14.5 Intermediate Applied Macroeconomics

Exam #2

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1. Borrowing and Lending in a 2-Period Model

Consider consumers who live for two periods. Their utility function is $U_1(c_1, c_2) = \log(c_1) + \frac{1}{1+\rho_1}\log(c_2)$ and they receive an exogenous stream of income every period denoted by y_1 and y_2, respectively. In the first period of their lives, consumers decide how much to consume and how much to save or borrow. The consumers can borrow and lend at an interest rate, r.

a. Write down the intertemporal budget constraint faced by the consumer.
b. Solve the maximization problem and determine the consumer's consumption in the first period.
c. Determine when a consumer is going to be a saver or borrower as a function of the interest rate and his or her own discount factor.

Now assume there is a second type of consumer in the economy who only differs in his or her individual discount factor such that his utility function is $U_2(c_1, c_2) = \log(c_1) + \frac{1}{1+\rho_2}\log(c_2)$, where $\rho_1 \geq \rho_2$. There is an equal number of consumers of each type in the economy. Also assume that net savings are zero in aggregate. In other words, there is no outside supply of deposits or bonds, and the consumers have to borrow from each other and lend to each other.

d. Determine the equilibrium interest rate. *Hint:* In equilibrium, the amount borrowed by the borrowers must equal the amount saved by the savers.

e. Verify, using the condition you derived in part c and the interest rate you derived in part d that indeed one type of consumer is going to be a borrower and the other type a saver.

2. Overlapping Generations Model

Consider an economy described by an overlapping generations model. Individuals live for two periods. In the first period of his life each individual works and earns a wage w_t, and decides how much to consume and how much to save and invest at a rate r_{t+1}. In the second period individuals consume all their savings. Assume no depreciation, no population growth, no technological progress and $A = 1$. The individual's utility function is $U(c_{1t}, c_{2t+1}) = \frac{c_{1t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{c_{2t+1}^{1-\theta}}{1-\theta}$ and the production function in per capita terms is $y_t = k_t^{\alpha}$. Assume markets are competitive and thus labor and capital earn their marginal products.

a. Determine the intertemporal budget constraint of each individual and use it to solve for the first-period consumption and saving.

b. When is the savings rate constant and what form does the utility function then take (i.e. what mathematical function does it equal)?

From now on assume that the saving rate is indeed constant (you do not need to have solved part (b) to continue).

c. Determine the relationship between k_t and k_{t+1}, show it in a graph, and determine the steady-state level of capital per capita, $k*$.

d. How is this steady-state level of capital per capita affected by a tax on the interest rate on savings (making the after-tax interest rate $r_{t+1}(1-\tau_{rt+1})$, where τ_{rt+1} is the tax rate)? You do not need to resolve the model to answer this question and you may disregard any implications it may have for government revenue.

e. How is this steady-state level of capital per capita affected by a tax on labor income (making the after-tax income $w_t(1-\tau_{wt})$, where τ_{wt} is the tax rate)? Again, you do not need to resolve the model to answer this question and you may disregard any implications it may have for government revenue.

3. Social Security reform.

Write a short paragraph in answer to each part.

a. What is the problem that current Social Security reform proposals are aiming to fix? Discuss demography as well as economics.

b. Summarize the reforms proposed by Diamond and Orszag.

c. Summarize the reforms proposed by Feldstein. Explain the difference between the two proposals in terms of a 2x2 matrix of unfunded and funded plans versus defined benefit and defined contribution plans.

d. Describe the 1983 Social Security reforms in terms of the 2x2 matrix. Explain briefly how Diamond analyzed this reform. What were his main assumptions and simplifications, and his major conclusions?

e. Under what circumstances will a move along the diagonal of the matrix (as in part c) increase the national capital stock? Under what circumstances will it fail to do so?