14.05 Intermediate Applied Macroeconomics
Problem Set 2

Distributed: September 29, 2005 TA: José Tessada
Due: October 6, 2005

Question 1 The Solow Model and "War, What is it Good For?" (Exam # 1 Fall 2004)

Consider a standard Solow model described by the following equations:

$$\begin{aligned} Y &= (1-\tau)\,K^{\alpha}\,(AL)^{1-\alpha}, \qquad 0<\alpha<1 \\ \dot{K} &= sY,\ \frac{\dot{L}}{L}=0,\ \frac{\dot{A}}{A}=g \end{aligned} \tag{1}$$

Note: τ is a government tax on output, and all tax revenue collected by the government is spent on an ongoing war that does not contribute to either output or capital stock of the economy.

(a) Define capital per effective worker as $k = K/AL$, and derive an expression for its evolution over time: dk/dt.

(b) Derive the long run equilibrium level of capital per effective worker, k^*, and the steady state output per-effective-worker, y^*, where $y = Y/AL$. Suppose the war takes an unfortunate turn for worse, and the government must increase the tax rate in order to buy more tanks. What will be the effect of increasing the tax on y^*? *[Hint: it will be easier to interpret y^* if g is in the denominator.]*

(c) Now suppose that the tax on output also hurts individual's incentives to invent new technologies. Specifically, assume that the growth rate of technology, g, is given by $g = b\,(1-\tau)^{1/\alpha}$ where $b > 0$. What is the new steady state level of output per-effective-worker, y^*? What is the effect of an increase in the tax on y^* now? Describe in words the two opposing effects of the tax on y^*. *[Hint: Be sure to talk about the effect of the tax on 'actual investment' and 'break-even investment'.]*

(d) Continue to assume $g = b\,(1-\tau)^{1/\alpha}$, and recall that consumption is given by $C = (1-s)\,Y$. What is the growth rate of consumption in the steady state? How will the increase in the tax rate affect the growth rate of consumption?

(e) Suppose there is an upcoming election between Candidate A and Candidate B:

   (i) Candidate A says: "I will end the war, and I will eliminate the tax ..." So, if Candidate A is elected, the war will end and τ will fall to 0, such that $g = b$.

   (ii) Candidate B says: "I will also and the war, but I will spend the tax revenues to improve education, etc." So, if Candidate B is elected, the war will end, τ will remain unchanged, but the growth rate of technology will increase, such that $g = B\,(1-\tau)^{1/\alpha}$ where $B > b$.

   What will be the new steady state growth rate of consumption under each candidate's plan? Under what condition will it be optimal for citizens to vote for Candidate B under the assumption that individuals only care about overall consumption growth? Explain in 1 or 2 sentences.

Question 2 Endogenous Growth, Balanced Growth Path, and Scale Effects (Exam # 1 Fall 2004)

Consider the following model:

$$
\begin{aligned}
Y &= [(1-a_K)\,K]^{\alpha}\,(AL)^{(1-\alpha)}, \qquad 0<\alpha<1 \\
A &= B\,(a_K K)^{\gamma}, \qquad 0<a_K<1,\ B>0 \\
\dot{K} &= sY \\
\frac{\dot{L}}{L} &= n
\end{aligned}
$$

(a) In 1-2 sentences, explain the two purposes capital, K, serves in this economy. What type of endogenous growth model is this?

(b) Let g_Y, g_K, and g_A represent the growth rates of output (Y), capital (K), and technology (A). Derive an expression for each growth rate.

(c) Find an expression for $\frac{\partial g_K}{\partial t}$ that is a quadratic function of only g_K and other constants.

(d) Assume $n > 0$. With this assumption, what must be true about γ in order for this economy to have a positive balanced growth path (BGP)? In 1-2 sentences explain the intuition behind this restriction on γ.

(e) What is g_Y along the BGP of the economy $n > 0$? Let $y = Y/L$ represent output-per-capita. What is the growth rate of output-per-capita along the BGP? How does an increase in population growth rate, n, affect the growth rate of output-per-capita? In 1-2 sentences, explain the intuition behind why we have this population "scale effect" on growth.

Question 3 Essays Based on the Assigned Readings

Be concise, go straight to the point unless explicitly required to link different papers. Write less than 10 lines per question.

Long answers makes it harder for the grader to find the right arguments, thus restrain yourself from using too many words.

(a) According to Acemoglu, Johnson and Robinson ("Colonial Origins...'), what institutions differed across colonial countries, why did they differ, and how did this matter for growth?

(b) How has the world distribution of income-per-capita evolved the latter half of the 20th century according to Charles Jones? In particular, do we see convergence or divergence in income-per-capita between countries? Or, do we see both? What does the Solow model predict we should see for countries that share the same investment rates, depreciation, and population growth rate?

(c) What is the main structural difference Jeffrey Sachs points out between the economies of East Asia and those of Eastern Europe and the former Soviet Union (EEFSU)? According to Sachs, what is the link between this difference and the need for "shock therapy" in EEFSU compared to East Asia?

(d) Svejnar presents an analysis of the performance of the transition economies. In his analysis he looks at the evolution of the Gross Domestic Product (GDP), among many other variables. Why do we see differences between these countries? Relate Svegnar's discussion to the new growth theories we have discussed.