

## 14.05 Intermediate Applied Macroeconomics Problem Set 3

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### Question 1 Savings and Taxation: Income and Substitution Effects

Assume an individual lives for two periods. Let  $C_1$  and  $C_2$  denote the consumption in the two periods of life. In the first period of life, the individual receives an income of  $Y_1$ , and in the second period of life, income is given by  $Y_2$ . The interest rate for both savings and borrowing is  $r$ . Assume that  $Y_1 < Y_2$ .

- (a) Write out the (intertemporal) budget constraint for this individual.
- (b) Draw the budget constraint from part (a), and draw an example utility curve (indifference curve) tangent to the budget constraint that represents an individual that is borrowing. Be sure to label your axes, the slope of the budget constraint, the  $(Y_1, Y_2)$  point and the utility curve.
- (c) Will an increase in the interest rate,  $r$ , increase or decrease savings for this borrower? You must discuss your answer by describing both the income and substitution effects. *[Hint: Remember that borrowing is just negative savings in this case. Thus, a reduction in borrowing is an increase in savings.]*

Now assume that the government decides to implement a tax on financial operations such that the real interest rate is given by  $(1 - \tau)r$ , where  $\tau$  is the tax rate, and  $r$  is the real rate before taxes; this rate applies both to savers and borrowers. Assume that the government simply throws away the tax revenue it collects.

- (d) write out the new budget constraint and draw a graph similar to part (b). What is the new slope of the budget constraint?
- (e) Again, assume that the individual is borrowing. How will an increase in the tax rate  $\tau$  affect savings? Explain your answer using both the income and substitution effects.
- (f) Now assume that the individual is a saver. How will an increase in the tax rate  $\tau$  affect savings now? Again, explain your answer using both the income and substitution effects.

## Question 2 Consumption “Smoothing”

Consider an individual who lives for  $T + 1$  periods. Let  $c_t$  denote the consumption in period  $t$ . Denote by  $y_t$  the exogenous income the agent receives in period  $t$ . She has the following intertemporal utility function

$$U(c_0, c_1, \dots, c_T) = \sum_{t=0}^T \frac{1}{(1 + \delta)^t} u(c_t), \quad \delta > 0 \quad (1)$$
$$u' > 0, u'' < 0$$

where  $\delta$  is the subjective discount rate. Assume, for simplicity, that the interest rate  $r = 0$  and the subjective discount rate  $\delta = 0$ . The agent has wealth  $a_0$  to begin with in the first period ( $t = 0$ ).

- (a) Write down the intertemporal budget constraint for this agent.
- (b) Write down the agent’s optimization problem to obtain the optimal consumption plan. Describe how consumption changes with time, does it increase? decrease? *[No need to determine the actual consumption level, just describe how it evolves on time.]*
- (c) How important is the assumption that  $r = \delta = 0$  for your answer in (b)? Can we change part of the assumption and still get the same result for consumption?

Now assume that the agent’s income is constant in each period,  $y_t = \bar{y} \quad \forall t$ .

- (d) What is the optimal consumption level in each period of life now? How much does the individual save in period  $t = 0$ ?
- (e) Now suppose that the government announces in  $t = 0$  that will tax individuals. The government implements a tax on income in each period of life such that the agent receives  $(1 - \tau)\bar{y}$  each period, where  $\tau = 1/(1 + T)$ . How much does the tax reduce the agent’s per-period income? How much does optimal consumption fall at  $t = 0$ ?
- (f) Now assume that instead of a constant tax rate on each period, the government sets a tax rate  $\tau = 1$  in  $t = T$  and 0 in all the earlier periods. How much does the tax reduce the agent’s per-period income? How much does optimal consumption fall at  $t = 0$ ?
- (g) Compare your answers in parts (e) and (f). Explain the intuition.