (c) Consider the case in which there is no productivity growth (A constant). Without loss of generality assume that $A = 1$. In this case, the per-worker and per-effective-worker variables are identical. Assuming that n evolves as in part b). Determine the long run equilibrium of this economy, that is, find the equilibrium value of k, and determine the growth rates of Y and K. Show your results in a graph.

   (d) Compate the economy of part c) with an economy with one in which population grows at a constant rate n_0. Is capital per worker higher or lower in this case? use a graph to illustrate your answer. Explain the economic intuition behind your result.

   (e) Assume now that A grows at an exogenous rate g, while population growth is still as in part b). Write an expression for the growth rate of capital per effective worker. How is capital per effective worker evolving over time? Explain the intuition behind your result.

2. **Transmission of Technologies to the South through Learning by Doing from Northern Technologies:** Suppose we have two regions in the world, the North and the South. In each region, output is given by $Y_i(t) = K_i(t)^{\alpha}[A_i(t)(1 - a_{Li})L_i]^{1-\alpha}$, and capital accumulation by $\dot{K}_i(t) = s_i Y_i(t)$, where $i = N, S$ (North or South). The North develops new technologies according to $\dot{A}_N(t) = Ba_{LN}L_N A_N(t)$. The South relies on Learning by Doing from Northern technologies to improve its own technology. That is, the South is not able to develop new technologies by itself. In particular, we assume $\dot{A}_S(t) = \mu a_{LS}L_S[A_N(t) - A_S(t)]$, where $A_N(t) > A_S(t)$. Note that this implies that, the larger the rechnological gap between the South and the North, the more the South learns from the Northern technology. We assume labor to be constant in both countries. The fraction of labor devoted to R&D in region $i = N, S$ corresponds to a_{Li}. You can think about this model as the North doing R&D and the South, not being able to innovate, devoting labor to imitation. The equations for A_N and A_S summarize this intuition.

   (a) What is the long-run growth rate of Northern output per worker?

(b) Define $Z(t) = \frac{A_S}{A_N}$. Find an expression for $\dot{Z}(t)$ as a function of $Z(t)$ and the parameters of the model. Is $Z(t)$ stable. If it is, what value does it converge to? *Hint: Differentiate $Z(t)$ with respect to time and use the expressions for $\dot{A}_i$. Then write everything in terms of $Z(t)$.*

(c) What is the long-run growth rate of output per worker in the South? What are the parameters that affect it? Is that realistic?

(d) Assume $a_{LN} = a_{LS}$ and $s_N = s_S$. What is the ratio of output per worker in the South to output per worker in the North in the Balanced Growth Paths (BGP)? Does your opinion about the relevance of the model change? *Hint: Write expressions for the dynamics of capital per unit of effective worker for both countries. Substitute the growth rate for A_N and A_S with the expressions you found and use the assumptions $a_{LN} = a_{LS}$ and $s_N = s_S$. Is there any difference at all between the 2 equations?*

(e) According to the model, does imitation benefit the South, the North or both? (compare to a case where there is no innovation at all in the South). What crucial determinant of innovation discussed in class is missing in this model?

3. **Briefly answer the following questions**

(a) Describe the main features of the evolution of the world income distribution according to Jones.

(b) According to Hall and Jones: What is the main source of differences in income per capita across countries? What is the ultimate cause of these differences?

(c) How do Engerman and Sokoloff explain the differences in economic performance between North America and Latin America? Be precise about the mechanism. How would you test their theory?

(d) What is the explanation proposed by Acemoglu et al for the observed differences in output per capita across former colonies? How does this explanation differ from the Engerman and Sokoloff theory?

(e) Why do Acemoglu et al argue that the results obtained by Sachs using latitude are probably misleading? What does Sachs argue about Acemoglu et al results?

(f) Many observers of the world economy were claiming at the beginning of the 90's that the Asian "tiger economies" - Hong Kong, Singapore, South Korea, Taiwan - were growing due to an exceptional increase in productivity. Young (1994) demonstrated that this was not the case. Explain Young's methodology and conclusions.

(g) There is a clearly positive relationship in the data between population growth and the population level. Discuss how Kremer explained this and its relationship with Malthus. Also argue what extra ingredients do we need to justify the fall in the positive relationship in recent years.