

14.05 Intermediate Applied Macroeconomics

Problem Set 1

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Question 1 Comparative Statics in the Solow Model

Describe how each of the following developments affects the break-even and actual investment lines in our basic diagram for the Solow model. [See Figure 1.2 in Romer, page 15 for an example diagram.] Analyze the effect on the steady state values of k . Give intuition for your results.

- The savings rate s goes up.
- The population growth rate goes up.
- There is an improvement in the quality of the food provided to children so they are more productive when adults [Hint: simplify your life and think that people become adults instantaneously.]
- In one line: is there any policy that can generate permanent changes in the **growth rate** of the product per capita?

Question 2 An Extension of the Solow Model

Consider an economy that has access to a production technology $Y = K^\alpha L^{1-\alpha}$, with $\alpha \in (0, 1)$, where the savings rate is exogenously given and equals s , the population grows at a constant rate n , and depreciation rate δ is 0. In this country a fraction $u \in (0, 1)$ of the total population does not work because they do not offer hours of work (thus population is not equal to employment and labor supply because some inhabitants are “inactive”).¹

- Using the production function derive an expression for the product per capita as a function of the equilibrium participation rate $(1 - u)$. [Hint: start by writing down the relation between population and employment/labor force.]
- Using the same definition of long run equilibrium as in the Solow model, find the long-run equilibrium values for the product per capita and the product per worker. Compare them to the values in the version of the Solow model covered in the lectures, what are the differences?
- Suppose now that the government decides to implement a labor market reform that increases the equilibrium participation rate (reducing u to u'). How do the long-run equilibrium values for the product per capita and product per worker change after the reform? Is there any permanent effect on the growth rates of the same variables?
- A well known fact in many countries is the marked increase in the participation rate of women during the second half of the 20th century.

Using the results you just derived in this questions give a concise explanation about the effects of this increased participation on income per capita and its growth rate, both in the short run (transition and impact) and in the long run (stationary equilibrium values).

¹Notice that labor force and employment are equal in this case because there is no unemployment.

Question 3 Growth Accounting Calculation

The goal of growth accounting is to learn where growth comes from. In particular, we can use it to estimate the extent to which growth comes from capital accumulation, greater labor force participation, or technology improvements. Standard growth accounting uses time-series data on factor quantities to determine the sources of an economy's growth. In the below problem, you will do a sample growth accounting for Chile and Ireland.

- (a) Go to the Penn World Tables online at datacentre2.chass.utoronto.ca/pwt56/. Please note that for this problem set, you need to use version 5.6 of the Penn World Tables. From version 5.6 of the Penn World Tables, obtain the following three datasets for both Chile and Ireland.
- The Real GDP per capita (in 1985 International prices), 1970 and 1990.
 - Real GDP per worker (in 1985 International prices), 1970 and 1990.
 - Capital Stock per Worker (in 1985 International prices), 1970 and 1990.
- Using the data, calculate the annualized growth rate for each of these three variables from 1970-1990. Include these growth rates and the above data in a table.
- How does Chile's Real GDP per capita growth rate compare to Ireland's? Is the gap for the Real GDP per worker growth rates smaller or bigger? What could explain this?
- (b) Compare the Real GDP per capita growth rates for the periods: 1970-1975, 1975-1980, 1980-1985, and 1985-1990. Give a brief description. What can you say? Just two lines, add a table with the values.
- (c) Now, assume a generic production function: $Y(t) = F(A_t, K_t, L_t)$. Show that if factors are paid their marginal products, so that $F_K = r$ and $F_L = w$, (where $r =$ interest rate and $w =$ wages), then we can express the growth rate of output as follows:

$$\frac{\dot{Y}}{Y} = s_K \frac{\dot{K}}{K} + s_L \frac{\dot{L}}{L} + s_R \quad (1)$$

$$\text{where} \quad s_R = \frac{A}{Y} \frac{\partial Y}{\partial A} \frac{\dot{A}}{A}$$

And, s_K and s_L refer respectively to the factor shares of capital and labor in total income. The s_R is called the Solow Residual and usually considered to capture the contribution of increases in total factor productivity (TFP). [*Hint: First totally differentiate the generic production function with respect to time. Then, use the fact that the factor share of capital in total income is defined as $s_K = rK/Y$. The definition for the factor share of labor is similar.*]

- (d) Assuming all income is paid to either of the two factor inputs, Labor and Capital, it must be true that $Y = rK + wL$. Thus, dividing by Y , we have $1 = s_K + s_L$. Use this and equation (1) to derive the following expression:

$$\frac{\dot{y}}{y} = s_K \frac{\dot{k}}{k} + s_R \quad (2)$$

$$\text{where} \quad y = Y/L$$

$$k = K/L$$

- (e) In many empirical studies, the share of income paid to capital, s_K is estimated to be around 0.35. Use this fact, equation (2) from Part (d), and your calculated growth rates of RGDP per worker and capital per worker from Part (a), to find the Solow Residual for both countries. Is the difference between the countries very big? What drives Ireland's growth in this period?