Three Essays in Financial Economics

by

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ABSTRACT

In Chapter 1, I propose that abnormal returns generated by price momentum can be explained away within the framework of an existing risk factor model such as the Fama-French three-factor model. Two features of a systematic factor, weakly positive autocorrelation and the leverage effect, generate a small positive alpha in the factor portfolio scaled by its own past returns. The momentum portfolio magnifies this alpha by taking long positions in stocks with highly positive (negative) betas and short positions in stocks with highly negative betas given a positive (negative) realized factor return. Time-varying stock betas enhance the degree of magnification significantly. I demonstrate that a simulated market in which asset returns obey the three-factor model can produce realistic momentum dynamics and substantial abnormal profits comparable to those found in the data. In empirical tests, I show that a replicating portfolio with time-varying betas accounts for 84% of the mean return and 75% of the alpha of the value-weighted momentum portfolio. Among firms larger than the NYSE median, momentum has negligible return and alpha after taking into account the dynamic replicating portfolio. Among small firms, the addition of a financial distress factor is sufficient to explain away momentum alpha.

Chapter 2 confirms the idea that momentum is a dynamic portfolio of existing risk factors and suggests that momentum profit should be positive and may be justified as compensation for risk. Using select factors extracted from principal component analysis, I can construct replicating portfolios that match the equally-weighted momentum returns and even achieve a higher Sharpe ratio, whereas previous attempts based on the Fama-French factors failed to do so. Analyzing the replicating portfolio reveals that momentum derives its returns from a diverse base of existing risk factors, proportional to the amount of cross-sectional dispersion in risk premia they can explain. Momentum returns are positive because it tends to have positive loadings on stocks with positive risk premia and vice versa. The numerous and constantly changing sources of momentum profit make it unlikely that a parsimonious model of a few long-lasting risk factors can completely explain momentum; in that sense, momentum may yet be considered a quasi-risk factor.

In Chapter 3, a joint work with Eung Jun Brandon Lee, we present three sets of empirical results pertaining to cross-sectional patterns in stock returns associated with various accounting ratios such as return on assets, return on equity, turnover ratios of accounts receivable and payable, and gross and net profit margins. First, we show that recent changes in these accounting ratios, rather than their levels, are responsible for large returns spreads. Second, we document "fundamental momentum", long-short portfolios formed by sorting on recent changes in these accounting ratios have significant alphas after controlling for Fama-French three-factor and Carhart four-factor models. Third, we examine the findings of Chordia and Shivakumar (2006) who conclude that the well-known price momentum effect is a manifestation of earnings momentum. We find, on the contrary, that price momentum is not fully explained nor subsumed by earnings momentum.

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Chapter 1

Explaining Momentum within an Existing Risk Factor Model

1 Introduction

Price momentum is a well-documented stock market phenomenon in which stocks with high recent returns ("winners") tend to outperform those with low recent returns ("losers") in subsequent months. The momentum portfolio that buys winners and short-sells losers based on stock returns within the past year is held for up to one year forward. It is a persistent and puzzling fact that such a zero-investment portfolio generates an abnormal positive profit, which is highly significant even after controlling for well-established systematic risk factors. Since it was first described in Jegadeesh and Titman (1993), momentum has become somewhat of an enigma and its genesis a hotly debated topic. Fama and French (1993, 1996) successfully explained a large portion of the cross-sectional variation in stock returns using a three-factor model (henceforth FF3) consisting of market (MKTRF), size (SMB) and value (HML) factors; however, they had to concede that their model was unable to explain momentum. I will argue that the Fama-French three-factor model *can* in fact account for a large portion of momentum's abnormal returns, on its own, without any additional assumptions or theories.

My explanation for momentum is a mechanical one where the observed positive alpha from the unconditional factor regression is not real but arises due to mismeasurement of factor betas. The momentum portfolio does not have constant loadings on risk factors but rather highly time-varying ones that depend on past realizations of factor returns. When the factors exhibit positive autocorrelation and leverage effect (past returns negatively predicting squared future returns), momentum inherits and amplifies these predictability features. Such features, in turn, cause the unconditional factor regression model to generate an artificial alpha, even if there is actually none. This mechanism is powerful in that only a small amount of predictability in the risk factors is needed to generate a large unconditional alpha in the momentum portfolio; it is also universal in that it works for any set of assets that obeys a linear risk factor model.

Assuming that the Fama-French three-factor model is true, I can construct a replicating portfolio for momentum. It is a portfolio of risk factors scaled by a weighted average beta of firms chosen into the momentum portfolio; the weight assigned to the beta of each firm is equal to the weight that momentum assigns to its past returns. With this replicating portfolio, I can explain away 84% of the mean return and 75% of the FF3 alpha of the value-weighted momentum portfolio. The residual return and alpha are statistically indistinguishable from zero. This result is significant because it demystifies momentum within the confines of an existing and well-accepted multifactor pricing model. Among small firms in which momentum is not as well explained by the replicating portfolio, the addition of a financial distress factor can close the gap. The empirical evidence I will present below paints a complete picture of momentum within a parsimonious framework.

It is important to understand the exact mechanism that gives rise to momentum alpha and identify the parameters that determine its magnitude. The mechanism can be divided into two parts, one that links momentum betas to past factor returns and the other that links factor predictability to alpha. At each time the portfolio is formed, stocks are sorted based on their recent returns, which are determined by their systematic returns (the realized factor returns times individual firm betas on those factors) plus idiosyncratic returns. Given that a factor has a positive realized return, winners are more likely to be firms with high betas on that factor, while losers are more likely to be ones with low betas. The momentum portfolio would then have a highly positive beta on the factor. The opposite is true given a negative realized factor return. The magnitude of the realized factor return also determines the extent to which a firm's beta determines its rank in the past return sort. Therefore, momentum's portfolio beta on that factor is time-varying and proportional to the realized factor returns.

An unconditional factor regression specification assigns constant betas to a portfolio, even when it has time-varying betas. The beta estimates are roughly equal to the average portfolio betas over the sample period. If the time-varying betas are positively correlated with factor returns, the regression yields a positive intercept estimate, or alpha, since $E[wF] - E[w]E[F] = \text{cov}(w, F) > 0$. In addition, if the portfolio beta is negatively correlated with the squared factor returns, there is an additional source of positive alpha. These two sources of biases have been previously explored in the conditional CAPM literature: the first was stated and emphasized in Dybvig and Ross (1985) and Jagannathan and Wang (1996); the second was mentioned in Jagannathan and Wang (1996) and Lewellen and Nagel (2006) but not explicitly considered until Boguth, et al. (2011). Boguth, et al. (2001) showed that a number of biases, including the aforementioned two, can exist when computing the alpha of the momentum portfolio. I will show that those two sources of alpha originate from the factor structure and argue that they alone are responsible for generating the momentum alpha we observe in the data.

Momentum betas inherit the ability to predict factor returns and return volatilities from features that already exist within the factor structure. The three Fama-French factors, for example, exhibit varying degrees of positive autcorrelation and leverage effect. Even though the strength of the correlation is weak, the magnitude is statistically significant such that it is possible to generate unconditional alpha from trading the factors themselves. Momentum takes full advantage of this feature from all the factors it loads on by amalgamating and magnifying factor-level alphas. Since only stocks at the extreme ends of the return sort are selected into the momentum portfolio, its beta on a risk factor is a large multiple of the past factor returns.

The size of the multiplier depends on the cross-sectional dispersion of individual firm betas and the size of the idiosyncratic volatility. In simulation, increasing the former and reducing the latter both lead to increases in momentum alpha. With values calibrated from real data, the simulated market produces a momentum portfolio that is very similar to the dynamic replicating portfolio in the empirical analysis, particularly in terms of FF3 alpha and correlation with the actual momentum portfolio. This experiment, along with the empirical reconstruction of momentum profits from the factor-level correlations, provides a comprehensive analysis of the mechanism that I described and proves that it works as intended in both a controlled environment and historical data.

The work most closely related to mine is that of Grundy and Martin (2001), who adjusted the momentum portfolio by time-varying exposures to common risk factors and found that the adjustment significantly reduced variability of returns. They also found that momentum profit and alpha *increased* after controlling for time-varying betas. Chordia and Shivakumar (2002) cited this conclusion but raised doubt about its validity. They used a set of lagged macroeconomic variables to predict one-month-ahead returns. The predicted part of returns can largely explain momentum profits, giving hope that a conditional pricing model may yet prove a viable path. In addition, Wang and Wu (2011) performed a similar exercise as Grundy and Martin (2001) with an expanded sample and showed that the FF3 model can explain 40% of momentum profits. My empirical procedures are similar to those of Grundy and Martin (2001) and Wang and Wu (2011); my results are consistent with the latter in support of conditional risk adjustment, as opposed to the former.

My results run counter to Grundy and Martin's for a number of reasons. One is that they defined momentum by equally-weighting top and bottom deciles of stocks sorted on past returns, so that the portfolio is dominated by a few very small firms. I examine the entire cross-section and show that the replicating portfolio works well for most firms except the very small. Another difference is that Grundy and Martin estimated pre-formation firm betas using a five-year window, which systematically underestimates the magnitude of the momentum portfolio betas. I will show that individual firms have time-varying betas affected by persistent shocks, and a short window of two years leads to much more accurate estimates. GM also estimated post-formation firm betas using a rather unusual five-month window and constructed an ex post replicating portfolio that yields a negative alpha. I repeat the same exercise using a two-year estimation window and find that the ex post replicating portfolio has a highly positive FF3 alpha, similar to the ex ante replicating portfolio.

Most existing explanations for momentum fall into one of two categories: behavioral or rational. The behaviorists argue that price continuation is irrational and is the result of cognitive biases such as under-reaction. Barberis, Shleifer and Vishny (1998) and Hong and Stein (1999) both offer models of investors' bounded rationality that leads to under-reaction to news. The former has investors suffer from representativeness heuristics of Tversky and Kahneman (1974) and incorrect beliefs about the earnings process; the latter assumes that investors follow simple trading strategies conditional on a limited information set. Both cause investors to react slowly news initially and then possibly overreact over the next months, leading to the observed short-term momentum. Fully rational investors are either not present in the market or unable to correct the mispricing due to limits to arbitrage. The problem is expected to be most pronounced among small, illiquid firms.

There is an ongoing debate about whether transaction costs are substantial enough to eliminate momentum profits in practice. Lesmond, Schill and Zhou (2003) estimated that trading costs exceed momentum returns and argued that the strategy is unprofitable due to its selection of small stocks that are especially costly to trade. Korajczyk and Sadka (2006), however, pointed out that the previous conclusion only applies to equally-weighted momentum portfolio that heavily favors small firms. They estimated trading costs using price-impact models and found that value-weighted and liquidity-weighted momentum portfolios remain profitable even after accounting for transaction costs. Momentum among large and liquid firms, it seems, is the more puzzling phenomenon that cannot be easily justified with a limit-to-arbitrage argument. In this respect, my explanation for momentum is particularly successful.

There are also attempts at a fully rational framework for momentum. Johnson (2002) proposed one in which a firm's log market value $\log(V)$ is a convex function of a priced state variable p. Then the firm's beta, $d\log V(p)/dp$, is a positive and increasing function of p. An increase in p leads to both an increase in V, i.e., positive recent returns, and higher subsequent returns due to the firm's now higher beta. Sagi and Seasholes (2006) expanded on Johnson's model and interpreted the upper portion of the convex function as risky growth opportunities within firms. Garlappi and Yan (2011) modeled firms as having lower systematic risks as they approach the default boundary, rationalizing the lower portion of the convex function. The link between momentum and financial distress risk, focusing on small firms with large recent losses, has also been explored empirically in Dichev (1998), Campbell, Hilscher and Szilagyi (2008) and Avramov, et al. (2012). My results can be interpreted as being consistent with this literature: for large firms in which financial distress does not come into play, the Fama-French three-factor model is quite capable of explaining momentum profits on its own; for small firms, the addition of a financial distress factor proves sufficient.

Compared to convex value function models, my proposed mechanism for momentum is more flexible because it is not specific to firm equity. Asness, Moskowitz and Pederson (2009) found that the momentum phenomenon exists internationally across many asset classes including bonds, currencies and commodities. In addition, momentum profits are positively correlated across asset classes. These observations are natural implications of my mechanism as long as these different groups of assets share some common risk factors. The same cannot be said about the convex value function models. While they contribute valuable insight into momentum of a particular market, namely the stock market, my explanation is more likely to be the common driving force behind momentum everywhere.

In the next section, I will specify the conditions under which a factor model can generate momentum endogenously, then perform simulation exercises to gauge the magnitude of the momentum alpha that can be produced and explore how various parameters affect it. Sections III will be devoted to empirical results that confirm the intuition and simulation results in the previous section. Section IV will break down the time-varying replicating portfolio to the factor level and illustrate the importance of each component in the alpha-generating and magnification processes. Section V will focus on the unexplained portion of momentum alpha in small firms and examine the effect of an additional factor on financial distress. Section VI will conclude.

2 Theory and Simulation

2.1 Unconditional Factor Regression and Alpha

I assume a market in which asset prices follow a multifactor model such as Ross (1976)'s APT model, i.e., all asset returns obey

$$r_{it} = r_f + \sum_{k=1}^{K} \beta_{ik} f_{kt} + \epsilon_{it}$$

where β_{ik} is the permanent constant beta of Asset $i \in \{1, 2, \ldots, I\}$ on the kth systematic factor. f_{kt} is the return of the kth systematic factor at time t and ϵ_{it} is an i.i.d. noise term. This is an arbitrage-free world with perfect information and no friction. Any static or dynamic zero-investment trading strategy P using a subset of available assets will have the following returns

$$r_{Pt} = \sum_{k=1}^{K} w_{kt} f_{kt} + \epsilon_{Pt}$$

where w_{kt} is the portfolio's weight or beta on the kth factor and ϵ_{Pt} the idiosyncratic component orthogonal to f_{kt}'s and w_{kt}'s. For a static trading strategy where $w_{kt} = \bar{w}_k$, a regression of r_{Pt} on the factor returns will yield an alpha of zero. The interesting case is one in which w_{kt} varies with time. The returns to the dynamic trading strategy can then be broken down into a number of terms:

$$r_{Pt} = \sum_{k=1}^{K} \tilde{w}_{kt} f_{kt} + \sum_{k=1}^{K} \bar{w}_k f_{kt} + \epsilon_{Pt} \tag{1}$$

where

$$\bar{w}_k = E\left[w_{kt}\right], \quad \tilde{w}_{kt} = w_{kt} - \bar{w}_k$$

The alpha of the trading strategy relative to the risk factors is simply the sum of the alphas generated from each component of Eq. 1. Since constant-beta portfolios and the idiosyncratic component do not produce any alpha, the overall alpha of the strategy is the sum, over k, of the alpha estimated from the following regression:

$$\tilde{w}_{kt} f_{kt} = \alpha_k + \sum_{j=1}^{K} \beta_{kj} f_{jt} + \eta_t$$

Let $F_t = \left[\begin{array}{ccc} f_{1t} & \cdots & f_{Kt} \end{array} \right]'$, $\mu = \hat{E}\left[F_t\right]$ ($K \times 1$ vector of the empirical expected factor returns) and $\Omega = \hat{E}\left[(F-\mu)(F-\mu)'\right]$ ($K \times K$ empirical variance-covariance matrix of the factors). Then, omitting the t subscripts, we have

$$\begin{bmatrix} \hat{\alpha}_k \\ \hat{\beta}_{k1} \\ \vdots \\ \hat{\beta}_{kK} \end{bmatrix} = \begin{bmatrix} 1 & \mu' \\ \mu & \Omega \end{bmatrix}^{-1} \begin{bmatrix} \text{cov}\left(w_k, f_k\right) \\ \text{cov}\left(w_k, f_k f_1\right) \\ \vdots \\ \text{cov}\left(w_k, f_k f_K\right) \end{bmatrix}$$

and

$$\hat{\alpha}_k = \left(1 + \mu'\Omega^{-1}\mu\right) \text{cov}\left(w_k, f_k\right) - \mu'\Omega^{-1}\text{cov}\left(w_k, f_k F\right) \tag{2}$$

where

$$\text{cov}(w_k, f_k F) = \begin{bmatrix} \text{cov}(w_k, f_k f_1) \\ \text{cov}(w_k, f_k f_2) \\ \vdots \\ \text{cov}(w_k, f_k f_K) \end{bmatrix}$$

Each $\hat{\alpha}_k$ is a linear function of the covariances. Of particular importance are $\text{cov}(w_k, f_k)$, the covariance between the portfolio's exposure to factor k and factor k's returns, and $\text{cov}(w_k, f_k^2)$, the covariance between the portfolio's exposure to factor k and its squared returns. Since the variances of factor correlations are magnitudes smaller than those of factor return and return volatility, the contributions of the covariance terms $\text{cov}(w_k, f_k f_j)$ $(k \neq j)$ to the portfolio alpha are negligible. So,

$$\hat{\alpha}_k \approx \left(1 + \mu' \Omega^{-1} \mu\right) \text{cov}(w_k, f_k) - \left[\mu' \Omega^{-1}\right]_k \text{cov}\left(w_k, f_k^2\right) \tag{3}$$

The coefficients on the two covariance terms depend on the covariance structure of the factors. The decomposition of alpha into the two covariance terms is well-known, and equations similar to Eq. 3 have appeared previously in Jagannathan and Wang (1996), Lewellen and Nagel (2006) and Boguth, et al. (2011). In the simplest case where all factors are uncorrelated, the above expression is reduced to

$$\begin{aligned} \hat{\alpha}_k &= \left(1 + \sum_{k=1}^{K} \frac{\mu_k^2}{\sigma_k^2}\right) \text{cov}(w_k, f_k) - \frac{\mu_k}{\sigma_k^2} \text{cov}\left(w_k, f_k^2\right) \\ &\approx \text{cov}(w_k, f_k) - \frac{\mu_k}{\sigma_k^2} \text{cov}\left(w_k, f_k^2\right) \end{aligned} \tag{4}$$

The equality comes from the fact that when factors are uncorrelated, $\text{cov}(w_k, f_j f_k) = 0$ for all $j \neq k$. The further simplification comes from the empirically observed fact that μ_k^2/σ_k^2 is close to zero. For example, each of the Fama-French factors has $\mu_k^2/\sigma_k^2 < 0.02$. Even though the second term in Equation 4 containing $\text{cov}(w_{Pk}, f_k^2)$ may seem much smaller than the first term due to f_k^2, the kurtotic nature of the Fama-French factors, in particular MKT and HML, makes it important as well. The term is also magnified by the multiplier μ_k/σ_k^2, which is around 2-3 in the data.

The alpha of the overall trading strategy is then

$$\hat{\alpha} \approx \sum_{k=1}^{K} \left[\text{cov}(w_k, f_k) - \frac{\mu_k}{\sigma_k^2} \text{cov}\left(w_k, f_k^2\right) \right]. \tag{5}$$

Each component of Equation 4 may be small, but when they come together they rise above statistical and economic significance. Momentum is a trading strategy that takes full advantage of all of these components to generate a significant abnormal profit.

Two crucial links bring momentum into the picture. The first is that the beta of the momentum portfolio on a factor is proportional to the realized return of that factor over the period in which past returns are used to rank stocks. When a factor has a positive (negative) realization, stocks with positive betas experience higher (lower) returns, keeping everything else constant. Positive-beta stocks are then more likely to be selected into the winner (loser) part of the momentum portfolio, and the opposite is true for low-beta stocks. The winner-minus-loser position is then proportional to past factor realization, i.e., $w_k \approx M f_{k-}$, where the minus sign in the subscript denotes the previous period (for momentum, it's $t-12$ to $t-2$). It follows that

$$\hat{\alpha} \approx M \sum_{k=1}^{K} \left[\operatorname{cov}\left(f_{k-}, f_k\right) - \frac{\mu_k}{\sigma_k^2} \operatorname{cov}\left(f_{k-}, f_k^2\right) \right]. \tag{6}$$

The terms within the bracket depend entirely on the factor structure itself, and M acts as a magnifier. The size of M depends on the volatility of the idiosyncratic risk and the dispersion in betas. The former is true because the idiosyncratic component of a firm's return competes with the systematic component in the return sort. It mixes up the ranking and reduces the spread between the betas of winners and losers. The latter is true because momentum selects stocks at the top and bottom of the sort, so high dispersion increases the spread between the betas of winners and losers.

The second link is the autocorrelation structure of factors that fosters a small amount of predictability. In order for momentum to generate a positive alpha, it is helpful if past returns of a factor have predictive power in two ways: first, it predicts positively future returns; second, it predicts negatively future return volatility. I will show in the empirical section below that these two conditions are indeed satisfied for the three Fama-French factors in the data. It must be noted that not all factors must exhibit these traits; the ones that do contribute to a positive alpha to the momentum portfolio while those that don't offset some of the positive alpha. While not true for the Fama-French factors, it is conceivable that an alternative factor structure can generate a negative momentum alpha.

The two aforementioned links magnify the small alpha born out of the factor structure and transform it into a significant one in the momentum portfolio. Therefore, momentum is certainly capable of having a positive alpha in theory; the question is whether that alpha is large enough, statistically significant and accounts for the momentum alpha we observe in the data. Through simulation and empirical analysis, I will prove that the answer is, for the most part, yes.

2.2 Simulation

A simulated stock market provides the ideal controlled environment to test the hypothesis that an existing factor structure can generate abnormal momentum returns. Simulation can also predict the amount of momentum alpha that we should observe in the data and enable comparative statics that is not possible to perform empirically.

I will use the CAPM and the Fama-French three-factor model as the reference factor structure. I create a market with 1,000 assets ($i = 1, \ldots, 1000$), all starting with a capitalization of $v_{i0} = 1$ and permanent betas on the systematic factor(s) drawn from a distribution. One period is equivalent to one month in reality. The market operates for 1032 months, which correspond to the period of July 1926 to June 2012 for which FF3 returns are available on Wharton Research Data Services (WRDS)[1]. At time t, each asset $i \in \{1, 2, \ldots, 1000\}$ has a return of

$$r_{it} = \beta_{mi} r_{mt} + \epsilon_{it}$$

assuming CAPM and, alternatively,

$$r_{it} = \beta_{mi} r_{mt} + \beta_{si} r_{st} + \beta_{hi} r_{ht} + \epsilon_{it}$$

with FF3. Subscripts m, s and h stands for MKTRF (market-minus-riskfree), SMB (small-minus-big or size) and HML (high-minus-low or value) factors, respectively. The risk-free rate is assumed to be zero, and ϵ_{it} is drawn from an i.i.d. $\mathcal{N}(0, \sigma_\epsilon^2)$ distribution. The assumption of a zero risk-free rate can also be interpreted as all firms paying out r_f of the firm value as dividend each month. Factor returns are taken as the historical factor realizations; therefore, they are identical across different iterations of the simulation. The reason for using actual returns is to leave the factor structure intact while controlling for everything else, so that the effect we see must be originated in the factor structure and nothing else. The capitalization of the firm after time t is

$$v_{i,t} = v_{i,t-1} (1 + r_{it})$$

In order to address the possibility that a small portion of the firms may grow disproportionally large and dominating the portfolio weight, a few randomly chosen firms each period have their capitalizations set to the median firm size in the market. This procedure in reality does not have a material impact on the results.

Betas are either drawn from independent normal distributions or an empirical distri-

[1] URL: https://wrds-web.wharton.upenn.edu/wrds/

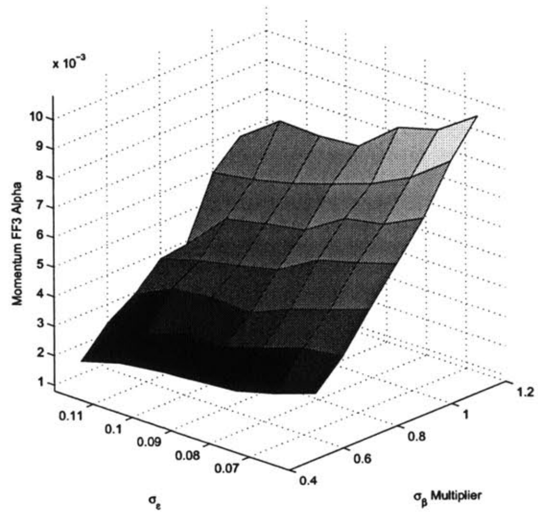
bution. The latter method is obtained by collecting betas estimated from time-series FF3 regressions of all individual stocks in the CRSP database with a monthly return history of five years or more. The three-factor betas are drawn together to preserve their correlation structure. This joint distribution best describes the long-run average betas of individual firms. The former method takes the standard deviation of all long-run average betas, which are approximately $\{0.5, 0.9, 0.9\}$ for the three factors, respectively. Then market betas are drawn from $\mathcal{N}(1,\ 0.5^2)$ while SMB and HML betas are drawn from $\mathcal{N}(0,\ 0.9^2)$. If we look at the volatilities of short-term betas, then they are much higher at $\{0.8, 1.3, 1.3\}$. The three scenarios, betas from empirical distribution and two different independent normal distributions, are tested to see whether the dispersion in beta matters and how the joint distribution of betas differs from the independent normal distribution case.

The standard deviation of the idiosyncratic risk, σ_ϵ, is either a constant from the set $\{0.06, 0.12\}$ or a time-varying value that equals the value-weighted root mean square error (RMSE) of a two-year rolling-window FF3 regression at each time t. The two constants are chosen to represent the mean RMSE for large and small firms. Since momentum portfolio typically consists of the smallest firms in any group of stocks, the higher idiosyncratic volatility of 12% per month more accurate reflects reality. Since the empirically estimated σ_ϵ varies significantly across time, a third scenario using empirical idiosyncratic risk is also included in the analysis to see whether the time-varying quality affects momentum alpha. During each month, σ_ϵ is taken to be the equally weighted RMSEs, across all firms, of firm-level rolling-window FF3 regressions. The average standard deviation is about 10%.

2.3 Simulation Results

Table 1 shows the mean returns, Sharpe ratios and factor regression coefficient estimates for various scenarios in the Fama-French three-factor simulation. Table 2 shows the same statistics the CAPM simulation. Each column represents a different parameter pair of betas' joint distribution and idiosyncratic volatility. The statistics are averaged over one hundred iterations of the simulation, though the variations between iterations are very small such that the statistics are all similar to the average values. The most striking observation from these tables is that momentum strategies exhibit consistently large and significant profits and unconditional alphas across different specifications. For the three-factor model, the alphas range from as low as 0.27% per year to as high as 0.86% per month. Table 2 illustrates that just the market factor itself can already generate a meaningful alpha, about half that seen in the three-factor simulation with comparable parameters. However, this does not mean that the market is more important than SMB or HML in the three-factor setting. When multiple

Figure 1: FF3 Alpha of the Simulated Momentum Portfolio



Plot of the FF3 alpha of the simulated momentum portfolio as a function of σ_ϵ, monthly volatility of the idiosyncratic component of firm returns and σ_β, the dispersion in the permanent firm betas on the FF3 factors. The y-axis is the multiplier applied to the benchmark σ_β of 0.8 for MKT and 1.3 for SMB and HML, i.e., $M = 0.5$ means σ_β of 0.4 for MKT and 0.65 for SMB and HML. The z-axis is the unconditional FF3 alpha of momentum averaged over 100 iterations of the simulated market.

factors are present, they compete for the attention of the momentum portfolio. Therefore, each factor is expected to contribute less than it would on its own. In fact, the three factors contribute to the alpha roughly equally in the data.

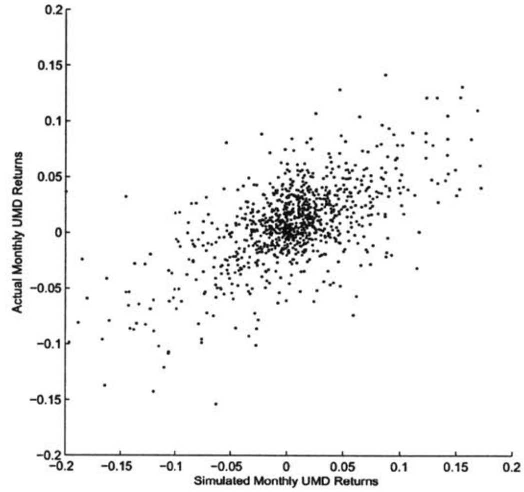
The amount of momentum profit and alpha depends on a set of familiar parameters. First and foremost is the dispersion in individual firm betas, which determines the degree of magnification that momentum can deliver from the factor-level correlations. Comparing the first two panels of Table 1, we can see the difference between using high (short-term) and low (long-term) standard deviations of betas. While dispersions in the "high" case is less than twice that in the "low" case, the resulting alpha is more than twice as high when $\sigma_\epsilon = 12\%$. Higher dispersion not only has the benefit of creating a larger spread but also cuts through the noise (idiosyncratic risk) better. Moments other the standard deviation do not seem to matter much, as seen in the comparison between the first and the third panels. Using the long-run empirical joint distribution of beta is similar to only using the standard deviations.

Also important is the volatility of the idiosyncratic risk, as it mixes up betas in the stock return ranking, reducing the spread in betas on which momentum profit is derived. The average level of this volatility seems to be most important, as a 1% increase in monthly volatility reduces momentum alpha by about 0.05% per month or 0.6% per year. The time variation of the volatility, though observed in the data, does not seem to have a material impact. Simulation using the empirical time-varying RMSE looks similar to that using the average of 10%. Figure 1 shows the joint effect of σ_ϵ and σ_β on momentum alpha. The x-axis is the monthly σ_ϵ and the y-axis the multiplier on the benchmark σ_β of 0.8 for MKT and 1.3 for SMB/HML. The z-axis is the average momentum alpha of the simulation based on the aforementioned parameter set over 100 iterations. Other than minor variations, alpha appears to be a monotonic function of the two parameters, decreasing in σ_ϵ and increasing in σ_β.

Among all combinations of the two parameters above, I will take as benchmark the case where $\sigma_\beta = \{0.8, 1.3, 1.3\}$ and $\sigma_\epsilon = 12\%$ per month. The reason for former choice is that momentum is based on one-year returns in the past, which depend on the short-term betas more than the long-term ones when betas are time-varying. In this case, there is indeed a significant discrepancy. As I will show in the empirical section below, the long-term estimates of betas are unreliable and lead to an alpha in the momentum replicating portfolio that is too small, just like the first panel in Table 1. The reason for the latter choice also comes from empirical analysis, based on the observation that firms selected into the momentum portfolio, winners or losers, tend to be small firms within the group. Therefore, it is prudent to use a larger idiosyncratic volatility. The benchmark case generates an alpha of 0.57% per month on average, a number that is highly significant both statistically and economically. Figure 2 plots the simulated momentum returns against actual momentum returns. The correlation between the two is quite strong at 69.4% despite the fact that the simulation consists of randomly generated firms with random betas and idiosyncratic risks. Additionally, Panel A of Table 4 presents statistics of the time-varying betas of the simulated momentum portfolio on the FF3 factors. The statistics are averaged over 100 iterations using the benchmark specifications. They will be compared to their empirical counterparts in the next section.

While the magnitude of alphas depends on a few of parameters, the existence of it depends crucially on the factor structure. Table 3 illustrates what happens when the factor structure is altered, while other parameters are fixed to the benchmark case. If the factor returns are drawn at random from historical values rather than in the order that they appeared (Column 2), then momentum alpha instantly vanishes. This is proof that the factor structure, as it exists in the Fama-French factors, is of paramount importance. On the other hand, only factor autocorrelation up to a certain horizon matters since momentum is a short-term phenomenon. I perform a randomized block bootstrap (Column 3) using the technique by Politis and Romano (1994) to see whether the alpha disappears if returns are randomly drawn by blocks and then stitched together. At an average bootstrap sample size of 24 periods (2 years), most of the momentum alpha is preserved. This is an indication

Figure 2: Simulated vs. Actual Momentum Returns



Scatterplot of actual monthly momentum returns against simulated momentum returns. The correlation between the two return series is 69.4%. The simulated returns are based on a typical iteration of the benchmark case (σ_β ={0.8, 1.3, 1.3} and $\sigma_\epsilon = 12\%$).

that the factor structure at the two-year horizon is the leading determinant of momentum profits.

I can push the experiment further and isolate the effect of factor autocorrelation. This is achieved by modeling factor returns as AR(12) processes. The parameters are estimated using historical data and then used to generate sequences of factor returns that are different between iterations of the simulation but have similar autogressive properties at a horizon of one year or less. The leverage effect is eliminated, so the momentum portfolio should have a smaller but still significant alpha; Column 4 of Table 3 shows exactly this result. The addition of contemporaneous correlation between the factors appears to have little impact on the resulting momentum alpha, so the results of this extension are omitted.

3 Main Empirical Results

3.1 Data Description

The main data sources are the CRSP (Center for Research in Security Prices) dataset for monthly stock returns and the Fama-French factor data, both of which are available on WRDS (Wharton Research Data Services). The entire CRSP universe of firms is used; it covers the period from January 1925 to December 2011. The Fama-French factors are available from January 1926 to June 2012. To be included in a ranking for portfolio formation, stocks must meet two conditions during the formation month: first, they must be traded on the New York Stock Exchange, the American Stock Exchange or the NASDAQ Stock Market; second, they must have an ending price per share of at least $1. The latter rule is aimed at eliminating penny stocks suffering extreme liquidity problems and trading frictions. The cutoff does affect the amount of alpha generated from the momentum portfolio, but only slightly and does not materially affect any of the results below. Only very small firms are affected; large firms are defined as those above the NYSE median capitalization, so their momentum is virtually unaffected by this rule.

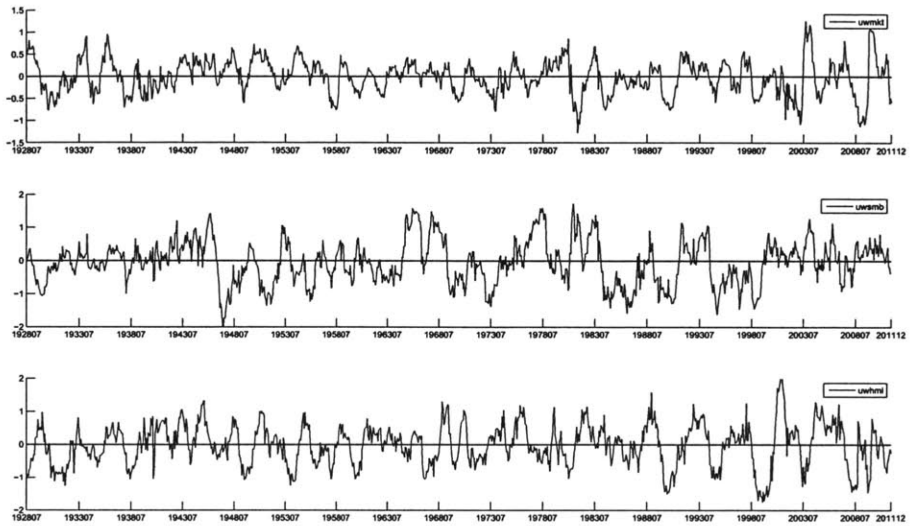
3.2 Variants of the Momentum Portfolio

To maintain consistency throughout this paper, I will define the momentum portfolio (also known as UMD or up-minus-down) by ranking stocks based on their cumulative returns in the second to 12th months prior to portfolio formation, i.e., the most recent one-year return excluding the most recent month. This is the canonical definition for momentum and is used in most papers on the subject as well as for the momentum factor data available on WRDS and Ken French's website. The most recent month is excluded due to short-term return reversal, which is a phenomenon distinct from momentum and is the consequence of market microstructure effects according to Jagadeesh and Titman (1995). Table 5 lists the most common variations of the momentum portfolio based on the set of stocks and portfolio weights used in the ranking. It is immediately apparent that large and small firms experience momentum differently, namely that momentum profit and alpha are much higher among small firms. The same discrepancy exists, to a lesser extent, between value-weighted and equally-weighted momentum portfolios based on sorts over the entire sample. Small firms dominate in the equally-weighted portfolio due to their more dispersed returns, while large firms dominate in the value-weighted portfolio due to their size.

3.3 Momentum Replicating Portfolio

For most of the empirical exercises below, I will assume that the Fama-French three-factor model adequately describes the cross-section of returns for all assets. This is strictly speaking an incorrect assumption, as the FF3 regressions leave large residuals that exhibit both cross-sectional and time-varying patterns indicative of additional latent systematic factors. However, the omission of latent factors does not invalidate the results below if their influences on pricing is limited. Since the mechanism I have described works for any factor structure, the results should at least be interpreted as a partial description of reality based

Figure 3: Momentum Betas on the Fama-French Factors



Plot of the betas of the momentum portfolio on the FF3 factors through history, computed as the weighted-average betas of individual firms chosen into the momentum portfolio. The weights are equal to the weights momentum assigns to the returns of these firms.

on one of the most widely accepted risk factor model available. I will argue that this partial picture is actually quite close to the full picture.

The momentum portfolio can be decomposed into two components: the systematic component that loads exclusively on the Fama-French factors and the idiosyncratic component orthogonal to them. The systematic component can in turn be decomposed into three factor components, each of the form $w_t f_t$, where f_t is the return of the factor and w_t is the time-varying weight of the momentum portfolio on that factor. w_t is taken as the weighted average beta of individual stocks on the corresponding factor. For the value-weighted momentum portfolio, the weights are either v_{it} (the "winner" group), 0 (the middle group) or $-v_{it}$ (the "loser" group). I estimate betas of individual stocks from a two-year rolling window, starting at one month prior to the portfolio formation date. The short window offers protection against time variations in betas, in exchange for noisier estimates compared to longer windows. However, since the betas are averaged across hundreds of stocks given the wide top-30% and bottom-30% design of the momentum portfolio, measurement error is attenuated at the portfolio level.

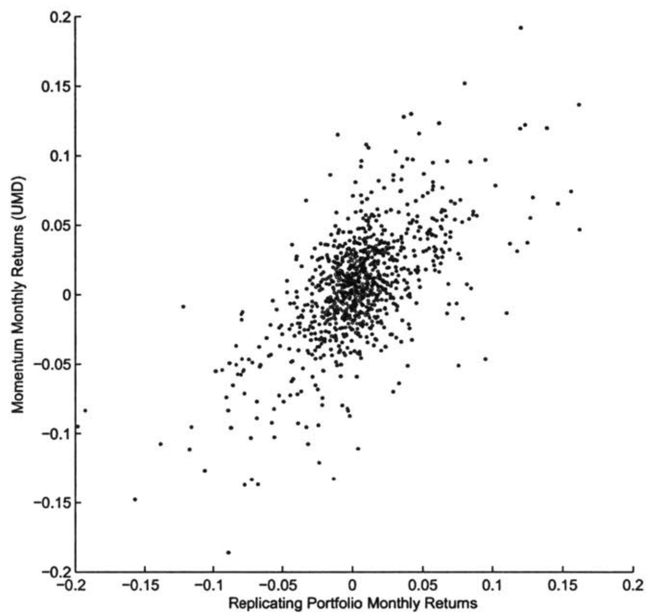
Panel B of Table 4 contains the summary statistics of momentum's loadings on the three factors. The average betas are close to zero and may paint a misleading picture that momentum is weakly correlated with the three factors or is not very volatile. In fact, momentum has high betas on the three factors judging from the high standard deviations of 0.4 to 0.6 and extreme values of magnitude well exceeding 1. However, since the loadings switch frequently from large positive values to large negative values and vice versa, as seen in Figure 3, the mean conceals the highly volatile time variation. These rapidly switching betas cause problems in the typical factor regression model because it can only capture the average loadings of a portfolio on a set of factors, leading to a positive alpha estimate. The statistics in Panel B can be compared with those in Panel A from the simulated market. The standard deviation, maximum and minimum values and correlations with factor returns are similar between the two panels, meaning that the simulated market captures well the time series dynamics of momentum. Other moments such as mean, skewness and kurtosis, are somewhat different, but they do not affect alpha, which mostly depends on the comovement between betas and factor returns.

The first three columns of Table 6 show positive and significant alphas between 0.1% to 0.2% per month from factor portfolios scaled by time-varying loadings. Altogether, the replicating portfolio generates a Fama-French alpha of over half a percent per month. This figure, as well as a Sharpe ratio of about 10% compares favorably with the value-weighted momentum portfolio or similarly momentum in large firms. When the replicating portfolio is subtracted from the value-weighted momentum portfolio, the residual portfolio has a return of less than 0.1% per month, compared with momentum's 0.5%, and an insignificant FF3 alpha of 0.19%, compared with momentum's highly significant 0.75%. The momentum strategy is no longer profitable after controlling for the replicating portfolio.

This result is not uniform in the cross-section, however. For firms larger than the NYSE median in terms of capitalization, the story is more or less the same since they dominate in the value-weighted momentum portfolio. The first panel of Table 7 shows that for them, the replicating portfolio explains away most of the abnormal returns. Small firms, on the other hand, have a 30% higher momentum alpha but a 10% lower momentum return volatility. This additional alpha poses a challenge for the three-factor framework because the time-varying factor dynamics of large and small firm momentum portfolios are rather similar. As a result, the replicating portfolio is able to explain less than half of the alpha among small firms. On the other hand, the group of small firms represents a tiny proportion of the market; their total capitalization is on average only 6% of that of the market. In addition, previous literature has shown that only momentum in large, liquid firms is profitable after accounting for transaction costs.

When large and small firms are given equal weight, as in the canonical half-half (HH) momentum portfolio, then half of the momentum alpha is explained (Panel 3 of Table 12). An additional observation is that in the 20 years since 1990, the replicating portfolio has

Figure 4: Momentum and Replicating Portfolio Returns



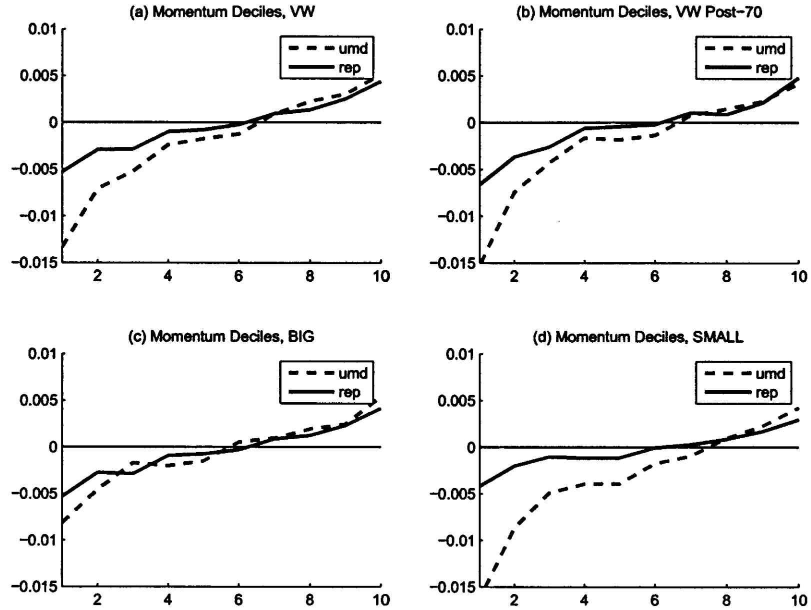
Scatterplot of the monthly weighted-average momentum returns and those of the replicating portfolio. The correlation between the two series is 69.5%.

performed much better while the performance of the momentum portfolio remains roughly the same. The alpha of the residual portfolio is reduced to 0.25% and no longer significant. One may interpret this result as an improvement in the efficiency of the overall market, particularly among small stocks. The residual alpha may be the result of mispricing of small stocks not corrected in the past due to limits to arbitrage. Over time, the market has become more efficient; the residual alpha has shrunk and may continue to shrink in the future.

3.4 Alternative Factor Regressions

A possible critique of the above replicating portfolio is that the post-formation betas of the momentum portfolio are systematically mismeasured. Since the replicating portfolio is a zero-investment trading strategy, it can be arbitrarily scaled to obtain any amount of alpha desired. For instance, if the beta on each factor is overestimated by a factor of $k > 1$, then the replicating portfolio would give an alpha $k\alpha$ greater than the "true" alpha. Then the replicating portfolio would suddenly appear to generate a higher alpha. The converse may also occur where the replicating portfolio with underestimated betas would appear to explain too little. Such concern can be alleviated with an alternative factor regression where

Figure 5: FF3 α of Momentum and Replicating Portfolios by Deciles



Plot of the FF3 alphas of deciles portfolios based on past return sorts, with the alphas of the corresponding replicating portfolios. BIG/SMALL represent subgroups of stocks whose capitalizations are larger/smaller than the NYSE median.

momentum returns are regressed on returns of each systematic component of the replicating portfolio. In essence, this regression measures the performance of the momentum portfolio relative to not the original systematic factors but the dynamic replicating portfolios based on these factors. As seen in Table 8, the alphas from these regressions are roughly equal to the difference between momentum alpha and that of the replicating portfolio. Since the alpha estimates from these alternative regressions do not change when the regressors are scaled by constants, they confirm that there is no systematic bias in the estimates of individual firm betas and momentum portfolio betas. The coefficient estimates on the three systematic components are different from 1 due to the fact that the factors are correlated and also that the estimated momentum betas are bound to contain noise and different from the "true" betas. In addition, the alpha estimates cannot be realistically obtained, unlike the residual alpha from momentum minus the replicating portfolio, since the betas are fitted from the entire sample and cannot be known at the portfolio formation time.

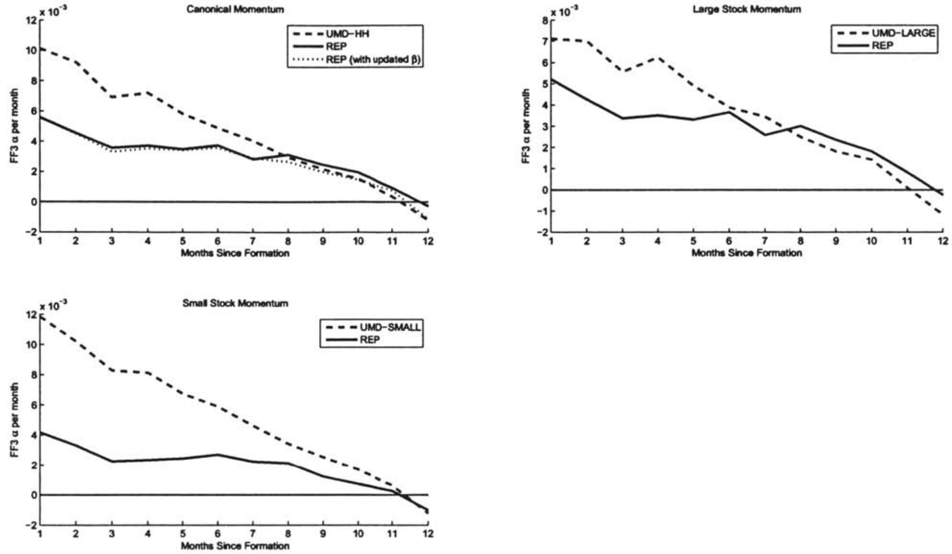
3.5 Decile Portfolios

Analysis of returns in the cross-section offers more sights into where the conditional beta model succeeds and where it is lacking. Just as the momentum portfolio can be replicated with a time-varying factor portfolio, each decile portfolio from the past return sort can also be replicated in the same manner. The replicating decile portfolios exhibit monotonically increasing alphas from about -0.5% in Decile 1 to zero in Deciles 5 and 6 to about $+0.5\%$ in Decile 10 (Figure 5a). The symmetrical nature of the alpha as a function of the decile contrasts with that for the momentum deciles, which drop much more below zero at Decile 1 than it rises above at Decile 10. At the lower deciles, the replicating portfolios only capture half of the magnitude of momentum's negative alphas, but they fare better at the higher deciles. The discrepancy can mainly be attributed to the small stocks. If the decile portfolios are formed separately for large and small stocks, as shown in Figure 5(c)(d), it is clear that the replicating portfolios perform admirably for large stocks by matching the alpha curve almost perfectly. Accounting for time-varying betas, there is no statistically significant momentum effect for large stocks. The same cannot be said for the small stocks, as the replicating portfolio struggles to keep up at the lower deciles. Figure 5(d) confirms that the highly negative returns of the stocks suffering the worst recent returns present the main challenge to my mechanism applied to the Fama-French three-factor model.

3.6 Longer Holding Periods

The abnormal returns of the momentum portfolio not explained by the replicating portfolio diminish over the next six months and then disappear altogether. The first subplot of Figure 6 traces the performance of the canonical half-half momentum and the corresponding replicating portfolio relative to the Fama-French factors in the year following formation. The portfolio composition changes slightly over time as a small proportion of stocks are delisted. Their weights are then spread out among the remaining stocks according to the originally weighting scheme. The weights of the replicating portfolio on the factors are computed from individual stock betas estimated during the formation period, which are not revised in the subsequent months. The dotted line represents the alphas through time of the same replicating portfolio, but with continually updated beta estimates. The updated replicating portfolio looks similar to the original replicating portfolio. Momentum alpha decreases nearly linearly from about 1% to zero in 12 months, while the replicating portfolio holds steady at around 0.4% for the first half a year, then matches momentum alpha and drops to zero in the later half. Even though the replicating portfolio only explains away half of the momentum FF3 alpha in the first month after formation, it explains about two-thirds

Figure 6: FF3 α of Momentum and Replicating Portfolio over Time



Plot of the FF3 alphas of the canonical (half-half), large/small stock momentum portfolios and their corresponding replicating portfolios over the 12 months after formation. The solid line represents the replicating portfolio based on firms betas estimated using a two-year rolling window prior to the formation month; the dotted line represents the replicating portfolio based on up-to-date firm betas estimated with a two-year rolling window relative to the current month.

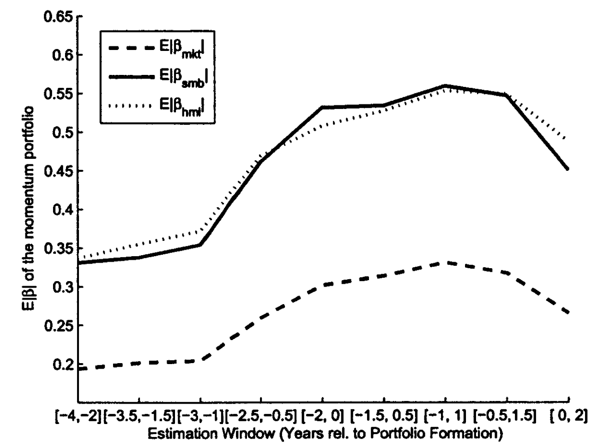
of it over the 12 months after formation. Moreover, momentum alpha becomes insignificant within six months after controlling for the replicating portfolio.

The second and third subplots of Figure 6 repeat the exercise for large and small firms. For large firms, the difference between momentum alpha and the replicating portfolio alpha is small and never statistically significant. In essence, there are no abnormal returns to this momentum portfolio. For small firms, however, the replicating portfolio only explains part of the momentum alpha.

4 Alpha Generation

Grundy and Martin (2001) explored the idea of explaining momentum using time-varying exposures to existing systematic factors. They concluded that while factor models can explain a large portion of the variability in momentum returns, they "cannot explain their mean returns." In fact, after subtracting the dynamic replicating portfolio, the alpha of the momentum portfolio increases in most specifications rather than decreases. The author attributed the negative alpha of the replicating portfolio to the negative autocorrelation of the underlying factors. The logic is sound since momentum is a magnified version of a portfolio whose time-varying weight on a factor is a magnified version of the factor's past

Figure 7: Beta Estimation Window



Plot of the average magnitude of betas on the three factors, $E|\beta|$, of replicating portfolios based different estimation windows for individual firm betas. The window $[a, b]$ means that firm betas are estimated in monthly FF3 regressions in the period that begins at Year a and ends at Year b relative to the momentum portfolio formation period.

returns, and positive (negative) autocorrelation of the factor leads to a positive (negative) alpha. The authors, however, have missed several important aspects of the issue, including the effects of sample selection, time-varying stock betas, the leverage effect of factors and the nonlinear relationship between past returns and momentum loadings. They together have a dramatic effect on the dynamic replicating portfolio, namely that it consistently generates a large and positive alpha and explains a substantial proportion of momentum profit; they also lead to a more accurate understanding of how such alpha comes to be.

4.1 Time-Varying Betas

Individual stock betas vary over time, and the selection of stocks based on past returns takes this time-varying nature into account. If we assume that the true stock betas are constant over time, then the window during which stocks betas are estimated prior to portfolio formation should not systematically bias the betas of a portfolio consisting of many stocks. Figure 7 is a clear illustration to the contrary: it plots the average magnitude of the momentum portfolio's betas on the three factors computed from individual firm betas estimated during different intervals. There are nine scenarios, each of which is a two-year estimation window starting at six months apart. The data points in the middle represent betas estimated from the most recent two years. There is a noticeable rise of over 50% in the magnitude of these betas from a year ago. Individual stock betas are clearly not constant over time.

If stock betas change over time, then the momentum portfolio can take advantage of this feature. Since stocks are ranked based on past year's returns which are products of past factor returns and betas during the past year, those with highly positive or negative *recent* betas, not long-run average betas, are chosen into the momentum portfolio. Regardless of the exact time-series dynamics of the individual stocks, those that have recently experienced a shock that made its betas more positive or more negative are more likely to be chosen. Therefore, the magnitude of momentum's betas on the three factors should be higher than what are implied by long-run betas of individual stocks. According to Figure 7, it is higher by between 50% and 60%. This large difference adds a new dimension to the momentum portfolio and has three important implications. First, as long as there is time variation in stock betas, momentum betas are more volatile and its abnormal returns magnified. Time variation in stock betas is not a controversial claim: previous studies such as Harvey (1989), Ferson and Harvey (1991, 1993) and Ferson and Korajczrk (1995) offer evidence of this property. Time-varying betas magnify the momentum portfolio by offering a wider range of realized betas at any given time. Momentum seizes this opportunities and picks stocks with the more extreme recent betas.

The second implication is that long run betas of individual stocks are bound to lead to poor replicating portfolios. Table 12 compares the performance of replicating portfolios based on different estimation windows. The 2-4 year window and the five-year window (used by Grundy and Martin, 2001) are similar in that they both give estimates of long-term average betas; they both produce replicating portfolios whose alphas are too small. The recent two-year window, as expected, takes into account the recent changes in stock betas picked up by the momentum portfolio, and produces an alpha that is significantly higher.

The third implication is that the most recent two-year window, if anything, *underestimates* the dispersion in betas and produces an alpha in the replicating portfolio that is *too low*. The two-year window includes an extra year outside of the most recent year that the momentum sort considers and biases the estimated betas in the winner and loser groups towards zero. It is clear from the rightmost panel of Table 12 that going from a $[-5, 0]$ window to a $[-2, 0]$ window, the resulting alpha increases monotonically when fewer of the more distant returns are included in the FF3 regression. In addition, running the regression on a smaller sample produces more noise, which lowers the covariance between momentum betas and future factor returns (and volatility). Therefore, the true momentum betas must be higher in terms of magnitude and more precise than the ones inferred from the firm-level factor regressions. The amount of momentum alpha that can be explained by the mechanical channel is even stronger than the empirical results suggest.

There is also evidence that the changes in stock betas are persistent over time. In Figure

7, the average magnitude of the momentum betas increases by over 50% in the two years prior to portfolio formation but decreases by less than 20% in the two years that follow, still significantly higher than the previous average level. In fact, a replicating portfolio constructed using post-formation betas, while clearly not a tradable strategy, yields a Fama-French alpha of about 0.5%. This result refutes the possible criticism that the recent shocks to stock betas are temporary and that those betas revert to the original level immediately after the formation period; instead, it suggests that such shocks are persistent so that the pre-formation short-window estimation is the correct method.

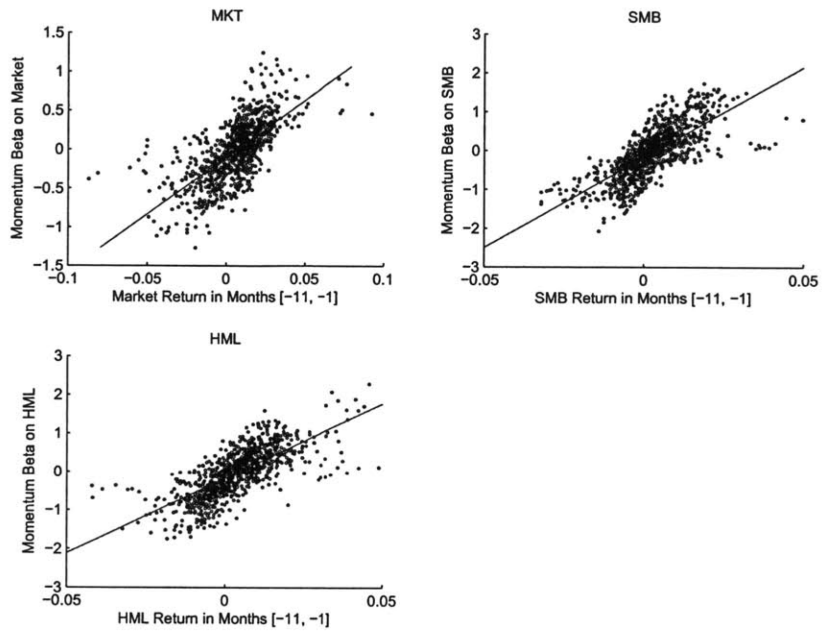
The two-year post-formation window also avoids the "overconditioning" problem associated with short-window contemporary beta estimation identified by Boguth, et al. (2011). Both that paper and Grundy and Martin (2001) illustrated that momentum alpha appears the same or larger when evaluated against a dynamic replicating portfolio formed on firm betas estimated with a short post-formation window, e.g., daily data over a month or a few months. Estimating betas at different frequencies may also contribute to this apparently puzzling result. Table 12 shows that a two-year post-formation window is a sensible choice that yields an alpha in the post-formation replicating portfolio similar to that generated by its pre-formation counterpart. The consistency of the window length and the resulting alpha are testaments to the validity of this method as opposed to the high-frequency short-window approach.

4.2 Autocorrelation and the Leverage Effect

The root cause of abnormal return in the replicating portfolio is the factor structure itself. Two effects are at work to generate a positive alpha: the slightly positive autocorrelation and the leverage effect. Since factor autocorrelation is weak, it is susceptible to outliers influencing the coefficient estimate, particularly when the factor is highly kurtotic. The first look can be deceiving: the market factor and the HML factor have autocorrelation coefficients of 1.5% and 0.9%, respectively, and both are insignificant. SMB is the only factor with a significant autocorrelation coefficient of 6.5%. They seem to contradict the fact that each factor component of the replicating portfolio generates a significantly positive alpha. However, these low estimates are due to outliers. Removing around 5% of the most extreme realizations of past factor returns changes the estimates dramatically. Market, SMB and HML now have autocorrelation coefficients of 6.9%, 10.5% and 7.6%, respectively. All are significant at 1%.

These extreme realizations of factor returns, while able to skew the autocorrelation estimates at the factor level, turn out to have little impact on the replicating portfolio.

Figure 8: Scatterplots of Momentum Betas and Past Factor Returns



Scatterplots of momentum betas against past factor returns with OLS slope estimates, showing the downward bias caused by outliers.

The reason is that as the realized factor return becomes larger in magnitude, its ability to overcome idiosyncratic returns and align winners and losers with betas increases. After a certain point, however, the marginal impact diminishes and eventually disappears since winners and losers are already well aligned with betas. Momentum's loading on a factor is roughly a linear function of the past factor return in the majority of the periods when the return is close to zero. In periods with extremely large factor realizations, however, the actual loading is much smaller in magnitude than that linear function would suggest. Figure 8 illustrates the problem where a few outliers reduce significantly the slope of the fitted line through the momentum betas and factor returns scatterplot. Robust regressions using Huber weights, for example, reduce the influence of outliers and result in higher slope estimates. This problem will affect conditional factor regression models that use past returns as instrumental variables for betas. Although betas are allowed to vary over time, the linear nature of the regression means that the estimated betas will be less volatile than in reality. As a result, the alpha estimate has a positive bias. Estimating conditional betas directly using individual firm betas avoids this problem and is the superior approach.

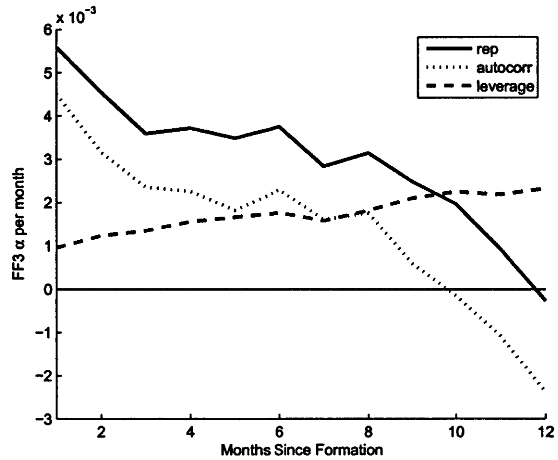
Even though the relationship between factor returns and momentum betas is not linear, a linear function estimated from the core sample, i.e., with outliers removed, proves to be a suitable approximation because the vast majority of factor realizations are close to the origin. A simple linear regression between the two variables yields an R^2 of around $50-60\%$. More elaborate methods, including nonlinear ones proposed by Grundy and Martin (2001), do not improve on the predictive power. GM assumed normal distributions for all returns as well invariant distribution of betas and idiosyncratic component of returns; none of these are true in the data. A linear relationship allows the dissection of the momentum loading on a factor w, into the past factor returns $F_{t-12,t-2}$ times a multiplier M. The exact mechanism by which the features of the factor structure lead to momentum alpha is then revealed.

A solution that is ad hoc but effective is to remove the few outliers and look at the "core" factor structure and relationship between past returns and momentum betas. The two panels of Table 13 show the stark contrast between the full sample and truncated sample statistics. Periods in which the absolute values of factor returns exceeding a cutoff point are omitted in the truncated analysis. For the three factors, the thresholds are 0.04, 0.03 and 0.03, respectively. Altogether only 5% of the sample is removed, but the difference this makes is substantial. The autocorrelation coefficients become significant for market and HML and more significant for SMB. The full-sample "autoregressive portfolios", formed by factors scaled with their own past returns, have insignificant alphas for SMB and HML, which are at odds with the fact that the corresponding components in the replicating portfolio have significant alphas. After the outliers are removed, however, both of these autoregressive portfolios generate positive and significant alphas.

Table 14 gives a detailed account of momentum alpha down to the factor structure level, made possible with the analysis of the core sample without outliers. Most of the alpha of the autoregressive portfolio $F_{t-12,t-2}F_t$ is the sum of the covariance between $F_{t-12,t-2}$ and F_t and the covariance between $F_{t-12,t-2}$ and F_t^2. The former is the factor autocorrelation and the latter the leverage effect. All three factors have significant core autocorrelation, while the market has a significant leverage component accounting for 36% of its autoregressive portfolio alpha. The alphas of the autocorrelation portfolio, though small on the order of 10^{-5}, become magnified with the large multiplier M, which is estimated by regressing momentum beta on the past returns of the corresponding factor. The result is a significantly positive alpha around 0.15% for each component of the replicating portfolio. In total, factor autocorrelation and the leverage effect contribute an alpha of 0.48% per month, which is more than 85% of the total amount in the replicating portfolio. The remainder is a combination of several minor effects including the slight predictability of factor covariance, the nonlinearity of the momentum betas as a function of past factor returns and the predictability of the residual portion.

It would appear, upon first glance, that the leverage effect is very small, only a fifth of

Figure 9: Contribution of the Autocorrelation and Leverage Effects



Plot of FF3 alpha of momentum broken down into contributions from return predictability and volatility predictability, according to Eq. 5, over a period of one year after portfolio formation.

the autocorrelation effect. This is the case mainly because market is the only factor with significant leverage effect. Over the next 12 months, however, the situation changes. Figure 9 plots the breakdown of momentum alpha into the two effects, according to Eq. 5, over a period of one year after portfolio formation. While factor autocorrelation diminishes quickly down to zero and even into negative territory, the leverage effect holds steady and actually increases its contribution to the portfolio alpha to over 0.2% per month. In the year following formation, the average alpha generated from the leverage effect is about the same as that from autocorrelation.

5 Residual Alpha

About 40% of the momentum alpha that is not accounted for by the replicating portfolio is concentrated among small, losing stocks. They represent firms that are already small but have suffered large losses recently; these are firms on the verge of being delisted and possibly going into bankruptcy. Taking advantage of this profit opportunity is difficult and costly, as one must take a short position in these often illiquid stocks and rebalance the portfolio every month. The replicating portfolio has already removed what would otherwise appear to be easier and cheaper arbitrage opportunity, i.e., to take a long position in recent winners among large firms and reap an abnormal positive profit. The remaining portion presents a high barrier to entry for potential arbitrageurs. Regardless of the limits to arbitrage argument, however, the question remains of why these small losing firms suffer substantially lower-than-expected returns.

5.1 Calm and Turbulent Periods

One possibility is that the Fama-French model is misspecified, and there exist important latent systematic risk factors that generate positive alpha among small firms. These latent factors would be concealed in the idiosyncratic risk portion of the three-factor regression. If the Fama-French model is assumed to be the correct pricing model, then the momentum portfolio should have high alpha during times when past factor returns are large in either direction and low otherwise. According to Table 9, this is true for the replicating portfolio but not for the momentum. If we divide the last 80-some years into two halves based on the absolute value of market returns in the one year prior to portfolio formation, we can see that the replicating portfolio generates an alpha of 0.7% per month conditional upon high market action and 0.4% per month otherwise. In the meantime, the momentum portfolio generates an alpha of 0.8% per month conditional upon high market action but 1.0% per month otherwise. The results are similar when conditioning on the realizations of all three factors jointly (Panel 4). Here a turbulent period is one in which the realized factor return of at least one of the factors lies outside of the 10th-90th percentile range of historical returns for that factor. As in the previous case with the market factor, the replicating portfolio performs admirably during turbulent times but is less effective during calm times. This striking difference shows when the replicating portfolio succeeds and when it fails: it functions as expected and kicks into high gear whenever there is a significant movement in one of the factors. This factor becomes the dominant one in the winner and loser selection, and subsequent momentum returns are strongly tied to its subsequent returns. When there is little movement in all of the factors, winners and losers are chosen based on the "idiosyncratic" portion of their returns, which likely contains latent factors. The influence of these latent factors is quite strong, creating a return spread of about 0.6% per month on top of what the replicating portfolio can provide.

There is also the possibility that the return spread during calm periods is caused by behavioral biases. Since the major systematic risk factors have barely moved in the last 12 months, investors have difficulties judging the relative performance of stocks. The regression of individual stock returns on the three factors yields very noisy estimates because of the low variation in the regressors, so investors must focus on other aspects of the firms. Therefore, the recent returns of individual stocks become more important, as investors put more weight on recent news. Then behavioral biases such as under-reaction to news are likely exacerbated, leading to a significant alpha not accounted for by the systematic risks.

5.2 Idiosyncratic Risks and Mismeasured Betas

More evidence of possible missing factors can be seen in Table 15, where the cross-section of stocks is divided according to two indicators. First is the R^2 of the firm-level FF3 regressions that is a proxy for the proportion of returns that can be attributed to exposure to the FF3 factors. The lower the R^2, the higher the idiosyncratic risk relative to the FF3 model. Latent factors almost certainly exist, and their influences are summarized in the idiosyncratic component. The second indicator is the normalized distance between the firm-level betas estimated from the more recent FF3 regression ($[-2, 0]$ year window) and those estimated from a more distant regression ($[-4, -2]$ year window). Given a set of recent estimates $\left(\hat{\beta}_{mkt}^{[-2,0]}, \hat{\beta}_{smb}^{[-2,0]}, \hat{\beta}_{hml}^{[-2,0]}\right)$ and earlier estimates $\left(\hat{\beta}_{mkt}^{[-4,-2]}, \hat{\beta}_{smb}^{[-4,2]}, \hat{\beta}_{hml}^{[-4,2]}\right)$, the distance between them is

$$\Delta\beta = \sqrt{\left(\hat{\beta}_{mkt}^{[-2,0]} - \hat{\beta}_{mkt}^{[-4,-2]}\right)^2 / \sigma_{\beta_{mkt}}^2 + \left(\hat{\beta}_{smb}^{[-2,0]} - \hat{\beta}_{smb}^{[-4,-2]}\right)^2 / \sigma_{\beta_{smb}}^2 + \left(\hat{\beta}_{hml}^{[-2,0]} - \hat{\beta}_{hml}^{[-4,-2]}\right)^2 / \sigma_{\beta_{hml}}^2}$$

A large distance means that the firm has recently experienced a larger shock to its betas, making it a more likely candidate to the momentum portfolio. A long-run estimate of betas using a long window will not be able to detect the significance of this change. Stocks are divided into four groups based on the medians of these two measures.

Comparing the high and low R^2 groups, we can see that the momentum profits and alphas are roughly the same between the two. This should not be observed if the idiosyncratic component is truly idiosyncratic and contains no latent factors. For stocks where the FF3 component dominates, the influence of the latent factors is limited and the dynamic replicating portfolio explains a larger portion of momentum returns; for stocks where there is high probability of latent factor influences, the explanatory power of the replicating portfolio is reduced. Within the high R^2 group, the residual momentum is much smaller for stocks with lower $\Delta\beta$. This is because the firm-level betas estimated using the short-window FF3 regression still have a bias towards zero and contain substantial estimation noise (from 4.1). The high $\Delta\beta$ group suffers more from this problem, so its actual residual alpha should be lower than the stated number. As expected, the mechanical explanation works well for the set of stocks where the FF3 model works well.

5.3 Financial Distress Factor

Financial distress risk is often linked to momentum in the literature and has been rationalized as a systematic risk in Garlappi and Yan (2001). They showed that when stocks are grouped by different levels of default probabilities, the momentum portfolio produces significantly positive Fama-French alphas in all groups but higher alphas in groups with higher default risk. Since distressed firms are most likely to be small ones who have recently experienced highly negative returns, it is possible that financial distress risk may indeed explain some or all of the residual alpha.

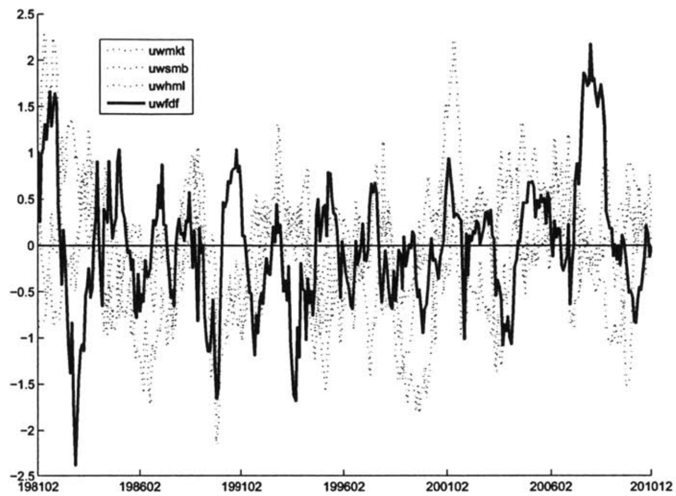
To construct a financial distress factor (FDF), I first estimate each individual stock's default probability in a fashion similar to Campbell, Hilscher and Szilagyi (2010). At the end of each year, I perform a panel logistic regression of the form (Campbell, et al. 2010 Eq. 1)

$$P_t\left(Y_{i,t+1}=1\right)=\frac{1}{1+\exp\left(-\alpha-\beta x_{i,t}\right)}$$

using all available COMPUSTAT firm data prior to that point. $x_{i,t}$ is a vector of firm characters: net income to total asset ratio (NITA), total liabilities to total asset ratio (TLTA), log excess return relative to the S&P 500 index in the most recent month (EXRET), standard deviation of returns in the past three months (SIGMA), relative size of the firm relative to the S&P 500 index (RSIZE), cash and short-term investments over the market value of total assets (CASHMTA), market-to-book ratio (MB) and log price per share winsorized above \$15 (PRICE). All accounting indicators are computed from the most recent quarterly report. $Y_{i,t+1}$ is an indicator that takes a value of 1 if the firm defaults in the next 12 months with a delisting code of 4XX (liquidations) and 5XX (dropped or stopped trading). The most recent year is excluded because the firms' future prospects during the next year are not known. The coefficient estimates are then applied to each firm to predict the likelihood that it will be delisted in the next year.

At the end of each month, all stocks are assigned and then sorted by distress probabilities implied by the most recent logistic regression results and their current firm characteristics. A portfolio is formed by taking a long position in the firms with the 30% lowest distress probabilities and a short position in those with the 30% highest distress probabilities. The return on the financial distress factor is the return of the aforementioned portfolio in the subsequent month. Since COMPUSTAT only has coverage starting in the 1970s, and the number of firms available is spotty until the late 70s, the financial distress factor is available from 1979 to 2010. During this period, the canonical "half-half" momentum portfolio produces a FF3 alpha of 0.86% per month. Controlling for the distress factor in a four-factor regression, momentum still yields a significant alpha of 0.63% per month (Table 16). As expected, momentum has a loading of 0.44 on the distress factor, whose positive expected return helps to reduce its alpha. However, it is important to note that the unconditional Fama-French model with the distress factor added to it does not explain all of momentum's abnormal profits; for that, the dynamic replicating portfolio is necessary.

Figure 10: Momentum Beta on the Financial Distress Factor

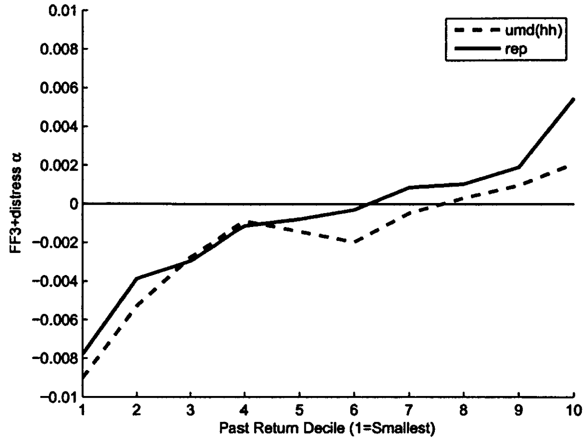


Plot of the betas of the momentum portfolio on the financial distress factor (and the FF3 factors) from 1980 to 2010.

The replicating portfolio retains an alpha of 0.57% after controlling for the additional factor. The alpha has hardly changed because the replicating portfolio is simply a weighted factor portfolio and therefore unrelated to distress. Momentum and its replicating portfolio now generate almost the same amount of alpha, and the difference is tiny and insignificant. Distress appears to be the sole source of the residual alpha. A decile plot (Figure 11) shows exactly what has changed when the fourth factor is added. In the lower deciles, momentum loads negatively on the distress factor, so the abnormal return is much less negative than before. The replicating portfolio can now match the left tail. In the higher deciles, the replicating portfolio outperforms momentum slightly, but the difference is not statistically significant.

I am agnostic about whether the distress risk factor arises due to behavioral biases or compensation for risks. Garlappi and Yan (2011) would argue that firms take on less systematic risk as they approach default, so the lower returns of distressed firms are justified in terms of risk exposures. On the other hand, a behavioral argument in which investors flee from failing firms due to disastrous recent performances, causing fire sale and contributing to further price decline, may also justify this observed effect. in any case, it would be very difficult for arbitrageurs to take advantage of this "arbitrage" opportunity. Regardless of what causes distress risk, results in this section highlight the fact that the dynamic replicating portfolio based on the Fama-French factors is distinct from it and that the four factors together are just enough to explain all of momentum profits. The implications are twofold.

Figure 11: FF3 + FDF α of Momentum and Replicating Portfolio by Deciles



Plot of the return deciles on which the canonical (half-half) momentum portfolio is based, with the corresponding returns of the replicating portfolio.

First, the breakdown between the mechanical effect from the Fama-French factor structure and the financial distress effect is clear. Second, the sum of the two represents the totality of momentum's abnormal returns, eliminating the need for other mechanisms and risk factors.

6 Conclusion

I have shown through simulation and empirical tests that a multifactor asset pricing model is capable of explaining a large portion of momentum profits without resorting to behavioral biases and additional latent systematic risks. Two features inherent in factor structures, positive autocorrelation and the leverage effect, allow for the creation of small, positive alphas in factor portfolios where the weights are equal to past returns. Momentum loads selectively on factors depending on their realized returns and magnifies alphas by choosing stocks with highly positive and negative betas in a long-short portfolio. Momentum's exposure to each factor is roughly the product of the past return of the factor and the dispersion of individual stock betas on that factor in the cross-section. The former provides two sources of positive alpha, while the latter provides magnification. The time-varying nature of individual stock betas is very important, as momentum gains additional magnification power by selecting stocks who have experienced large shocks to their betas recently, not ones with large average betas.

During the first month after portfolio formation, the replicating portfolio based on time-varying loadings on the factors is capable of explaining half of canonical momentum's Fama-

French alpha. For the value-weighted momentum portfolio, however, the replicating portfolio explains 75% of its FF3 alpha, with the remaining portion statistically insignificant. Explaining the value-weighted momentum is arguably more important, since previous literature has shown that such portfolio, dominated by large stocks, is still profitable after accounting for transaction costs, whereas the equally-weighted portfolio representing small stocks is not. During the year after formation, the replicating portfolio explains an increasingly large portion of momentum's abnormal profit until it reaches 100% in Month 8. The role of the leverage effect is small in the beginning relative to the large alpha generated by factor autocorrelation. However, the latter is short-lived, and the leverage effect becomes stronger and dominates the autocorrelation effect six months from formation. The two sources contribute equally to the abnormal return of the momentum replicating portfolio over time.

The remaining alpha not explained by the replicating portfolio can be attributed to the underperformance of very small firms with recent losing streaks. Financial distress risk appears to be the sole factor at work: a distress factor based on firms' predicted failure rate can explain away the remaining alpha completely. The four-factor model demonstrates the power of the conditional replicating portfolio: it is capable of explaining away the entirety of the momentum returns, whereas an unconditional four-factor regression still leaves a highly significant alpha and suggests that the model is inadequate.

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Tables

Table 1: Simulated Momemtum Based on the Fama-French Three-Factor Model

A simulated stock market with 1,000 stocks is operated for 1,032 months, corresponding to July 1926 to June 2012 in history. Each stock starts with a capitalization of 1 and is assigned a permanent set of three Fama-French betas drawn from either independent normal distributions with means $\{1,0,0\}$ and standard deviations $\{0.5,0.9,0.9\}$ (low std) or$\{0.8,1.3,1.3\}$ (high std) for mkt, smb and hml factors, respectively, or from the empirical joint distribution of long-term betas from all stocks in CRSP. Returns follow the Fama-French three-factor model with i.i.d. idiosyncratic risk drawn from the distribution $\mathcal{N}\left(0,\sigma_\epsilon^2\right)$, where σ_ϵ is either a constant or the average RMSE in the FF3 regressions of all stocks during that period in history. The factor returns are taken from and fixed at the realized historical values. Dividend yield is equal to the risk-free rate. At the end of each month, stocks are sorted into deciles by their cumulative returns in the past 12 months (omitting the most recent month). The momentum portfolio is formed by taking a value-weighted long position in the top three deciles and a value-weighted short position in the bottom three deciles. The simulated market is repeated 100 times, and all statistics are averages across simulations.

β	independent normal (low std)			independent normal (high std)			long-run empirical		
σ_ϵ	6%	EM	12%	6%	EM	12%	6%	EM	12%
$\bar{r}$	0.0036**	0.0023*	0.0018	0.0069***	0.0049**	0.0042*	0.0034**	0.0024*	0.0018
	[0.0016]	[0.0013]	[0.0012]	[0.0029]	[0.0025]	[0.0023]	[0.0017]	[0.0014]	[0.0013]
$\bar{r}/\sigma$	0.0639	0.053	0.0477	0.0787	0.0691	0.0595	0.0682	0.0444	0.0451
FF3 α	0.0046***	0.0032**	0.0027**	0.0086***	0.0065***	0.0057***	0.0051***	0.0038***	0.0032***
	[0.0015]	[0.0012]	[0.0011]	[0.0026]	[0.0022]	[0.0020]	[0.0016]	[0.0013]	[0.0011]
β_{mkt}	-0.25**	-0.20**	-0.19**	-0.41**	-0.35**	-0.35**	-0.29***	-0.23***	-0.23***
	[0.1]	[0.08]	[0.08]	[0.17]	[0.15]	[0.14]	[0.10]	[0.09]	[0.08]
β_{smb}	0.39**	0.26**	0.28**	0.61**	0.47**	0.50**	0.28*	0.19	0.22*
	[0.16]	[0.12]	[0.12]	[0.26]	[0.21]	[0.21]	[0.15]	[0.12]	[0.11]
β_{hml}	-0.12	-0.09	-0.10	-0.19	-0.17	-0.18	-0.16	-0.12	-0.13
	[0.19]	[0.16]	[0.16]	[0.30]	[0.27]	[0.27]	[0.19]	[0.16]	[0.16]
Avg R^2	9.9%	8.5%	10.3%	9.1%	8.7%	10.5%	10.5%	9.8%	11.6%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 2: Simulated Momemtum Based on the CAPM

A simulated stock market with 1,000 stocks is operated for 1,032 months, corresponding to July 1926 to June 2012 in history. Each stock starts with a capitalization of 1 and is assigned a permanent set of market betas drawn from either independent normal distributions with mean of 1 and standard deviation of 0.5 (low std) or 0.8 (high std), or from the empirical distribution of long-term market beta from all stocks in CRSP. Returns follow the CAPM with i.i.d. idiosyncratic risk drawn from the distribution $\mathcal{N}\left(0, \sigma_\epsilon^2\right)$, where σ_ϵ is either a constant or the average RMSE in the CAPM regressions of all stocks during that period in history. The market returns are taken from and fixed at the realized historical values. Dividend yield is equal to the risk-free rate. At the end of each month, stocks are sorted into deciles by their cumulative returns in the past 12 months (omitting the most recent month). The momentum portfolio is formed by taking a value-weighted long position in the top three deciles and a value-weighted short position in the bottom three deciles. The simulated market is repeated 100 times, and all statistics are averages across simulations.

β	independent normal (low std)			independent normal (high std)			long-run empirical		
σ_ϵ	6%	EM	12%	6%	EM	12%	6%	EM	12%
$\bar{r}$	0.0013	0.0009	0.0007	0.0030	0.0023	0.0017	0.0016	0.0010	0.0007
	[0.0010]	[0.0008]	[0.0007]	[0.0019]	[0.0016]	[0.0014]	[0.0011]	[0.0009]	[0.0008]
$\bar{r}/\sigma$	0.0360	0.0208	0.0435	0.0562	0.0404	0.0449	0.0418	0.0376	0.0209
FF3 α	0.0022**	0.0016**	0.0015*	0.0046**	0.0037**	0.0032**	0.0027**	0.0020**	0.0016*
	[0.0010]	[0.0008]	[0.0007]	[0.0020]	[0.0016]	[0.0014]	[0.0012]	[0.0009]	[0.0008]
β_{mkt}	-0.15	-0.13	-0.12*	-0.26	-0.24	-0.23	-0.18	-0.15*	-0.14*
	[0.09]	[0.08]	[0.07]	[0.17]	[0.15]	[0.14]	[0.11]	[0.09]	[0.08]
Avg R^2	7.5%	7.9%	8.2%	6.0%	7.1%	8.2%	7.5%	8.5%	8.7%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 3: Simulation with Randomized Fama-French Factors

The benchmark simulation setup is one with $T = 1032$, betas drawn from independent normal distributions with high standard deviations $\{0.8, 1.3, 1.3\}$ and i.i.d. idiosyncratic risk drawn from the distribution $\mathcal{N}(0, 0.12)$. All other scenarios have this setup and $T = 3000$ plus the following difference. The randomized factor scenario uses factor returns drawn individually from the historical distributions of factors without replacement. The random block bootstrap scenario uses factor returns drawn in random length blocks (average length = 24, or 2 years) without replacement. The generated factor scenario has factor returns generated from an AR(12) process, where the AR parameters are calibrated from historical factor returns.

Case	Benchmark	Randomized Factors	Random Block Bootstrapped Factors	Generated AR(12) Factors
$\bar{r}$	0.0042*	0.0016	0.0031**	0.0052***
	[0.0023]	[0.0016]	[0.0012]	[0.0006]
$\bar{r}/\sigma$	0.0595	0.0148	0.0765	0.1227
FF3 α	0.0057***	0.0002	0.0047***	0.0039***
	[0.0020]	[0.0015]	[0.0012]	[0.0006]
β_{mkt}	-0.35**	0.00	-0.29***	0.07**
	[0.14]	[0.07]	[0.08]	[0.03]
β_{smb}	0.50**	0.25	0.35*	0.17
	[0.21]	[0.17]	[0.12]	[0.11]
β_{hml}	-0.18	0.20*	-0.17	0.14***
	[0.27]	[0.11]	[0.15]	[0.04]
Avg R^2	10.5%	4.5%	8.9%	3.6%

[ ]: Newey-West standard errors with 6 lags (except OLS in Row 6); */**/***: statistically significant at 10/5/1%

Table 4: Summary Statistics of Momentum Betas

Panel A contain statistics of the momentum portfolio betas (w) on the FF3 factors in the simulated market with the benchmark parameter values, averaged over 100 runs. Panel B contain the same statistics of the momentum portfolio betas estimated from the data. At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns in the past two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The portfolio betas (w) are the value-weighted average betas of the individual stocks chosen into the momentum portfolio.

A. Simulations (Benchmark Case)

	Univariate Statistics							Correlations				
	Mean	Median	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum	$MKT_{t-12,t-2}$	$SMB_{t-12,t-2}$	$HML_{t-12,t-2}$	w_{SMB}	w_{HML}
w_{MKT}	0.1079	0.1624	0.5432	-0.3845	-0.0672	-1.5070	1.4758	**78.32%**	21.43%	9.84%	15.50%	12.58%
w_{SMB}	0.0970	0.1247	0.7795	-0.1350	-0.1463	-2.1616	2.2189	21.43%	**79.47%**	5.50%		2.82%
w_{HML}	0.2161	0.2404	0.8759	-0.0853	-0.3876	-2.2934	2.5062	6.57%	-3.56%	**78.40%**		

B. Data

	Univariate Statistics							Correlations				
	Mean	Median	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum	$MKT_{t-12,t-2}$	$SMB_{t-12,t-2}$	$HML_{t-12,t-2}$	w_{SMB}	w_{HML}
w_{MKT}	-0.0017	0.0154	0.3692	-0.0641	0.3378	-1.2980	1.2147	**66.18%**	15.16%	14.02%	9.12%	14.74%
w_{SMB}	-0.0542	-0.0548	0.6444	0.0899	-0.1975	-2.0127	1.7449	10.83%	**72.23%**	4.93%		5.31%
w_{HML}	-0.0147	0.0327	0.6120	-0.0517	-0.0055	-1.6924	2.2288	11.56%	8.03%	**72.31%**		

Table 5: Variants of Momentum

At the end of each month, stocks are sorted into deciles by their cumulative returns in the past 12 months omitting the most recent month. The momentum portfolio is formed by taking a long position in the top three deciles and a short position in the bottom three deciles; it is then held for one month forward. Each column is a different method of weighting stocks in the long and short portions of the portfolio: VW - value-weighted; EW - equally weighted. B/S/HH - stocks are divided into two groups based on whether their capitalizations are larger or smaller than the NYSE median that month; a value-weighted momentum portfolio is formed for each group, B as the large cap group and S as the small cap group. HH = (B+S)/2.

Weights	VW	EW	HH	B	S
$\bar{r}$	0.0049***	0.0085***	0.0065***	0.0046***	0.0085***
	[0.0015]	[0.0014]	[0.0014]	[0.0015]	[0.0015]
$\bar{r}/\sigma$	0.1044	0.1798	0.1428	0.0955	0.1723
FF3 α	0.0075***	0.0117***	0.0094***	0.0071***	0.0118***
	[0.0013]	[0.0012]	[0.0012]	[0.0013]	[0.0013]
β_{mkt}	-0.16**	-0.22***	-0.20***	-0.15**	-0.26***
	[0.07]	[0.06]	[0.06]	[0.07]	[0.06]
β_{smb}	0.07	-0.11	-0.02	0.07	-0.24
	[0.09]	[0.09]	[0.08]	[0.09]	[0.06]
β_{hml}	-0.44***	-0.44***	-0.42***	-0.45***	-0.39***
	[0.12]	[0.13]	[0.12]	[0.12]	[0.13]
Adj. R^2	16.4%	23.29%	20.01%	16.10%	20.79%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 6: Replicating Portfolio with Time-Varying Beta

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns in the past two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the weighted average betas of individual stocks selected into the momentum portfolio; the weights are identical to those assigned by the momentum portfolio. Each factor component of the replicating portfolio, REP(Factor), is the factor itself scaled by momentum's beta on that factor. The replicating portfolio (REP) is the sum of the three components. The residual portfolio (RES) is the difference between the value-weighted momentum portfolio (UMD-VW) and the replicating portfolio (REP).

	REP(MKT)	REP(SMB)	REP(HML)	REP	RES	UMD-VW
$\bar{r}$	0.0007	0.0020***	0.0014*	0.0041***	0.0008	0.0049***
	[0.0008]	[0.0007]	[0.0008]	[0.0014]	[0.0012]	[0.0015]
$\bar{r}/\sigma$	0.0306	0.1041	0.0596	0.0964	0.0234	0.1044
info ratio	0.0350	0.1042	0.0931	0.1382	0.0551	0.1726
FF3 α	0.0015**	0.0020***	0.0021***	0.0056***	0.0019	0.0075***
	[0.0008]	[0.0006]	[0.0007]	[0.0012]	[0.0012]	[0.0013]
β_{mkt}	-0.12***	-0.01	-0.09**	-0.21***	0.05	-0.16**
	[0.04]	[0.01]	[0.04]	[0.06]	[0.04]	[0.07]
β_{smb}	0.01	0.03	0.14**	0.18	-0.12	0.07
	[0.03]	[0.07]	[0.07]	[0.13]	[0.07]	[0.09]
β_{hml}	-0.03	-0.01	-0.14	-0.17	-0.27***	-0.44***
	[0.04]	[0.03]	[0.1]	[0.13]	[0.05]	[0.12]
Adj. R^2	8.9%	0.2%	10.2%	10.1%	8.0%	16.4%
				corr(UMD-VW, REP)		69.50%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 7: Replicating Portfolio by Size and Time Period

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns in the past two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles.The momentum portfolio betas are the weighted average betas of individual stocks selected into the momentum portfolio; the weights are identical to those assigned by the momentum portfolio. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. "REP", the replicating portfolio, is the sum of the three components. "RES", the residual portfolio, is the momentum portfolio minus the replicating portfolio. "Big"/"Small" are momentum portfolios formed on stocks larger/smaller than the NYSE median. "HH" (half-half) is the portfolio that gives equal weight to the big and small momentum portfolios. "Jan. Excluded" means that all months of January are removed from the sample.

Size/Period	Big		Small		HH		HH, ≥1990		HH, Jan. Excluded	
	REP	RES	REP	RES	REP	RES	REP	RES	REP	RES
$\bar{r}$	0.0039***	0.0007	0.0035**	0.0050***	0.0037***	0.0029**	0.0043	0.0021	0.0030**	0.0049***
	[0.0014]	[0.0012]	[0.0014]	[0.0013]	[0.0013]	[0.0011]	[0.0031]	[0.0026]	[0.0014]	[0.0012]
$\bar{r}/\sigma$	0.0928	0.0191	0.0965	0.1192	0.0886	0.0814	0.0822	0.0566	0.0730	0.1554
info ratio	0.1320	0.0536	0.0960	0.2094	0.1213	0.1500	0.1293	0.0691	0.1173	0.1799
FF3 α	0.0052***	0.0018	0.0042***	0.0076***	0.0047***	0.0047***	0.0060**	0.0025	0.0045***	0.0055***
	[0.0012]	[0.0012]	[0.0012]	[0.0013]	[0.0012]	[0.0011]	[0.0027]	[0.0025]	[0.0013]	[0.0012]
β_{mkt}	-0.21***	0.05	-0.26***	0.02	-0.23***	0.03	-0.43***	0.10	-0.24***	0.01
	[0.06]	[0.04]	[0.06]	[0.04]	[0.06]	[0.04]	[0.12]	[0.08]	[0.06]	[0.04]
β_{smb}	0.22*	-0.15**	0.43***	-0.53***	0.32**	-0.34***	0.45	-0.32*	0.32**	-0.15*
	[0.12]	[0.07]	[0.13]	[0.09]	[0.13]	[0.08]	[0.29]	[0.16]	[0.14]	[0.08]
β_{hml}	-0.17	-0.27***	-0.06	-0.33***	-0.12	-0.30***	-0.24	-0.08	-0.17	-0.21***
	[0.13]	[0.05]	[0.12]	[0.07]	[0.12]	[0.05]	[0.27]	[0.11]	[0.12]	[0.06]
Adj. R^2	10.5%	8.8%	14.6%	26.7%	12.5%	20.1%	18.8%	6.5%	14.9%	7.6%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 8: Alternative Three-Factor Regressions

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns in the past two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the weighted average betas of individual stocks selected into the momentum portfolio; the weights are identical to those assigned by the momentum portfolio. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. Momentum returns are then regressed on the returns of the three factor components.

	VW	Big	Small	HH
FF3 α	0.0094***	0.0071***	0.0118***	0.0075***
Alt. α	0.0039***	0.0019	0.0058***	0.0022*
	[0.0011]	[0.0012]	[0.0012]	[0.0011]
REP(MKT)	0.93***	0.91***	0.96***	0.92***
	[0.21]	[0.11]	[0.12]	[0.11]
REP(SMB)	0.44***	0.41***	0.47***	0.44***
	[0.08]	[0.08]	[0.10]	[0.08]
REP(HML)	0.82***	0.85***	0.78***	0.85***
	[0.11]	[0.10]	[0.12]	[0.1]
Adj. R^2	53.0%	49.0%	45.1%	8.0%

[ ]: Newey-West standard errors with 6 lags;

*/**/***: statistically significant at 10/5/1%

Table 9: Momentum and the Replicating Portfolio in Turbulent and Calm Periods

Turbulent periods for a factor are defined as ones in which the realized factor return lies outside of the 25th-75th percentile range of historical returns. The calm periods are the remaining periods. The "Any One" turbulent periods are ones in which the realized factor return of at least one of the factors lies outside of the 10th-90th percentile range of historical returns for that factor. UMD-HH FF3 α is the Fama-French three-factor alpha of the half-half momentum portfolio. REP α and β's are coefficient estimates from the unconditional FF3 regression of the replicating portfolio.

	Market		SMB		HML		Any One	
	Turbulent	Calm	Turbulent	Calm	Turbulent	Calm	Turbulent	Calm
UMD-HH FF3 α	0.0072***	0.0106***	0.0074***	0.0114***	0.0094***	0.0089***	0.0074***	0.0082***
	[0.0018]	[0.0013]	[0.0017]	[0.0016]	[0.0019]	[0.0013]	[0.0022]	[0.0011]
REP α	0.0066***	0.0038***	0.0044**	0.0069***	0.0052***	0.0053***	0.0057**	0.0030**
	[0.0018]	[0.0014]	[0.0018]	[0.0017]	[0.0019]	[0.0016]	[0.0023]	[0.0012]
β_{mkt}	-0.30***	-0.07	-0.20**	-0.23***	-0.31***	-0.01	-0.28***	0.01
	[0.08]	[0.05]	[0.09]	[0.07]	[0.08]	[0.06]	[0.08]	[0.05]
β_{smb}	0.01	0.39*	0.27	0.04	0.23	0.11	0.24	0.04
	[0.11]	[0.21]	[0.18]	[0.09]	[0.16]	[0.12]	[0.16]	[0.08]
β_{hml}	-0.04	-0.27**	-0.26	-0.02	-0.23	0.12	-0.27*	0.22***
	[0.18]	[0.14]	[0.18]	[0.14]	[0.16]	[0.09]	[0.16]	[0.07]
Adj. R^2	15.0%	16.4%	11.9%	10.6%	18.4%	1.4%	18.2%	15.70%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 10: Momentum and the Replicating Portfolio Based on Different Formation Periods

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns during the period specified in "Formation Period", where t is the formation month.$[t-11, t-6]$ means the most recent year omitting the most recent six months; $[t-6, t-1]$ means the most recent seven months less the most recent month. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the weighted average betas of individual stocks selected into the momentum portfolio; the weights are identical to those assigned by the momentum portfolio. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. "REP", the replicating portfolio, is the sum of the three components. "RES", the residual portfolio, is the momentum portfolio minus the replicating portfolio. "VW" and "HH" represent the value-weighted and half-half-weighted momentum portfolios, respectively.

Formation Period	$[t-11, t-6]$				$[t-6, t-1]$			
Weighting Scheme	VW		HH		VW		HH	
	REP	RES	REP	RES	REP	RES	REP	RES
$\bar{r}$	0.0047***	0.0012	0.0044***	0.0028***	0.0013	0.0004	0.0012	0.0021*
	[0.0017]	[0.0011]	[0.0016]	[0.001]	[0.0016]	[0.0013]	[0.0015]	[0.0012]
$\bar{r}/\sigma$	0.1055	0.0377	0.1030	0.0888	0.0288	0.0101	0.0260	0.0567
info ratio	0.1225	0.0704	0.1168	0.1469	0.0618	0.0454	0.0534	0.1198
FF3 α	0.0052***	0.0022**	0.0048***	0.0043***	0.0027*	0.0017	0.0023*	0.0041***
	[0.0014]	[0.0011]	[0.0014]	[0.0010]	[0.0014]	[0.0012]	[0.0013]	[0.0011]
β_{mkt}	-0.07	0.03	-0.10	0.01	-0.20***	0.00	-0.23***	-0.01
	[0.08]	[0.03]	[0.08]	[0.03]	[0.06]	[0.05]	[0.06]	[0.05]
β_{smb}	0.27***	-0.13**	0.32***	-0.28***	0.04	-0.09	0.16	-0.29***
	[0.1]	[0.06]	[0.1]	[0.05]	[0.19]	[0.13]	[0.19]	[0.1]
β_{hml}	-0.21	-0.22***	-0.15	-0.22***	-0.08	-0.28**	-0.05	-0.29***
	[0.16]	[0.05]	[0.15]	[0.05]	[0.13]	[0.11]	[0.13]	[0.11]
Adj. R^2	6.4%	6.9%	7.1%	15.1%	6.3%	7.1%	7.3%	16.6%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 11: Fama-French Alphas of the Past Return Decile Portfolios and the Replicating Portfolios

For Panels 2-3, stocks are divided into two groups based on whether their capitalizations are larger (B) or smaller (S) than the NYSE median. Within each size group, stocks are sorted into deciles by their cumulative returns in the past 12 months omitting the most recent month. Each decile portfolio is then held for one month forward. UMD columns list the FF3 alphas of the decile portfolios; REP columns list the FF3 alphas of the replicating portfolios, formed using the same methodology as the momentum replicating portfolio. Figure 5 has the graphical representation of this table.

Size	VW		Big		Small	
	UMD	REP	UMD	REP	UMD	REP
Smallest	-0.0133***	-0.0053***	-0.0081***	-0.0053***	-0.0163***	-0.0042***
	[0.0014]	[0.0011]	[0.0012]	[0.0011]	[0.0014]	[0.001]
2	-0.0071***	-0.0029***	-0.0046***	-0.0027***	-0.0087***	-0.002***
	[0.0011]	[0.0008]	[0.0009]	[0.0008]	[0.001]	[0.0007]
3	-0.0052***	-0.0028***	-0.0017**	-0.0029***	-0.0049***	-0.001*
	[0.0009]	[0.0007]	[0.0007]	[0.0007]	[0.0008]	[0.0006]
4	-0.0024***	-0.0010**	-0.0020***	-0.0009*	-0.0039***	-0.0012**
	[0.0007]	[0.0005]	[0.0006]	[0.0005]	[0.0007]	[0.0005]
5	-0.0017***	-0.0008*	-0.0015**	-0.0008*	-0.0039***	-0.0011**
	[0.0006]	[0.0004]	[0.0006]	[0.0004]	[0.0007]	[0.0005]
6	-0.0012**	-0.0002	0.0005	-0.0003	-0.0017***	-0.0001
	[0.0005]	[0.0003]	[0.0006]	[0.0003]	[0.0006]	[0.0005]
7	0.0009	0.0009**	0.0010*	0.0009**	-0.0009	0.0003
	[0.0006]	[0.0004]	[0.0005]	[0.0004]	[0.0006]	[0.0005]
8	0.0023***	0.0014***	0.0019***	0.0012***	0.001	0.0009
	[0.0006]	[0.0005]	[0.0007]	[0.0005]	[0.0006]	[0.0006]
9	0.003***	0.0025***	0.0025***	0.0023***	0.0023***	0.0017***
	[0.0007]	[0.0007]	[0.0008]	[0.0007]	[0.0007]	[0.0006]
Biggest	0.0051***	0.0044***	0.0054***	0.0041***	0.0042***	0.0030***
	[0.0012]	[0.0011]	[0.0012]	[0.0011]	[0.001]	[0.001]

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 12: Momentum Replicating Portfolios Based on Various Beta Estimation Windows

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns during different windows. The first three scenarios all involve two-year windows; they are nonoverlapping periods starting at 47 months prior to, 23 months prior to and one month ahead the formation month, respectively. The last scenario involves a longer window of the most recent five years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the value-weighted average beta of stocks in the long portion minus that in the short portion. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. The replicating portfolio is the sum of the three components. In the first, second and fourth scenario, the replicating portfolio is a tradable strategy.

REP	[-4, 2]	[-2, 0]	[0, 2]	[-5, 0]	[-4, 0]	[-3, 0]	[-2.5, 0]
$\bar{r}$	0.0021***	0.0041***	0.0019	0.0025**	0.0027**	0.0030**	0.0037***
	[0.0008]	[0.0014]	[0.0013]	[0.0012]	[0.0012]	[0.0013]	[0.0014]
$\bar{r}/\sigma$	0.0805	0.0964	0.0448	0.0732	0.0721	0.0733	0.0874
info ratio	0.0857	0.1382	0.1426	0.1117	0.1162	0.1180	0.1309
FF3 α	0.0022***	0.0056***	0.0052***	0.0037***	0.0041***	0.0045***	0.0052***
	[0.0008]	[0.0012]	[0.0011]	[0.001]	[0.0011]	[0.0012]	[0.0012]
β_{mkt}	-0.09**	-0.21***	-0.23***	-0.17***	-0.21***	-0.23***	-0.22***
	[0.04]	[0.06]	[0.07]	[0.06]	[0.06]	[0.06]	[0.06]
β_{smb}	0.08	0.18	-0.08	0.16*	0.18*	0.16	0.17
	[0.07]	[0.13]	[0.08]	[0.10]	[0.10]	[0.12]	[0.13]
β_{hml}	0.05	-0.17	-0.41***	-0.13	-0.16	-0.17	-0.17
	[0.06]	[0.13]	[0.13]	[0.12]	[0.12]	[0.13]	[0.13]
Adj. R^2	3.3%	10.1%	27.8%	9.4%	12.2%	11.9%	11.0%
ρ (., umd)	52.4%	71.1%	84.4%	71.8%	72.1%	71.1%	70.0%

[ ]: Newey-West standard errors with 6 lags;

*/**/***: statistically significant at 10/5/1%

Table 13: Fama-French Factor Autoregressive Portfolios

The correlations are between a factor's average return this month and return in the past year (omitting the previous month). Outliers are defined as past returns whose values are in the top 5% percentile or the bottom 5% percentile of the entire sample of historical factor returns. The autocorrelation portfolio is a factor portfolio with weights equal to the past returns; at time t it has a return of $r_{t-12,t-2}r_t$. The tradable portfolio has weights $r_{t-12,t-2}$ winsorized at the top and bottom 5% of historical factor returns up to the portfolio formation date. All point estimates and standard errors are 100 times their actual values.

	Full Sample			Outliers Winsorized			Tradable Portfolio		
	MKT	SMB	HML	MKT	SMB	HML	MKT	SMB	HML
$\rho\left[r_{t-12,t-2}, r_t\right]$	1.5%	6.5%**	0.9%	6.9%**	10.5%***	7.6%**			
	{0.64}	{0.04}	{0.78}	{0.03}	{0.00}	{0.02}			
$\rho\left[r_{t-12,t-2}, r_t^2\right]$	-24.1%***	4.1%	-6.2%**	-17.8%***	4.9%	-12.8%***			
	{0.00}	{0.04}	{0.05}	{0.00}	{0.13}	{0.00}			
FF3 α ×100	0.0125**	0.0018	0.0014	0.0077*	0.0023**	0.0027**	0.0090**	0.0021**	0.0025**
	[0.0053]	[0.0013]	[0.0019]	[0.0041]	[0.0011]	[0.0013]	[0.0041]	[0.0010]	[0.0012]
β_{mkt} × 100	-0.62	-0.04	-0.18	0.01	-0.02	-0.21*	-0.06	-0.03	-0.17
	[0.56]	[0.04]	[0.13]	[0.32]	[0.03]	[0.11]	[0.35]	[0.03]	[0.13]
β_{smb} × 100	0.50	0.48***	0.46**	0.22	0.39***	0.29**	0.34	0.46***	0.36**
	[0.38]	[0.16]	[0.19]	[0.21]	[0.14]	[0.14]	[0.24]	[0.13]	[0.16]
β_{hml} × 100	-1.28	0.04	0.09	-0.13	0.01	-0.25	-0.54	0.00	0.05
	[0.99]	[0.09]	[0.34]	[0.22]	[0.07]	[0.3]	[0.46]	[0.07]	[0.33]
Adj. R^2	10.28%	12.93%	4.78%	8.94%	12.92%	9.16%	3.35%	17.73%	4.41%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

{ }: p-values for $H_0 : \rho = 0; H_a : \rho \neq 0$

Table 14: Reconstruction of Momentum's FF3 Alpha from Factor Structure Autocorrelations

The alpha of the replicating portfolio is the sum of the alphas of the three factor components. Each component is roughly a $M\times$ magnified version of the factor autocorrelation portfolio, which is the factor itself scaled by its past returns, i.e., $F_{t-12,t-2}F_t$. The alpha of the factor autocorrelation is in turn approximately the sum of two components, the autocovariance of the factor $\hat{E}\left[F_{t-12,t-2}F_t\right]$and the covariance between the factor's squared returns and past returns $\hat{E}\left[F_{t-12,t-2}F_t^2\right]\times\left\{E\left[F_t\right]/\sigma^2\left[F_t\right]\right\}$. The factor covariances are estimated in a robust regression limiting the influence of outliers. The magnifier is estimated in a robust regression of momentum beta on a factor on the factor's past returns, also limiting the influence of outliers. The alphas of the autoregressive portfolios are similar to those in the right panel of Table 13.

	$\rho\left(r_{t-12,t-1},r_t\right)$	$\rho\left(r_{t-12,t-1},r_t^2\right)$	$\operatorname{cov}\left(r_{t-12,t-1},r_t\right)$	$\operatorname{cov}\left(r_{t-12,t-1},r_t^2\right)$	
MKT	6.9%	-17.8%	4.93E-05	-1.50E-05	
SMB	10.5%	4.9%	2.88E-05	1.16E-06	
HML	7.6%	-12.8%	2.12E-05	-2.95E-06	
	Autocorr.	Leverage	Autoreg. α	Magnifier (M)	α
MKT	5.08E-05	2.86E-05	7.94E-05	17.97	0.0014
SMB	2.97E-05	-2.61E-06	2.71E-05	61.06	0.0017
HML	2.18E-05	9.06E-06	3.09E-05	55.32	0.0017
				Momentum α	0.0048

Table 15: Momentum Replicating Portfolios Based on Recent $\Delta\beta$ and Regression R^2

At the end of each month, stocks are ranked according to two indicators: (1) the R^2 of the FF3 factor regression with the most recent two years (i.e., $[-2,0]$ years) of monthly returns and (2) the normalized distance between the beta estimates from the recent ($[-2,0]$ years) FF3 factor regression and estimates from the earlier ($[-4,-2]$ years) regression, i.e., $\Delta\beta=\sqrt{\left(\beta_{mkt}^{NEW}-\beta_{mkt}^{OLD}\right)^2/\sigma_{\beta_{mkt}}^2+\left(\beta_{smb}^{NEW}-\beta_{smb}^{OLD}\right)^2/\sigma_{\beta_{smb}}^2+\left(\beta_{hml}^{NEW}-\beta_{hml}^{OLD}\right)^2/\sigma_{\beta_{hml}}^2}$. The sample is divided into four subsamples by the medians of these two indicators. UMD-VW is the value-weighted momentum portfolio in the subsample and REP the dynamic replicating portfolio. Stocks with high R^2 are ones with lower idiosyncratic risks relative to the FF3 factors compared to stocks with low R^2; stocks with high $\Delta\beta$ are ones that have experienced larger shocks to their betas than stocks with low $\Delta\beta$.

	High R^2				Low R^2			
	High $\Delta\beta$		Low $\Delta\beta$		High $\Delta\beta$		Low $\Delta\beta$	
	UMD-VW	REP	UMD-VW	REP	UMD-VW	REP	UMD-VW	REP
$\bar{r}$	0.0070***	0.0048**	0.0037***	0.0032***	0.0072***	0.0055***	0.0036**	0.0006
	[0.002]	[0.0021]	[0.0014]	[0.0011]	[0.0017]	[0.0012]	[0.0015]	[0.0008]
FF3 α	0.0108***	0.0073***	0.0061***	0.0040***	0.0096***	0.0056***	0.0055***	0.0014*
	[0.0018]	[0.0018]	[0.0012]	[0.001]	[0.0017]	[0.0012]	[0.0014]	[0.0007]
β_{mkt}	-0.30***	-0.32***	-0.17***	-0.17***	-0.10	-0.13**	-0.16**	-0.1**
	[0.09]	[0.08]	[0.06]	[0.05]	[0.07]	[0.06]	[0.07]	[0.04]
β_{smb}	0.14	0.23	0.03	0.23**	0.02	0.22	0.09	0.08
	[0.12]	[0.2]	[0.07]	[0.1]	[0.13]	[0.17]	[0.08]	[0.05]
β_{hml}	-0.61***	-0.34*	-0.38***	-0.11	-0.46***	0.00	-0.31***	-0.11*
	[0.17]	[0.2]	[0.11]	[0.1]	[0.13]	[0.11]	[0.11]	[0.06]
Adj. R^2	21.7%	13.2%	16.3%	10.1%	10.0%	3.6%	11.0%	8.0%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 16: Momentum Replicating Portfolio relative to the FF3 Factors + Financial Distress Factor

The financial distress factor (FDF) is a long-short portfolio formed by sorting stocks on their predicted delisting probabilities, which are estimated from a logistic regression model using firm characteristics. At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns and the financial distress factor during the most recent two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the value-weighted average beta of stocks in the long portion minus that in the short portion. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. The replicating portfolio is the sum of the four components.

	UMD-HH		REP (FF3)		REP(FF3+FDF)	
$\bar{r}$	0.0066***		0.0049**		0.0064**	
	[0.0022]		[0.0021]		[0.0029]	
$\bar{r}/\sigma$	0.1516		0.1092		0.1266	
info ratio	0.2045	0.1461	0.1269	0.1464	0.2005	0.1885
FF3 α	0.0086***	0.0063**	0.0054***	0.0063**	0.0093***	0.0087***
	[0.0018]	[0.0029]	[0.0021]	[0.0025]	[0.0026]	[0.0029]
β_{mkt}	-0.18**	-0.11	-0.19*	-0.28**	-0.38***	-0.37***
	[0.09]	[0.08]	[0.10]	[0.11]	[0.10]	[0.11]
β_{smb}	0.11	0.27*	0.34	0.23	0.30	0.33
	[0.13]	[0.15]	[0.23]	[0.23]	[0.28]	[0.25]
β_{hml}	-0.35**	-0.30	-0.14	-0.25	-0.32	-0.30
	[0.17]	[0.18]	[0.22]	[0.25]	[0.23]	[0.25]
β_{fdf}		0.44**		-0.04		0.08
		[0.20]		[0.16]		[0.20]
Adj. R^2	7.0%	8.7%	7.3%	8.7%	14.0%	13.9%
ρ(., UMD-HH)			68.3%		68.8%	

[ ]: Newey-West standard errors with 6 lags;

*/**/***: statistically significant at 10/5/1%

Chapter 2

The Diverse and Ever-changing Sources of Momentum Profit

1 Introduction

Since first described in detail in Jegadeesh and Titman (1993), price momentum has been a well-known but puzzling phenomenon in finance. In many asset classes including stocks, bonds and commodities, assets that have high recent returns ("winners") tend to outperform those with lower recent returns in subsequent months. A zero investment portfolio that buys the aforementioned winners and short-sells losers tend to generate a persistently positive profit. Such profit remains positive even after controlling for well-established systematic risk factors. This raises the question of the origin of the momentum profit, whether it arises due to short-lived anomalous causes or represents compensation for systematic risks.

The answer to this question has proven elusive for several reasons. One is that we do not yet have a complete picture of momentum profit. Previous attempts to replicate momentum using dynamic portfolios of the popular Fama-French factors (henceforth FF3) were first deemed a failure by Grundy and Martin (2001) and then declared successful by Liu (2012). Success, however, is limited to large stocks, for which abnormal momentum profit can be reduced to a positive but insignificant level; for small stocks, the FF3 replicating portfolio has trouble matching momentum return. One may stop here and declare "mission accomplished" because small stocks are notoriously difficult and expensive to trade, rendering small-stock momentum impractical. A limits-to-arbitrage argument would be able to rationalize the continued existence of small-stock momentum. Still, the complete origin of momentum profit, why it is positive and more significant among small stocks, remains unknown. Moreover, residual momentum, a portfolio formed by sorting stocks on their residual returns relative to the Fama-French factors, appears more prominent than momentum itself according to Blitz, et. al. (2011). As long as we leave unexplained portions of momentum, we will forever wonder about what remains. Further research along that line amounts to a waiting game for new systematic factors to be discovered and widely accepted. Those factors can then be used to construct a momentum-replicating portfolio and its performance compared against the real thing. Given the constantly changing market dynamics and the potentially very large set of systematic risks, it is unlikely that we will ever know the complete and permanent set of risk factors as the benchmark against which momentum can be measured.

The second source of difficulty is that the definition of what constitutes systematic risks is unclear, and the set of known systematic risks is constantly debated. In a theoretical sense, a systematic risk factor is one that is correlated with the stochastic discount factor, which in turn is determined by the preferences of the representative consumer. All of these are highly abstract concepts that can only be tested indirectly in the data. In a more practical sense, a systematic risk factor can explain some of the cross-sectional variations in asset returns and in turn generates a nonzero risk premium. The Fama-French factors were first conceived as systematic risk factors in the practical sense since they function well in explaining cross-sectional returns over a long period of time. Subsequently, they were assigned deeper meanings to justify their practical usefulness, but those theoretical connections are up for debate. The Fama-French factors may be good proxies of some underlying "real" risk factors, but they may also include transient trends. Liu (2012) found that momentum's FF3 alpha can be traced to slightly positive autocorrelation among the Fama-French factors themselves, which could arguably be a sign of behavioral influences. At the same time, the size factor, which was originally thought to be a permanent systematic factor, has lost its significance in recent years. Our experience with identifying risk factors so far shows that we cannot even guarantee the factors we know to be fully rational and long-lasting.

This paper does not aim to explain momentum completely but rather take a step in the right direction. The first part is purely empirical, an attempt to replicate momentum better than previous efforts and eliminate the unknowns in residual momentum. Rather than relying on known systematic factors such as the FF3, I will use a model-free method, namely principal component analysis, to extract factors. This method has a well-defined limit: as the number of factors approaches the number of available assets in the market, the replicating portfolio by definition approaches perfect correlation with momentum. I will show that only a small set of factors is needed to replicate the difficult equally-weighted momentum portfolio with high precision, and all remaining factors make insignificant contributions.. I will introduce a refined selection technique that can substantially reduce the number of factors needed to match momentum profit. Several interesting results follow immediately: first and the most obvious is that there is no longer residual momentum. Momentum profit can be essentially characterized as the sum of the profits of the replicating component plus a number of terms that are essentially noise. Second is the fact that the momentum portfolio is not the most efficient implementation given its design: the replicating portfolio can yield superior return and Sharpe ratio, both in the first month following formation and over a longer horizon. In addition, momentum makes apparent "mistakes" in its implementation so that there exists a portfolio positively correlated with momentum but has a negative return.

Finding a set of factors that can explain momentum is only the beginning. The next task is to see whether these factors could shed a light on the mechanisms behind momentum and the factor structure that drives it. Across all factors, momentum indeed takes positions corresponding to the factors' recent returns, hence the highly time-varying loadings. Momentum casts a wide net and changes its portfolio composition constantly. Thus, there is a severe limit to how much a fixed set of few factors, such as the Fama-French factors, can explain. Momentum derives its profit in many pieces scattered about in the factor structure. The more a factor explains the cross-sectional dispersion in mean returns, the more the factor contributes to momentum. Since the portfolio tends to load positively (negatively) on stocks with positive (negative) risk premia, momentum is expected to achieve a positive return on average. If we believe that the time-varying risk premia, as proxied by long-run historical average returns, are justified as compensation for risks, then momentum profit can also be rationalized as fair compensation.

The idea of momentum not being a standalone risk factor but a portfolio of existing risk factors stands alongside other well-known behavioral and rational explanations. The behaviorists argue that price continuation is irrational and is the result of cognitive biases such as under-reaction. Barberis, Shleifer and Vishny (1998) and Hong and Stein (1999) both offer models of investors' bounded rationality that leads to under-reaction to news. The former has investors suffer from representativeness heuristics of Tversky and Kahneman (1974) and incorrect beliefs about the earnings process; the latter assumes that investors follow simple trading strategies conditional on a limited information set. Both cause investors to react slowly news initially and then possibly overreact over the next months, leading to the observed short-term momentum. Fully rational investors are either not present in the market or unable to correct the mispricing due to limits to arbitrage.

Fully rational frameworks for momentum includes Johnson (2002), who modeled a firm's log market value $\log(V)$ as a convex function of a priced state variable p. Then the firm's beta, $d\log V(p)/dp$, is a positive and increasing function of p. An increase in p leads to both an increase in V, i.e., positive recent returns, and higher subsequent returns due to the firm's now higher beta. Sagi and Seasholes (2006) expanded on Johnson's model and interpreted the upper portion of the convex function as risky growth opportunities within firms. Garlappi and Yan (2011) modeled firms as having lower systematic risks as they approach the default boundary, rationalizing the lower portion of the convex function. Both behavioral and rational explanations have merits and are backed by empirical evidence, albeit often in indirect forms. It is difficult to distinguish between the two because rational explanations require complete identification of the systematic risks that momentum profit rewards, while behavioral explanations require proof of the lack thereof. Both of which are not easy tasks. The decomposition of momentum into a series of dynamic factor portfolios does help build a common platform upon which the phenomenon can be better understood and various explanations tested.

The paper is organized as follows. In the next section, I will discuss in theoretical terms the mechanisms that drive momentum returns and the method of extracting factors from principal component analysis. Section 3 is devoted to the replication of equally-weighted momentum portfolio in the data, and Section 4 contains analysis of the results that link factor risk premia to momentum profit. Section 5 has additional tests on the replicating portfolio, and Section 6 concludes.

2 Theoretical Background

2.1 Factor Structure and Momentum Returns

As in Liu (2012), I modeled a market of many assets whose returns follow a multifactor model such as Ross (1976)'s APT model, i.e.,

$$r_{it} = r_f + \sum_{k=1}^{K} \beta_{ik} f_{kt} + \epsilon_{it}$$

where β_{ik} is the beta of Asset $i \in \{1, 2, \ldots, I\}$ on the kth systematic factor. f_{kt} is the return of the kth systematic factor at time t and ϵ_{it} is an i.i.d. noise term. This is an arbitrage-free world with perfect information and no friction. Any static or dynamic zero-investment trading strategy P using a subset of available assets will have the following returns:

$$r_{Pt} = \sum_{k=1}^{K} w_{kt} f_{kt} + \epsilon_{Pt}$$

where w_{kt} is the portfolio's weight or beta on the kth factor and ϵ_{Pt} the idiosyncratic component orthogonal to f_{kt}'s and w_{kt}'s. w can be a constant or vary over time. Rather than focusing on the alpha of the portfolio relative to the factor structure, I will instead focus on the mean return, which can be easily measured and model-free. Unlike the more complicated expression for the alpha, the mean return of the above portfolio is simply

$$\bar{r}_P = \sum_{k=1}^{K} E\left[w_{kt} f_{kt}\right]$$

Compared to the one for alpha, this expression is substantially simpler. Since at each time t, $w_{k,t+1}$ is already known, we can write the conditional version of the above expression:

$$E_t\left[\bar{r}_{P,t+1}\right] = \sum_{k=1}^{K} w_{kt} E_t\left[f_{k,t+1}\right] \equiv \sum_{k=1}^{K} w_{kt}\pi_{kt} \tag{1}$$

where π_{kt} is the (possibly time-varying) risk premium of factor k at time t. It follows that momentum return is positive if enough components in the summation in Eq. 1 are positive, i.e., w_{kt} and π_{kt} are positively correlated. In other words, as long as momentum more often takes positive positions on most systematic risks, we would expect a positive return. This condition is substantially easier to satisfy than the one for positive alpha. Given that a factor has a positive (negative) risk premium, its realized returns are more likely to be positive (negative). Therefore, momentum is more likely to take a positive (negative) position in it. So over time, w_{kt} and π_{kt} will in fact be positively correlated. It follows that momentum profit is expected to be positive.

In practice, we can proxy risk premium by the average historical factor return over a long horizon (e.g., 60 months), then we have

$$E_t\left[\bar{r}_{P,t+1}\right] \approx \sum_{k=1}^{K} w_{kt}\bar{f}_{k,t-60\to t-1} \tag{2}$$

If we are concerned about the most recent 12 months being unusual deviations from the long-run average, then we can also define the risk premium more conservatively with the $[-60,-13]$ window instead.

When we decompose momentum into a large number of replicating components, each of which represent a risk factor in the market, the factors that contribute the most to the portfolio mean return are ones whose $w_{kt}\pi_{kt}$ are the largest. Rather than a perfect replication, we can select a small subset of components with the highest risk premia and discard the rest. The reduced portfolio is much leaner and possibly more efficient due to the residual terms contributing negligible amount to the mean but positive amount to the variance. Since a higher risk premium affects both w and π, it has a quadratic influence on the covariance between them. Therefore, factors with larger risk premia have disproportionately larger roles in shaping the momentum profit. It is likely that only a small subset of factors is needed in the reduced portfolio to achieve a return similar to momentum's. An indirect test of Eq. 2 involves sorting a known set of factors, in descending order, by their risk premium estimates multiplied by the loadings of the momentum portfolio on them. Taking a small subset of factors should yield a replicating portfolio with the majority of the momentum profit explained. Then, taking successively larger subsets will yield replicating portfolio of increasing mean returns, but with the marginal increase quickly diminishing. This selection method can reduce a potentially large set of factors to a small set of relevant ones.

The above mechanism only works if past returns are good proxies for future returns, i.e., Eq. 2 holds. This can be tested directly in the data by comparing the mean factor returns going forward with the mean past factor returns across the set of available factors. If the future factor returns are well-explained by the past returns, it means that the historical risk premia predict future factor returns. If the coefficient is equal to one, then Eq. 2 holds exactly.

A related implication of Eq. 2 is that there could exist components for which w and π have opposite signs. Since the sign of w is determined by the 12-month past returns, momentum sometimes assigns negative weights to factors with positive expected returns. For factors with small risk premia, the effect is negligible; but for ones with large risk premia, the effect may be significant. When this happens, momentum returns next period will be impacted negatively. By selecting the most negative $w_{kt}\bar{f}_{k,t-60\to t-1}$, we may be able to construct a portfolio that is positively correlated with momentum yet has a negative return. The practical benefit of identifying this portfolio is that they can be subtracted from momentum to increase its return and Sharpe ratio simultaneously.

2.2 Systematic vs. Idiosyncratic Risks

I define systematic risk factors as ones who have a relatively large impact on the dispersion of mean returns (risk premia) in the cross-section. This definition is a relative one in the sense that there is no absolute boundary between systematic and idiosyncratic risks. A set of factors can be sorted based on their impact. Factors at the two ends are clearly systematic and idiosyncratic risks, while the ones in the middle are of uncertain quality. The amount of impact a factor has on the dispersion of risk premia depends on its own risk premium and the dispersion of betas on the factor among assets. The impact of Factor k is therefore

$$I_k = \sigma_{\beta_k} \cdot |\pi_k| \tag{3}$$

Both terms are necessary. By definition, all factors orthogonal to the market portfolio must have net betas in the cross-section equal to zero. Therefore, a higher dispersion of betas leads to more differences in mean returns among assets, ceteris paribus. A larger risk premium, of course, magnifies this effect. The dispersion term also normalizes the risk premium term since factors are zero-investment portfolios that can be arbitrarily scaled.

The sorting mechanism described in 2.2 is somewhat similar to the one that separates systematic risks from idiosyncratic risks. $|w|$ is proportional to the dispersion of betas in the cross-section, while $\bar{f}$ is a proxy for the factor risk premium. However, w is also proportional to the recent 12-month return of the factor, so a sort by Eq. 2 will favor assets that have a more significant return in recent months. On the other hand, since $\bar{f}$ is a multiplicative term, a factor will still only be chosen if its historical average return is sufficiently large.

2.3 Principal Component Analysis

I use principal component analysis to generate a pool of orthogonal factors. At each time t, a panel data consisting of cross-sectional excess returns (stock return - the risk-free rate) over a long history, e.g., 60 months, is transformed into a set of principal components

$$f_j = \sum_{i=1}^{I} \lambda_{ji} r_{it}^e$$

for $j = 1, \ldots, J$. Each factor f_j is automatically a portfolio of stocks and the risk-free asset, uncorrelated with all other factors by construction. The factors are by default numbered in the descending order of their eigenvalues, or equivalently their ability to explain the variances and covariances of returns in the cross-section. The first factor often resembles the market portfolio with a correlation of over 95% and is often capable of explaining well over half of the total variances.

Factors generated from PCA are not literally the factor structure because the procedure uses only limited data (return data). The real factor structure, should it exist, may look rather different. At any given time, the underlying "real" systematic factor may be captured by one or several PCA factors or none at all. The last scenario may occur if the factor is not volatile enough to explain a significant portion of cross-sectional returns. In the next period, that factor may correspond to some differently numbered PCA factors. Since PCA by definition explains most of the variations in returns, however, it is impossible for systematic risks to elude the PCA factor space altogether. A risk factor that hardly explains any cross-sectional returns over time certainly cannot be a relevant one. Therefore, the dynamically updated set of PCA factors should be an adequate starting point for identifying systematic risks.

A useful feature of the PCA factors is that subsets of factors can be chosen at will without affecting subsequent analysis such as beta estimation. In a time-series regression of any particular stock return,

$$r_t^e = \alpha + \sum_{s=1}^{S} f_{j(s),t} \beta_s + \epsilon_t$$

The point estimate for β_s only depends on r^e and $f_{j(s)}$, not any other f's when all the factors are uncorrelated:

$$\hat{\beta}_s = \frac{\text{cov}\left(r^e, f_{j(s)}\right)}{\text{var}\left(f_{j(s)}\right)}$$

For the purpose of replicating a portfolio, it is possible and easy to discard some factors. The resulting replicating portfolio is simply the full replicating portfolio minus the contribution from the discarded factors. Each component in the replicating portfolio is essentially independent to every other since the betas of each factor on all assets can be estimated independently. Discarding components of a replicating portfolio do lead to a loss of precision, for sure, but there may be a gain in efficiency.

This technique of selecting a subset of factors is particularly important for replicating momentum, since by construction it derives its profit from a large and diverse set of factors. As I will show in the next section, the selection process matters a great deal to how well the replicating portfolio matches the momentum portfolio.

3 Main Empirical Results

3.1 Data Description

The main data sources are the CRSP (Center for Research in Security Prices) dataset for monthly stock returns, which is available on WRDS (Wharton Research Data Services). The entire CRSP universe of firms is used; it covers the period from January 1925 to December 2011. The Fama-French factors, also available from WRDS, are used in certain subsections; the data run from January 1926 to June 2012. To be included in a ranking for portfolio formation, stocks must meet three conditions during the formation month. The first is that they must be traded on the New York Stock Exchange, the American Stock Exchange or the NASDAQ Stock Market. The second is that they must have an ending price per share of at least $1; this rule is aimed at eliminating penny stocks suffering extreme liquidity problems and trading frictions. The third rule, which is imposed by necessity, removes stocks without a complete 60-month return history. This rule ensures that principal component analysis has access to a complete panel. While the first two rules do not reduce the sample size significantly, the third one on average removes 28% of the stocks in the CRSP database. One may argue that requiring a five-year history takes out many small stocks that are often chosen by momentum. That is indeed the case. Nevertheless, all analysis in this paper assume that the universe consists of only stocks that satisfy these rules, so there is internal consistency. For example, the replicating portfolio has access to the same set of stocks as the momentum portfolio. Had complete historical data existed for the remaining stocks, they would be included in momentum and made available to the replicating portfolio at the same time.

3.2 Variants of the Momentum Portfolio

To compensate somewhat for omitting a large set of stocks, I will use the equally-weighted momentum portfolio as the default momentum. It is a more difficult variant of momentum to explain, as it heavily favors small stocks and results in higher return and Sharpe ratio compared to other momentum variants with different weighting schemes. On the other hand, the equal weights make factor analysis and portfolio formation much more straightforward.

As usual, I will define the momentum portfolio (also known as UMD or up-minus-down) by ranking stocks based on their cumulative returns in the second to 12th months prior to portfolio formation, i.e., the most recent one-year return excluding the most recent month. This is the canonical definition for momentum and is used in most papers on the subject as well as for the momentum factor data available on WRDS and Ken French's website. The most recent month is excluded due to short-term return reversal, which is a phenomenon distinct from momentum and is the consequence of market microstructure effects according to Jagadeesh and Titman (1995). Table 1 lists the most common variations of the momentum portfolio based on the set of stocks and portfolio weights used in the ranking. The large- and small-stock momentum portfolios are formed by first splitting the sample into two groups, stocks with capitalizations higher and lower than the NYSE median, then computing the value-weighted momentum in the subgroup. The half-half momentum portfolio is the simple average of the large- and small-stock momentum. It is immediately apparently that momentum in small stocks is notably stronger than that in large stocks.

3.3 The Replicating Portfolio

Momentum is a dynamic portfolio of existing risk factors, so in theory, if we can identify all relevant factors, we can replicate the momentum portfolio exactly in two simple steps: determining the betas of individual assets on each factor, inferring from them momentum's loadings on all the factors and composing a portfolio by attaching the appropriate weights to them. The replicating portfolio should then be perfectly correlated with the momentum portfolio and attains the identical returns. In practice, there are several difficulties. The most serious problem is that the set of relevant factors is not known. The Fama-French three-factor model has been widely accepted as a working solution to this problem. The FF3 factors, when applied to large stocks, can adequately explain the time variation of the momentum portfolio and a large portion of the mean return. For small stocks, however, they are not sufficient. Panel A of Table 2 shows the replication results. While the FF3 replicating portfolio can explain most of the time variation in equally-weighted momentum portfolio, it achieves a mere fraction of the mean return.

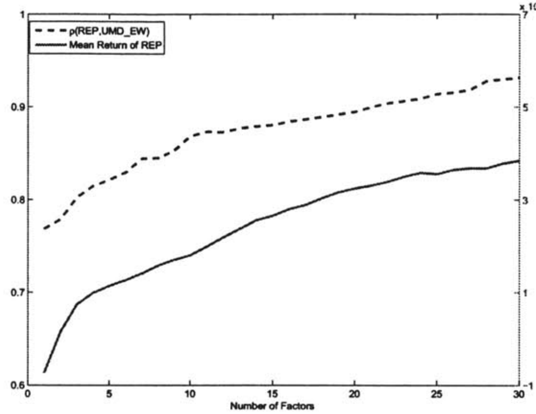
In general, it seems very easy to explain the time variation part of momentum: even the market component of the replicating portfolio is already 60.5% correlated with momentum. Explaining the mean return, however, is almost a completely separate task. In view of the dynamic nature of the momentum portfolio, this is not entirely a surprise. In any given period, momentum loads on all risk factors depending on their past returns and the dispersion of betas in the market. Since the market factor is by far the most prominent, momentum's time-series properties are inextricably tied to it. But since the dynamic portfolio takes positive and negative positions on it over time, the average position is close to zero. Momentum's mean return, the most puzzling part about it, is insulated from the constant ebbs and flows of market returns over time.

Therefore, it is unlikely that factors formed from the first few principal components can adequately describe momentum's mean return. They are deemed important because they explain the highest amount of variances among individual stock returns. For example, the market portfolio is often the first factor. In Panel B of Table 2, we can see that the replicating portfolio from the first three factors performs very similarly to the FF3 factors. The replicating portfolio is highly correlated with momentum but has a negligible mean return.

Unlike FF3, which stops at three factors, the principal component method produces a large number of potential factors. Using more factors to construct the replicating portfolio should improve the mean return of the replicating portfolio. The reason is that, in the theoretical limit when the frequency of the available return data exceeds the number of stocks in the market at any time, all stock returns can be expressed precisely as a linear combination of the principal components. That is, of course, not a meaningful result. Even though momentum loads on all risk factors, the question is whether there exists a small number of factors that can replicate momentum sufficiently well. In the current ordering (by eigenvalues), increasing the number of factors helps but not fast enough. Panel C of Table 2 contains the statistics of replicating portfolios of increasingly larger factor set. As expected, both correlation with momentum and mean return increases with the number of factors used. Still, the replicating portfolio can only explain a little over half of the 0.71% per month return of the momentum portfolio with 30 factors. Over 90% of the momentum variance has already been explained by this point.

The encouraging fact, as seen from Figure 1, is that the mean return of the replicating

Figure 1: Explanatory Power of the Replicating Portfolio with Successively More Factors



The correlation of the replicating portfolio with the equally-weighted momentum portfolio (mean return = 0.71%) and its mean return as the number of factors increases. The factors are portfolios formed from principal components of the 60-month return panel and sorted in descending order by their eigenvalues.

portfolio increases roughly monotonically as the number of factors increases. It suggests the possibility that at every point in time, some select factors are contributing to the mean return in significant ways while most others do not. Over time, these important factors rotate due to momentum putting more or less emphasis on them with its dynamic loadings. When not sorted in the correct manner, e.g., in a random order, the first few factors chosen are going to have importance just by luck in some periods but are not useful in others. As a result, the average is positive but small. As the factor set expands, the most relevant factors are more likely to be included somewhere in the set every period, so that the mean return increases steadily towards that of the momentum portfolio.

3.4 Subsample Momentum

Performing principal component analysis on a large panel is a computationally intensive task that can be simplified by noting that momentum works just as well with a subset of stocks as with the full cross-section. The momentum portfolio on a randomly selected subset is indistinguishable from the canonical momentum portfolio in almost all aspects, and the subset can be a fraction of the size of the cross-section. Table 3 shows the moments of the momentum portfolios formed on various subsets of stocks. For example, in Column 2, 5% of stocks are chosen at random (with a minimum of 200) each period, and they are capable of generating momentum portfolios that are on average identical to the canonical momentum portfolio in terms of mean return and over 90% correlated with it. One minor difference is that since the subset momentum averages over a small set of stocks, the returns are more volatile, leading to a slightly lower Sharpe ratio. This difference, along with minor sample variations, will affect the precision of the analysis below but not the main conclusions. Therefore, instead of extracting factors from the full cross-section, I will extract them from randomly selected subsets. The benefit is faster computation plus smaller factor portfolios.

The factors extracted from a random subset are also similar to the ones extracted from the full cross-section, albeit slightly more noisy. Table 4 compares the performance of a typical replicating portfolio built from a 5% subset to its full-sample counterpart (Panel C of Table 2). They are nearly the same. In terms of mean returns, there are no obvious advantages or disadvantages in using factors extracted from a subset.

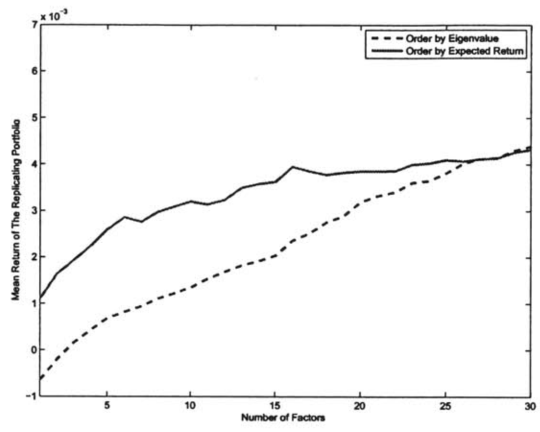
3.5 Factor Rearrangement and Refinement

Given a set of factors is generated each period and a replicating portfolio is formed on the first k factors, it may be possible to rearrange the factors in a particularly way first that will improve the efficiency of the first few factors. The obvious choice is to rank factors by their expected returns going forward, i.e., Eq. 2. The ranking is implemented as follows: the first 50 factors are retained from the principal component analysis; the beta of the momentum portfolio on each factor (w) and the factor's 60-month average returns (proxy for $E_t(f_{t+1})$) are determined; the factors are sorted in descending order of $wE_t(f_{t+1})$ and the first k factors are chosen.

If momentum return wf_{t+1} is positively related to its proxy $wE_t(f_{t+1})$, this ranking should shift the mass under the green curve in Fig. 1 to the left, meaning that the first few replicating components have become more efficient. This is indeed the case as illustrated by Table 5 and Fig. 2. Taking only first five components, the replicating portfolio achieves a mean return of 0.26% per month compared to the original 0.07%. The first three replicating components generate a mean return of 0.19% per month compared to FF3's 0.10% per month. The dramatic improvement in the first few factors proves that the expected returns of the factors, as proxied by the average historical returns, are positively correlated with returns going forward. This is the first piece of suggestive evidence that momentum's seemingly abnormal returns can be justified by compensation for systematic risks.

On a practical level, we still face the problem that there is unexplained residual momentum. With the current method, a sensible number of factors is unable to match the momentum profit in its entirety. As seen in Fig. 2, rearranging the factors shifts some positive mean returns to the first few components but does not affect the sum of the first 30 components. The problem may be caused by subsample noise. In any particular subsample,

Figure 2: Effect of Rearranging Factors on Replicating Portfolio Performance

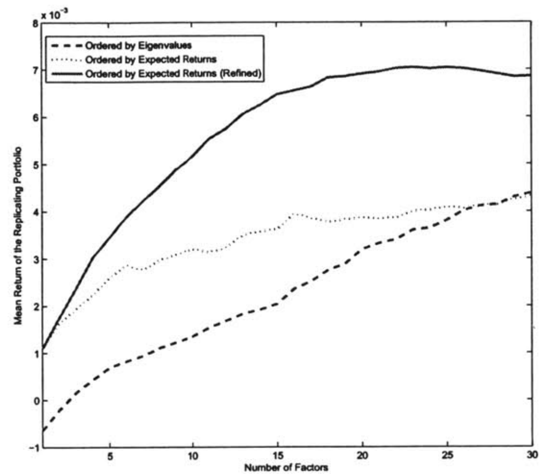


The mean return of the the replicating portfolio for the equally-weighted momentum portfolio as the number of factors increases. The factors are portfolios formed from principal components of the 60-month return panel of a typical 5% subset and sorted in descending order by their eigenvalues (dotted-blue) or by their expected return (solid-green).

some principal components may capture their corresponding risk factors well, while others do not. This occurs when the subset is not representative enough in some particular dimensions. Some risk factors may affect only a small portion of stocks during a given period, so a random subset is not guaranteed to capture them. It follows that the single-subset rearrangement strategy can be refined by comparing factor sets across a number of random subsets. Factors that are highly correlated across subsets are merged, while factors that are different are added to a pool. In practice, factors that are more than 70% correlated are merged while those that are less than 30% correlated are preserved. If two factors have correlation between 30% and 70%, the one that has a lower historical return is discarded. The large pool of factors is then sorted by their expected returns times their weights in the momentum portfolio, just as before, and the first k factors are chosen to construct the replicating portfolio.

The refinement produces a much more efficient factor set and better replicating portfolios. The last panel of Table 5 shows that the first five factors can now explain half of the mean return of the momentum portfolio; with about 20 factors, the replicating portfolio's mean return approaches the 0.71% per month level for equally-weighted momentum. After that, additional factor does not contribute additional mean return; in fact, they are ever slightly counterproductive. From Figure 3, we can see that this refined method substantially outperforms the previous methods. More importantly, it proves that it is possible to explain momentum in its entirety with a limited number of factors (in this case, 20). A comparison of the three panels in Table 5 also reveals that the refined method can overcome the noise

Figure 3: Refined Rearrangement of Factors



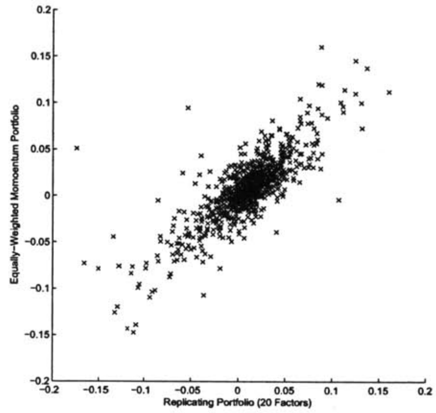
The mean return of the the replicating portfolio for the equally-weighted momentum portfolio as the number of factors increase. The factors are portfolios formed from principal components of the 60-month return panel of a typical 5% subset and sorted in descending order by their eigenvalues (dashed-blue) or by their expected return (dotted-green). The red line represents the refinement of the ordering-by-expected-return method.

introduced in subsampling by comparing factors across a number of subsamples.

With as few as 12 factors, the replicating portfolio now outperforms even the momentum portfolio itself in terms of Sharpe ratio. This result follows from concentrating the sources of mean return in the first few factors. The remaining factors, from 20th on, contribute nothing to the mean but positive amount to the variance. The implication is that momentum does not give the most efficient implementation given its design. When the expected return of a factor times its beta in the momentum portfolio is sufficiently small, the momentum portfolio should discard this component altogether, because the potential contribution to the return is not worth introducing the extra volatility.

In any case, we now have a set of 20-30 factors that can reliable reproduce both the time variation and mean level of momentum returns. The residual momentum is no longer a mystery because it is essentially white noise. Now we can treat this replicating portfolio (henceforth the "benchmark" replicating portfolio) as a decomposition of the momentum portfolio and turn our attention to analyzing the sources of momentum profit.

Figure 4: Scatterplot of Momentum Returns vs. the Benchmark Replicating Returns



The momentum portfolio is the equally-weighted variety. The replicating portfolio is constructed from 20 factors extracted from principal component analysis of the 60-month return panel using the refined method and sorted by their historical mean returns times their weights in the equally-weighted momentum portfolio. The correlation is 84.5%.

4 Sources of Momentum Profit

4.1 Dispersion in Mean Returns

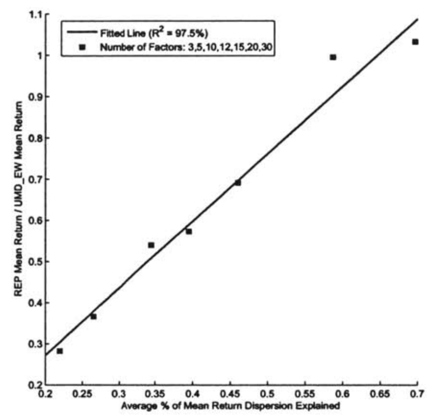
We have seen earlier a distinction between factors that can explain variances and covariances of assets and factors that can explain the spread in mean returns of assets. The former, represented largely by the market portfolio, is capable of explaining the time variation of momentum returns but not the mean. The latter, as seen in the previous section, can. The question is how well the ability of a factor to explain the spread in mean returns correlates with the factor's subsequent role in explaining momentum. For a given set of orthogonal factor set $F = \{f_1, \ldots, f_k\}$, I regress the historical excess returns of each stock r_{it} on factor returns to obtain the explained portion $\hat{r}_{it} = \sum_k \beta_{ikt} f_{kt}$. Then, I construct an indicator signifying the average ratio of the dispersion in mean return explained by the factor set to that in the actual cross-section:

$$R_F \equiv \frac{1}{T} \sum_{t=1}^{T} \sigma_t \left(\bar{r}_i\right) / \sigma_t \left(\bar{\hat{r}}_i\right)$$

where $\bar{r}_i$ and $\bar{\hat{r}}_i$ are both 60-month historical averages of r_{it} and $\hat{r}_{it}$, respectively, at time t. R_F is a proxy for the average amount of dispersion in mean return that the factor set F can deliver.

Figure 5 suggests a clear linear relationship between R_F and the mean return of the replicating portfolio. We can see that with only 60% of the dispersion in mean returns explained,

Figure 5: Scatterplots of the Dispersion Indicator vs. Replicating Portfolio Mean Return



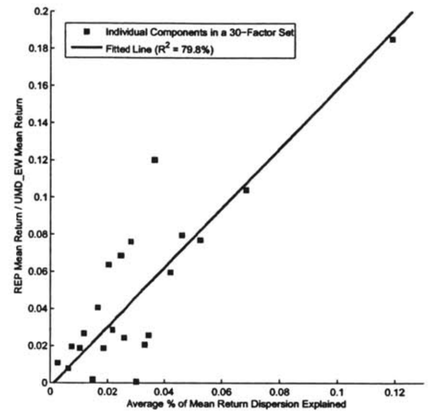
Each point represents a particular factor set with the number of factors equal to 3, 5, 10, 12, 15, 20 and 30 from left to right. The x-coordinate is the dispersion indicator R_F for that factor set and y-coordinate the mean return of the replicating portfolio divided by that of the momentum portfolio.

the replicating portfolio can already explain 100% of momentum's mean return. Reaching this goal without explaining all of the dispersion suggests the possibility of constructing momentum-like portfolios that are superior to momentum. As I will show below, this can indeed be done. It is also possible to break down a 30-factor portfolio into individual components and treat each as a factor set. Figure6 shows a noisier but still linear relationship between each factor's R_f and the mean return of the corresponding replicating component. The ability of a factor structure to explain the cross-sectional differences in mean returns among assets appears to be tightly related to its ability to explain momentum. Since R_F is also roughly the sum-product of the factors' historical risk premia and the cross-sectional dispersion of betas, we can also interpret the linear relationship in another light: factors that had nonzero risk premia in the past contribute to momentum profit according their historical impact on the dispersion of risk premia. Therefore, we may be able to argue that momentum returns are justified as risk premia or compensation for some risks.

4.2 Momentum Return as Compensation for Risk

In order to interpret momentum return as compensation for risk, we need to show that the factors involved commanded nonzero risk premia in the past and that momentum's returns can be justified given these past levels of risk premia. A straightforward test is to compute the replicating portfolio's historical risk premium and compare it against its return going forward. Since the portfolio consists of a number of components, the test can be applied to

Figure 6: Scatterplots of the Dispersion Indicator vs. Replicating Component Mean Return

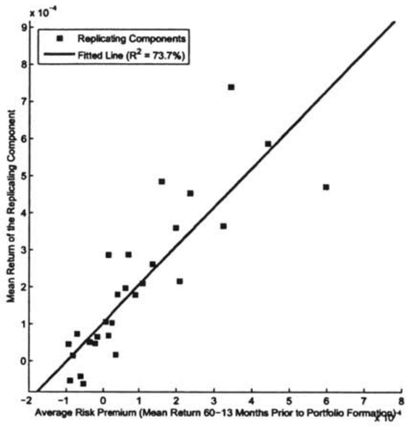


Each point represents a particular component within the 30-factor replicating portfolio. The x-coordinate is the dispersion indicator R_F for that factor and y-coordinate the mean return of the replicating component divided by that of the momentum portfolio.

individual components for additional comparisons.

For the benchmark replicating portfolio, its mean historical risk premium is 1.27% per month if measured with the most recent 60 months and 0.45% per month if measured with the most recent 60- to 13-month period. The most recent 12 months plays an important role because momentum's loading on a factor depends on the most recent 12-month return. The deviation of the recent return from the long-run average may be a temporary phenomenon that will be reversed or suggestive of a permanent change in the risk premium. The one-month forward return, about halfway between the 60- to 13- month historical mean and the most recent 12-month mean, seems to indicate influences from both the long-run risk premia and short-run fluctuations. When we look across individual replicating components in Figure 7, we see something more interesting: a one-to-one relationship between historical risk premium and the mean one-month forward return. The slope of the line in the figure is roughly one, yet the y-intercept is positive at 0.01%. There appears to be a constant 0.01% additional risk premium attached to each of the replicating component. Since the correlation between the long-run premia and short-term 12-month return across replicating components is about 96%, the aforementioned result suggests that about 0.41% of the momentum return can be traced directly to risk premia and 0.30% (a constant 0.01% per factor) to the influence of recent deviations.

Figure 7: Factor Risk Premia vs. Future Return



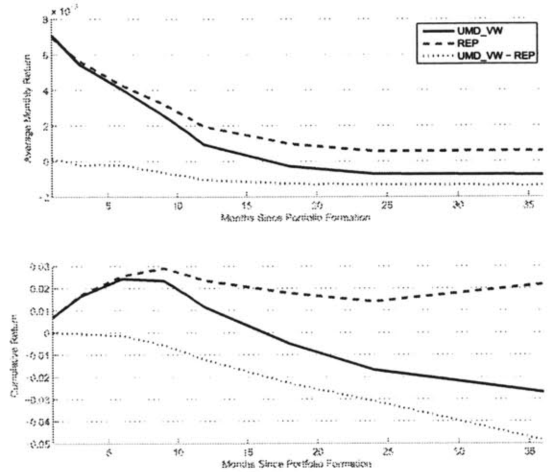
Each point represents a replicating component in the benchmark replicating portfolio. The x-axis is the long-run risk premium of the replicating component and the y-axis its average return one month after portfolio formation

4.3 Diminishing Returns

In addition to the first-month after portfolio formation, we can look further into the future to see the trajectory that momentum takes. It is well known that momentum profit declines quickly over time and even reverses slightly. The replicating portfolio matches the momentum portfolio in terms of the first-month-forward returns, but it also discards a large set of factors at the tail end. These residual components appear to be nothing more than white noise in the first month, but their subsequent performances are not trivial. Figure 8 plots the average monthly returns and cumulative returns of the momentum portfolio and the benchmark replicating portfolio over longer holding periods. The two portfolios diverge almost immediately after the first month, and the difference becomes more prominent as time goes on. The replicating portfolio consistently outperforms the momentum portfolio such that the residual portfolio consistently achieves a negative return. The superiority of the replicating portfolio in terms of the Sharpe ratio is even more spectacular, as seen in Table 6. By removing factors that do not contribute to the mean return, the replicating portfolio achieves a much higher rate of efficiency, often more than 50% higher than momentum, meaning that it achieves the same level of return with only 2/3 of the return volatility. In terms higher moments, the replicating portfolio is less skewed and less kurtotic than the momentum portfolio.

Inside the replicating portfolio, the individual component's performance over time also relates to its historical risk premium. Table 7 displays the predictability of historical risk premia on future returns over a number of periods. Starting at about the historical level at

Figure 8: Momentum and the Replicating Portfolio over Time



The average monthly and cumulative returns of the equally-weighted momentum portfolio (UMD_VW), the benchmark replicating portfolio (REP) and the residual (UMD_VW - REP) over a 36-month period after portfolio formation.

Month 1, risk factor returns decline uniformly over time. The slope coefficient decreases to less than 10% in two years but remains positive and highly significant. Across all factors included in the replicating portfolio, their historical risk premia have long-lasting impacts on their future returns. Unlike the momentum portfolio that experiences long periods of negative returns starting at Month 6, the replicating factors experience only slight amount of reversal and followed by leveling off.

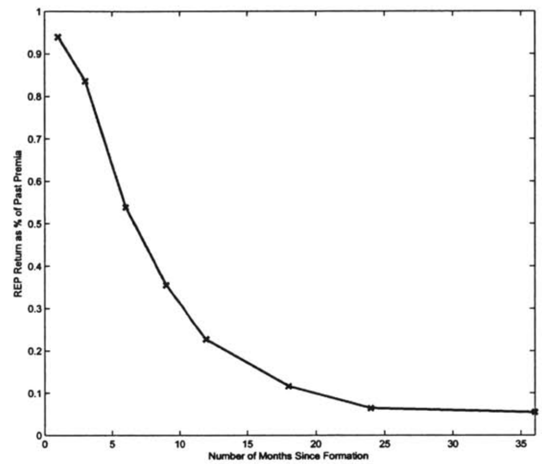
The difference in the long-term prognosis between the replicating factors and the residual factors is interesting. In the past, replicating factors looked more like systematic factors with significant observed risk premia while the residual factors look more like idiosyncratic risks. Over a long period of time, however, the systematic factors gradually fade into obscurity while the idiosyncratic risks start carrying a significant (negative) risk premium. The dynamic nature of the factor space seems to suggest that it is constantly in flux: systematic factors emerge only to disappear in the long run, replaced by new factors.

5 Additional Analysis

5.1 Durability of Factors

A crucial mechanism of the momentum portfolio is its ever-changing loadings on the entire set of risk factors in the market. The challenge, as seen in the replicating exercises above, is that momentum cannot be represented with one or a few factors. The strategy,

Figure 9: Return of the Replicating Portfolio over Time as Percentage of Past Risk Premia

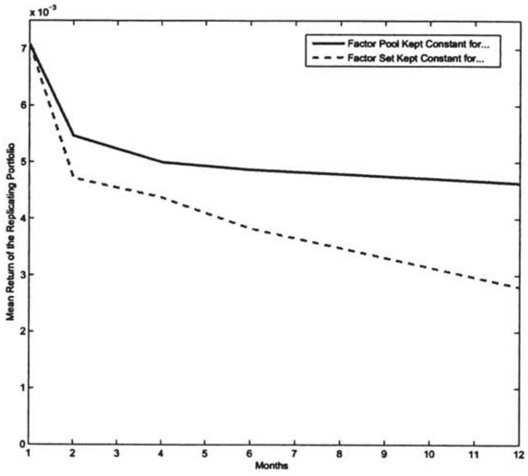


Slope coefficient estimate of the regression of the returns of the replicating components on their risk premium proxies over a number of holding periods. Each point is the average realized return of factors as a percentage of its supposed risk premium n months since formation. All estimates are statistically significant at 1%.

by definition, always casts a wide net across the entire spectrum of risk factors and collects returns from many sources at once. As I have shown, it is possible to tell which factors will more likely to be important in the future, so momentum can indeed be represented by a small, select set of factors, chosen out of a larger pool each period. However, the pool of factors also changes every period. Well known risk factors such as the Fama-French factors are not rebalanced every period. The size and value portfolios, for example, are only rebalanced once every year. The question arises as to whether momentum can potentially be replicated sufficiently well by a fixed set of factors that are only occasionally rebalanced.

Two conditions can be imposed on the factor set: the less restrictive version fixes the pool of available factors for a number of periods; the more restrictive version fixes the final set of selected factors for a number of periods. In the former case, a different set of factors can be chosen each period, leaving more flexibility. The first panel of Table 8 shows the statistics for the replicating portfolio when the factor pool is constructed once every two, four, six and 12 months. The loss of fidelity is about 35% when the factor pool remains stale for 12 months just like the Fama-French factors. A larger loss occurs when the final set of factors used is also fixed for 12 months. In this case, the replicating portfolio achieves only 40% of the momentum return. Still, this return of 0.28% is far superior to that of the replicating portfolio based on the Fama-French factors. There is still a possibility that a fixed set of "durable" factors can explain more of the momentum return, up to a limit of less than a half.

Figure 10: Replicating Portfolio with Infrequently Rebalanced Factors

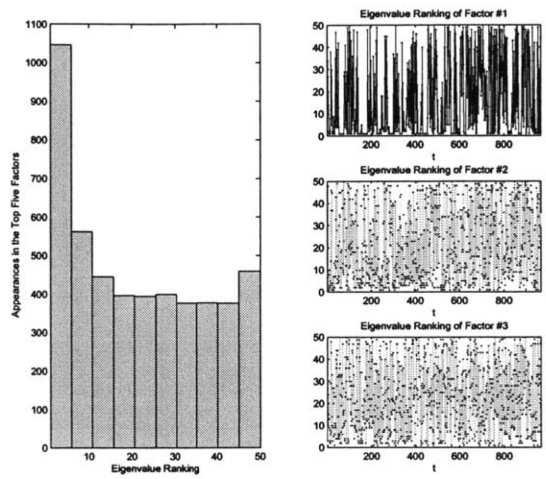


Mean return of the replicating portfolios that use factors rebalanced once every 2, 4, 6 and 12 months. The blue-solid line represents the scenario where the larger pool of factors is rebalanced infrequently but in each month a different subset can be chosen; the green-dotted line represents the scenario where the final selection of factors is rebalanced/reselected infrequently. The latter condition is more restrictive.

The reason that a small set of long-lasting factors is incapable of explaining momentum is that the momentum portfolio rapidly shifts its emphasis on factors. The weights that momentum places on each factor depend on the relative performance of that factor in the recent months. Given the sheer number of factors and the volatility of factor returns, the weights can shift drastically in just months. As the three time-series plots in Figure 11 show, there is virtually no pattern to which factors momentum selects over time, and it is not possible to predict the factor selection in the future. From the histogram, we can see that almost all factors have the same likelihood of being important to momentum, except the first factor, market, which as can be seen in the time-series plots is selected more often than others.

One may argue that the extracted factors are not the actual risk factors and that a permanent risk factor can exist and explain momentum: it just appears as different principal components at different times. However, since the extracted factors are orthogonal, the rapid shifting of factors seen in Figure 11 would imply that such a risk factor needs to be radically different over any short period of time. That factor would then be transient and momentum-like and very different from the permanent systematic risk factors like the Fama-French factors that we usually have in mind. Therefore, it is much more plausible that momentum is merely a dynamic portfolio that loads on a wide array of factors, not a few permanent ones. In that sense, we may still consider momentum as a distinct source of risks, a quasi-risk

Figure 11: The Rapidly Shifting Factor Set



On the left is the histogram of the eigenvalue rankings of the top five factors in the benchmark replicating portfolio. The vertical axis represents the number of times factors of certain rankings have been selected as the top five factors. On the right are time-series plots of the eigenvalue rankings of the top three factors chosen by the benchmark replicating portfolio.

factor. It is true that momentum is not a standalone source of risk distinguishable from all others out there, because it is merely a dynamic portfolio of existing risk factors; however, it cannot be represented as a portfolio, static or dynamic, of the Fama-French factors or a few stable and permanent factors. Rather, it summarizes different aspects of the large and changing factor space. Short of describing all of the systematic risks, which is probably an impossible task, momentum cannot be eliminated as an extraneous factor.

5.2 Counterproductive Factors

Up until this point, we have only considered factors that contribute most to the momentum portfolio. Since momentum loads on a wide array of factors, it also makes "mistakes" occasionally, taking the opposite position of what it should. This occurs when a factor has a positive (negative) risk premium but momentum takes a negative (positive) position in that. We can identify such factors by sorting the factor pool in ascending order of expected returns, which is the opposite of the replication exercise above. As illustrated in Table 9, the reverse order does yield a "counterproductive" portfolio (COUNTER) that has a expected return of -0.14% per month and yet is 44.2% correlated with momentum. When it is subtracted from the momentum portfolio, momentum's Sharpe ratio increases by over a third to 0.19. This result presents a simple method to enhance momentum profit with a complementary portfolio.

5.3 Residual Momentum

In Figure 8, we can see that the residual portfolio, defined as equally-weighted momentum minus the benchmark replicating portfolio, has almost zero return in the first month after formation, then negative and declining returns over time. Another way of looking at the residuals is to form a momentum-like portfolio based on the residual returns. The residual returns for an individual stock are returns not explained by factors used in the benchmark replicating portfolio. As expected, the residual momentum has an insignificant return. This conclusion is not surprisingly different from Blitz, et. al. (2011), who found significant residual momentum relatively to the FF3 model. Since the FF3 factors cannot adequately explain momentum, the residual portion has some momentum leftover. However, my method explains away all of the momentum profit, leaving no residuals. As seen in Table 10, the residual momentum portfolio has negative but insignificant returns not only in the first month but in all 24 months following portfolio formation.

5.4 Other Momentum Variants

I have focused on equally-weighted momentum for convenience and because the momentum phenomenon is the most difficult to explain among small stocks. Since the replicating method works on equally-weighted momentum, it should work on other momentum variants as well. Replication of two variants of particular interest, shown in Table 11, confirms this conjecture. The first panel refers to the scenario in which the momentum portfolio takes a long position in the top 10% winners and short position in the bottom 10% losers, as opposed to top 30% and bottom 30% for the regular momentum. Taking a smaller set of more extreme winners and losers yields a higher profit, but it is easily matched by the replicating portfolio. The replicating portfolio also generates a much higher Sharpe ratio due to the fact that the 10% momentum is noisier as it averages over fewer stocks. This exercise confirms that the replicating procedure works over the entire range of cross-sectional returns, including the extreme ends that prove problematic for any attempts at explaining momentum.

The second panel contains results for large-stock momentum. The setup is a little different here because the universe of stocks on which principal component analysis is performed is limited to large stocks (capitalization higher than the NYSE median) only. Using the full panel, including small stocks, to explain large-stock momentum, may amount to cheating as small-stock momentum overwhelms the large-stock counterpart. Restricting the universe of stocks to large stocks turns out not to be an issue. The replicating portfolio again matches momentum returns with ease and again achieves a higher Sharpe ratio.

5.5 Fama-French Plus

For all previous replication exercises, I have ignored the Fama-French factors, which are supposedly risk factors and so should be covered by the set of PCA-based factors. To separate the FF3 factors from the rest, I can first filter out the influence of the three factors and divide the replicating portfolio into the part for which they are responsible and the part orthogonal to it. Before the selection and replication process, I regress 60-month PCA factor returns on FF3 returns and take the residuals as the new PCA factors. Then I select factors just like before and add the final selection to the FF3 factors. The combined set is then used to replicate momentum. Not surprisingly, the contribution from the FF3 components is minor, and the addition of more PCA factors slowly bridges the gap between the returns of the replicating portfolio and those of momentum. As seen in 12, the replicating portfolios with 20 or 30 factors are almost identical to the benchmark replicating portfolio, meaning that isolating the FF3 components merely rearranges the factors. The FF3 factors contribute nothing in addition to the PCA-based factors.

5.6 The Conglomerate Factor

Since the set of factors in the replicating portfolio changes every period, it is difficult to trace any particular factor through time. We can, however, summarize the set of the factors important to momentum with the conglomerate factor. It is formed by first multiplying each factor used in the benchmark replicating portfolio by their 60-month historical mean returns. This procedure flips factors with negative risk premia so that all factors now have nonnegative risk premia. The factors are then further divided by the sum of the positive weights over individual stocks or the sum of the negative weights, whichever is greater. This normalization ensures that each factor is a zero-investment portfolio taking a long position in a stock portfolio and a short position in another. Finally, the conglomerate factor is formed as the equally-weighted portfolio of the normalized factors.

Table 13 displays the properties of the conglomerate factor versus the equally-weighted momentum portfolio. The two are 74.2% correlated but have many differences. The conglomerate factor has a higher mean return and Sharpe ratio, less skewed and kurtotic and less correlated with the Fama-French factors despite having a higher alpha. In some sense, the conglomerate is a momentum-like portfolio in that it tends to load positively (negatively) on factors with positive (negative) risk premia. However, it is more efficient than momentum and is an example of a momentum-like portfolio that outperforms momentum itself.

6 Conclusion

From the replicating exercises to the subsequent analysis, we have gained a better understanding of how momentum works. The nonparametric factor extraction method results in a replicating portfolio far superior to the one based on the Fama-French factors, achieving a higher correlation but more importantly matching momentum's seemingly impossibly high return. The existence of such a replicating portfolio is consistent with the idea that momentum is merely a dynamic portfolio of existing risk factors; it also removes the uncertainty about what the residual momentum really is. Best of all, this method does not rely on the exact factor structure being known, an almost impossible task. It does, however, impose a rather low upper limit on how much of the momentum profit a set of stable and permanent factors such as the Fama-French factors can explain.

The replicating portfolio, as it turns out, does a lot more than explaining momentum. Since it discards components that do not contribute to the mean return, it outperforms momentum in terms of the Sharpe ratio. Over a longer holding period, the replicating portfolio also achieves a higher return and Sharpe ratio compared to the momentum portfolio due to much less reversal. The residual portion generates slightly negative returns over time, and the residual momentum portfolio generates negative but insignificant returns. Analyzing the replicating components reveals that their returns going forward are directly proportional to their historical risk premia. In this sense, we may be able to attribute momentum profit as compensation for risks. At the same time, the simultaneous decline and slight reversal of all replicating components over a longer holding period indicates behavioral influences permeating through the entire factor structure.

The shortcoming of the principal component method is that the resulting factors are abstract constructs and have no inherent identities in the real economy. Ideally, we would like to know what each factor really means, but in practice there is no straightforward way to identify the complete set of systematic risks or an accepted set of criteria to evaluate it. The purpose of this paper has been to show that such a set, if we can find it at some point, *would* be able to replicate momentum in its entirety. Until then, we can still consider momentum as a quasi-risk factor that describes many of the systematic risks still unknown to us.

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Table 1: Variants of Momentum

At the end of each month, stocks are sorted into deciles by their cumulative returns in the past 12 months omitting the most recent month. The momentum portfolio is formed by taking a long position in the top three deciles and a short position in the bottom three deciles; it is then held for one month forward. Each column is a different method of weighting stocks in the long and short portions of the portfolio: VW - value-weighted; EW - equally weighted. LG/SM/HH - stocks are divided into two groups based on whether their capitalizations are larger or smaller than the NYSE median that month; a value-weighted momentum portfolio is formed for each group, LG as the large cap group and SM as the small cap group. HH = (LG+SM)/2.

Weights	EW	VW	HH	LG	SM
$\bar{r}$	0.0071***	0.0040***	0.0062***	0.0042***	0.0081***
	[0.0015]	[0.0015]	[0.0014]	[0.0015]	[0.0015]
$\bar{r}/\sigma$	0.1463	0.0851	0.1363	0.0889	0.1677
Skewness	-4.16	-2.75	-3.33	-2.68	-3.06
Kurtosis	39.93	24.81	31.53	24.13	27.5
FF3 $\hat{\alpha}$	0.0111***	0.0068***	0.0092***	0.0070***	0.0115***
	[0.0012]	[0.0013]	[0.0012]	[0.0013]	[0.0013]
$\hat{\beta}_{mkt}$	-0.21***	-0.18**	-0.21***	-0.18**	-0.23***
	[0.07]	[0.08]	[0.07]	[0.08]	[0.07]
$\hat{\beta}_{smb}$	-0.21**	0.08	0.01	0.08	-0.06
	[0.09]	[0.09]	[0.09]	[0.09]	[0.09]
$\hat{\beta}_{hml}$	-0.50***	-0.45***	-0.43***	-0.45***	-0.42***
	[0.15]	[0.14]	[0.14]	[0.14]	[0.14]
Adj. R^2	27.9%	17.7%	21.4%	17.6%	21.1%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 2: Replicating Portfolio with Time-Varying Beta

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the factor set in the past 60 months. For Panel A, the factor set is the Fama-French factors; for B and C, the factor set is the first nth orthogonal factors constructed from principal components of the 60-month historical return panel. The momentum portfolio betas are the average betas of individual stocks selected into the momentum portfolio. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. The replicating portfolio is the sum of the three components in Panel A and B. In Panel C, the replicating portfolio is the sum of the first nth PCA factors, The PCA factors are numbered in descending order of their eigenvalues.

Panel A	UMD-EW	FF3 REP	MKT REP	SMB REP	HML REP
$\bar{r}$	0.0071***	0.0010	-0.0005	0.0009	0.0006
	[0.0015]	[0.0012]	[0.0006]	[0.0006]	[0.0007]
$\bar{r}/\sigma$	0.1463	0.0292	-0.0287	0.0521	0.0332
$\rho(\cdot$, UMD-EW)		77.4%	60.5%	39%	57.4%
Panel B	**UMD-EW**	**PCA 1-3**	**PCA1**	**PCA2**	**PCA3**
$\bar{r}$	0.0071***	0.0007	-0.0007	0.0009***	0.0006***
	[0.0015]	[0.0010]	[0.001]	[0.0003]	[0.0002]
$\bar{r}/\sigma$	0.1463	0.0245	-0.0249	0.1186	0.1021
$\rho(\cdot$, UMD-EW)		80.3%	76.9%	-3.6%	38.4%
Panel C	**UMD-EW**	**PCA 1-5**	**PCA 1-10**	**PCA 1-20**	**PCA 1-30**
$\bar{r}$	0.0071***	0.0011	0.0018*	0.0033***	0.0039***
	[0.0015]	[0.0011]	[0.0011]	[0.0011]	[0.0012]
$\bar{r}/\sigma$	0.1463	0.0352	0.053	0.0887	0.0997
$\rho(\cdot$, UMD-EW)		82.2%	86.9%	89.5%	93.2%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 3: Subsample Momentum

Equally-weighted momentum portfolios are formed on random subsets of stocks at the end of each month. The first column, UMD_EW, contains the statistics of the full-sample equally-weighted momentum portfolio. Each subsequent column contains the statistics over 50 random subsamples of different sizes. The number of stocks in a subset is the maximum of $n\%$ of the total number of stocks and 200. There are 2235 stocks available on average but the number ranges from 350 in the early 1930s to 4500 by the end of 2011. The mean and standard deviation statistics are over the 50 random subsample momentum portfolios.

	UMD_EW	Subsample (% of total number of stocks)			
		5%	10%	15%	20%
Mean $\bar{r}$	0.0071	0.0071	0.0072	0.0071	0.0071
$\sigma(\bar{r})$		(0.0006)	(0.0005)	(0.0006)	(0.0005)
Mean $\bar{r}/\sigma$	0.1463	0.1372	0.1397	0.1392	0.1390
$\sigma(\bar{r}/\sigma)$		(0.0147)	(0.0110)	(0.0124)	(0.0120)
Mean σ	0.0485	0.0526	0.0514	0.0507	0.0505
Mean $\rho(\cdot$, UMD_VW)		0.9211	0.9441	0.9553	0.9621
Mean skewness	-4.16	-3.33	-3.63	-3.74	-3.85
Mean kurtosis	39.93	32.48	36.53	37.63	38.96

Table 4: Subsample Momentum Replication

In each period, a principal component analysis is performed on a 60-month panel of historical returns. In the first panel below, the full sample is used; in the second panel, a random 5% subsample is used, and the results shown are typical. The Replicating portfolios are constructed from the first few principal components, as ordered in descending order by their eigenvalues. PCA 1-N means the first N factors are used.

Full Sample	UMD-EW	PCA 1-5	PCA 1-10	PCA 1-20	PCA 1-30
$\bar{r}$	0.0071***	0.0011	0.0018*	0.0033***	0.0039***
	[0.0015]	[0.0011]	[0.0011]	[0.0011]	[0.0012]
$\bar{r}/\sigma$	0.1463	0.0352	0.053	0.0887	0.0997
$\rho(\cdot$, UMD-EW)		82.2%	86.9%	89.5%	93.2%
Subsample (typical)	**UMD-EW**	**PCA 1-5**	**PCA 1-10**	**PCA 1-20**	**PCA 1-30**
$\bar{r}$	0.0071***	0.0007	0.0014	0.0032***	0.0044***
	[0.0015]	[0.0010]	[0.0011]	[0.0011]	[0.0012]
$\bar{r}/\sigma$	0.1463	0.0223	0.0394	0.087	0.1108
$\rho(\cdot$, UMD-EW)		80.6%	83.4%	85.4%	88.4%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 5: Alternative Factor Selection Schemes

In each period, a principal component analysis is performed on a 60-month panel of historical returns. A random 5% subsample is used, and the results shown are typical. The Replicating portfolios are constructed from the first few principal components according to different orderings. The first panel uses the default ordering by eigenvalues. The second and third panels order factors by the expected returns of their corresponding replicating components. In addition, the third panel uses a refinement method that merges and selects factors from multiple random subsamples. PCA 1-N means the first N factors are used.

By eigenvalues	UMD-EW	PCA 1-5	PCA 1-10	PCA 1-20	PCA 1-30
$\bar{r}$	0.0071***	0.0007	0.0014	0.0032***	0.0044***
	[0.0015]	[0.0010]	[0.0011]	[0.0011]	[0.0012]
$\bar{r}/\sigma$	0.1463	0.0223	0.0394	0.087	0.1108
$\rho(\cdot$, UMD-EW)		80.6%	83.4%	85.4%	88.4%
By expected return	**UMD-EW**	**PCA 1-5**	**PCA 1-10**	**PCA 1-20**	**PCA 1-30**
$\bar{r}$	0.0071***	0.0026***	0.0032***	0.0039***	0.0043***
	[0.0015]	[0.0009]	[0.001]	[0.0011]	[0.0012]
$\bar{r}/\sigma$	0.1463	0.0959	0.1027	0.1068	0.1137
$\rho(\cdot$, UMD-EW)		72.7%	77%	78.9%	80.6%
By expected return (refined)	**UMD-EW**	**PCA 1-5**	**PCA 1-10**	**PCA 1-20**	**PCA 1-30**
$\bar{r}$	0.0071***	0.0034***	0.0052***	0.0069***	0.0069***
	[0.0015]	[0.0012]	[0.0012]	[0.0013]	[0.0013]
$\bar{r}/\sigma$	0.1463	0.0996	0.1372	0.1722	0.1708
$\rho(\cdot$, UMD-EW)		80.0%	83.4%	84.5%	84.2%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 6: Momentum and the Benchmark Replicating Portfolio Over Time

The momentum portfolio is the equally-weighted variety. The benchmark replicating portfolio is the 30-factor portfolio with factors chosen using the refined method (PCA 1-30 in Panel C of Table 5). The portfolios are held for the specified holding periods and the average monthly returns recorded below. Over time, some stocks are delisted and removed from both the factors and the momentum portfolio, with the remaining stocks reweighted equally.

Holding Period (Month)	1		3		6		9	
Portfolio	UMD-EW	REP	UMD-EW	REP	UMD-EW	REP	UMD-EW	REP
$\bar{r}$	0.0071***	0.0069***	0.0054***	0.0056***	0.0041***	0.0043***	0.0026**	0.0032***
	[0.0015]	[0.0013]	[0.0014]	[0.0010]	[0.0008]	[0.0008]	[0.0010]	[0.0007]
$\bar{r}/\sigma$	0.1463	0.1708	0.1873	0.2629	0.2185	0.306	0.1812	0.2841
$\rho(\cdot$, UMD-EW)		84.2%		78.8%		75.2%		74.5%
Holding Period (Month)	**12**		**18**		**24**		**36**	
Portfolio	UMD-EW	REP	UMD-EW	REP	UMD-EW	REP	UMD-EW	REP
$\bar{r}$	0.0010	0.0020***	-0.0003	0.0010**	-0.0007	0.0006	-0.0007*	0.0006*
	[0.0006]	[0.0006]	[0.0007]	[0.0005]	[0.0005]	[0.0005]	[0.0004]	[0.0003]
$\bar{r}/\sigma$	0.0785	0.1987	-0.0291	0.1319	-0.089	0.0861	-0.1416	0.1197
$\rho(\cdot$, UMD-EW)		73.8%		71.4%		74.3%		69.1%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 7: Predictability of Risk Premia on Future Returns

Cross-factor regression of the mean returns of replicating components on their estimated risk premia according to Eq. 2 over different holding periods.

Holding Period (Month)	1	3	6	9	12	18	24	36
Intercept	0.0001***	0.0001***	0.0001***	0.0001***	0.0000***	0.0000***	0.0000***	0.0000***
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
Slope	0.9399***	0.8362***	0.5394***	0.355***	0.2277***	0.1165***	0.0644***	0.0550***
	[0.0979]	[0.0968]	[0.0685]	[0.0926]	[0.0368]	[0.0271]	[0.0296]	[0.0168]
Adj. R^2	69.7%	76.4%	71.8%	60.4%	54.1%	41.0%	12.1%	12.5%

[ ]: Standard errors; */**/***: statistically significant at 10/5/1%

Table 8: Replication Based on Restricted Factor Pool and Set

The benchmark replicating portfolios are formed using the refined method in Table 5. Fixed factor pool (Panel A) means that the larger pool of factors from which a smaller subset is selected is not recomputed every month but rather every n months as specified in the subheading. Fixed factor set (Panel B) means that the final subset of factors is fixed for a number of periods and only refreshed once every n months.

Fixed Factor Pool	Benchmark	Allow Rebalancing Every...			
		2 Months	4 Months	6 Months	12 Months
$\bar{r}$	0.0069***	0.0055***	0.005***	0.0049***	0.0046***
	[0.0013]	[0.0011]	[0.0011]	[0.0011]	[0.001]
$\bar{r}/\sigma$	0.1708	0.1537	0.1428	0.1426	0.1397
$\rho(\cdot$, UMD-EW$)$	84.2%	75.9%	77.8%	71.5%	64.9%
Fixed Factor Set	**Benchmark**	**Allow Rebalancing Every...**			
		2 Months	4 Months	6 Months	12 Months
$\bar{r}$	0.0069***	0.0047***	0.0044***	0.0038***	0.0028**
	[0.0013]	[0.0013]	[0.0013]	[0.0013]	[0.0012]
$\bar{r}/\sigma$	0.1708	0.1238	0.1156	0.0995	0.0752
$\rho(\cdot$, UMD-EW$)$	84.2%	78.9%	78.3%	78.3%	73.2%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 9: Counterproductive Factors

At the end of each month, a large pool of factors is first generated from principal component analysis and then sorted in ascending order of the replicating components' expected returns. The first component is chosen and named the counterproductive portfolio. UMD-EW is the equally-weighted momentum and REP the benchmark replicating portfolio.

Portfolio	UMD-EW	REP	COUNTER	UMD-EW - COUNTER	REP - COUNTER
$\bar{r}$	0.0071***	0.0069***	-0.0014**	0.0085***	0.0083***
	[0.0015]	[0.0011]	[0.0006]	[0.0014]	[0.0013]
$\bar{r}/\sigma$	0.1463	0.1924	-0.0713	0.1946	0.2114
$\rho(\cdot$, UMD-EW$)$	100%	77.3%	44.2%	91.7%	49.0%
Skewness	-4.16	-2.82	-0.99	-5.19	-2.16
Kurtosis	42.93	31.84	31.05	59.83	25.48

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 10: Residual Momentum Portfolio

In each period, individual stock returns are regressed against the factors used in the benchmark replicating portfolio using the most recent 60 monthly return data. The intercept estimate, $\hat{\alpha}$, is then used to sort stocks into deciles. The residual momentum portfolio is formed by taking long positions in the top 30% of stocks and short positions in the bottom 30%. The stocks in each half of the portfolio are equally-weighted. The portfolio is then held for a number of periods specified below.

Residual Momentum	1 Month	3 Months	6 Months	12 Months	24 Months
$\bar{r}$	-0.0041	-0.0027	-0.0024	-0.0017	-0.0018
	[0.0025]	[0.0024]	[0.0022]	[0.0018]	[0.0014]
$\bar{r}/\sigma$	-0.0555	-0.0561	-0.0684	-0.0688	-0.1023
Skewness	0.78	1.29	0.32	0.27	0.30
Kurtosis	13.09	14.35	6.54	7.28	4.39

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 11: Replication of Other Momentum Variants

Panel A (Decile EW) contains statistics for the top 10% minus bottom 10% (as opposed to the canonical 30%) equally-weighted momentum portfolio and the corresponding replicating portfolios. Panel B (LG) contains statistics for the value-weighted momentum portfolio formed on large stocks and the corresponding replicating portfolios. Large stocks are stocks with capitalizations greater than the NYSE median. The replicating strategy is the same as the one used on the benchmark replicating portfolio throughout this paper.

Decile EW	UMD-EWD	PCA 1-5	PCA 1-10	PCA 1-20	PCA 1-30
$\bar{r}$	0.0116***	0.0051***	0.0082***	0.0114***	0.0117***
	[0.0022]	[0.0015]	[0.0016]	[0.0018]	[0.0018]
$\bar{r}/\sigma$	0.1627	0.1068	0.1517	0.1967	0.2016
$\rho(\cdot$, UMD-EWD)		72.8%	76.4%	78.5%	78.6%
Large Stocks	**UMD-LG**	**PCA 1-5**	**PCA 1-10**	**PCA 1-20**	**PCA 1-30**
$\bar{r}$	0.0042***	0.0026**	0.0038**	0.0050***	0.0049***
	[0.0015]	[0.0011]	[0.0012]	[0.0013]	[0.0013]
$\bar{r}/\sigma$	0.0889	0.0764	0.1009	0.1252	0.1238
$\rho(\cdot$, UMD-LG)		83.9%	86.0%	86.9%	86.7%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 12: Incremental Power of the Replicating Portfolio Relative to the FF Factors

In each period, PCA-based factors are regressed against the Fama-French three factors in a 60-month time-series regression. The residuals are sorted by their expected contribution to the replicating portfolio and the top factors (number specified below in the title row) are chosen. They are then added to the Fama-French factors to form the factor set used in the replicating portfolio. For example, FF+7 means seven additional factors are added so that 10 are used to replicate momentum.

Portfolio	UMD-EW	FF REP	FF+1 REP	FF+2 REP	FF+7 REP	FF+17 REP	FF+27 REP
$\bar{r}$	0.0071***	0.0010	0.0023*	0.0028**	0.0047***	0.0071***	0.0070***
	[0.0015]	[0.0012]	[0.0012]	[0.0012]	[0.0013]	[0.0014]	[0.0014]
$\bar{r}/\sigma$	0.1463	0.0292	0.0621	0.0750	0.1185	0.1732	0.1643
$\rho(\cdot$, UMD-EW)	100%	77.4%	77%	77.9%	79.8%	80.3%	80%
Skewness	-4.16	-2.47	-2.20	-2.56	-3.19	-2.95	-2.87
Kurtosis	42.93	32.51	27.97	31.74	40.61	35.75	34.06

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 13: The Conglomerate Factor

The conglomerate factor is formed by multiplying the factors used in the benchmark replicating portfolio by the signs of their mean 60-month historical returns and combining the results in an equally-weighted portfolio.

	$\bar{r}$	se	$\bar{r}/\sigma$	Skewness	Kurtosis	$\rho(\cdot$, UMD-EW)
UMD-EW	0.0071***	[0.0015]	0.1463	-4.16	42.93	
Congl. Factor	0.0146***	[0.0023]	0.2068	-2.30	19.82	74.2%

	FF3 $\hat{\alpha}$	se	$\hat{\beta}_{mkt}$	se	$\hat{\beta}_{smb}$	se	$\hat{\beta}_{hml}$	se	Adj. R^2
UMD-EW	0.0111***	[0.0012]	-0.21**	[0.07]	-0.21**	[0.09]	-0.50***	[0.15]	27.9%
Congl. Factor	0.0170***	[0.0023]	-0.01	[0.13]	0.13	[0.12]	-0.66***	[0.19]	11.3%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Chapter 3

Fundamental Momentum

1 Introduction

In this paper, we study cross-sectional patterns in stock returns associated with various accounting ratios. Accounting ratios of interest in this paper are measures of book returns, namely return on assets and return on equity, turnover ratios in accounts receivable and payable, and measures of profit margins, in terms of both gross and net profits. Our main results can be summarized in three parts. First, we find that long-short portfolios formed by sorting on these accounting ratios as well as their annual changes have large, positive average returns as well as significant α with respect to both Fama-French three-factor and Carhart four-factor models. We then ask the question of which of these two sets of variables, levels of accounting ratios or their changes, are more important in the cross-section of stock returns. We find significant evidence that changes in the accounting ratios are stronger determinants of stock returns than their levels. Second, motivated by the first finding, we document significant abnormal returns of portfolios formed by sorting on the changes in the accounting ratios through an extensive set of regressions with widely used risk factors. The two main portfolios in our study, long-short portfolios formed by sorting on changes in return on asssets and return on equity, exhibit α of 0.5% to 0.75% per month. Third, we find that the abnormal returns of the momentum factor can be explained by inclusions of the factors formed by sorting on changes in the accounting ratios. Chordia and Shivakumar (2006) report a similar finding and conclude from this regression evidence that price momentum is fully explained and subsumed by "earnings momentum", where they use the SUE portfolio[1] as an explanatory factor. We examine their finding by running double-sorts on past returns and change in return on assets or return on equity, and we find that there are large return spreads across past returns even after controlling for changes in return on assets or return on equity. Therefore, we conclude that the price momentum effect is not fully explained by earnings momentum, even though regression results of price momentum on earnings momentum might suggest otherwise.

Section 2 describes our data construction. Section 3 documents return performances

[1] SUE stands for standarized abnormal earnings and is formed by sorting on change in earnings standarized by standard deviation of quarterly changes. SUE portfolio is similar to our construction of factors based on changes in return on assets and return on equity.

of factors formed on accounting ratios and their changes, and then answers the question of whether levels or changes in the accounting variables are more important in the cross-section of stock returns. Section 4 continues the previous section by reporting significant and robust abnormal returns associated with factors based on changes in the accounting ratios. Section 5 first reports regression results of price momentum factor UMD on our accounting factors, and subsequently examines whether price momentum is fully captured by earnings momentum. Section 6 concludes.

2 Data

The main data sources are Compustat Annual and Quarterly Data Files for accounting variables and Center for Research in Security Prices (CRSP) monthly data for stock returns. Data covers the period from January 1975 to December 2010. The starting date is restricted by the availability of reporting date of quarterly earnings in Compustat (item RDQ) as well as poor data quality and limited number of data points in the earlier part of the Compustat database. We exclude financial firms (SIC codes 6000-6999), firms with negative book equity, and firms with share prices less than $10. The restriction on share prices is a conservative choice and is intended to alleviate the concerns that small companies are prone to poor accounting data and that abnormal returns may primarily be driven by illiquid stocks.

Accounting variables of interest are return on assets, return on equity, turnover ratios of accounts receivable and payable, and gross and net profit margins (often referred to as "the accounting ratios" in the rest of the paper). We denote them by lower case letters roa, roe, $arturn$, $apturn$, $gmarg$, and $nmarg$, respectively. Return on assets and return on equity measure rates of returns to entire stakeholders and equity shareholders, respectively. Return on assets, roa, is defined as income before extraordinary items (Compustat quarterly item IBQ) divided by one-quarter-lagged total assets (Compustat quarterly item ATQ). Similarly, return on equity, roe, is defined as income before extraordinary iterms (Compustat quarterly item IBQ) divided by one-quarter-lagged book equity (Compustat quarterly item CEQQ). Turnover ratios in accounts receivable and payable measure the number of times that balances of accounts receivable and payable are turned over, respectively. Accounts receivable turnover, $arturn$, is defined as revenue (Compustat quarterly item REVTQ) divided by average accounts receivable (Compustat quarterly item RECTQ) over the current and previous quarters. Similarly, accounts payable turnover, *apturn*, is defined as cost of goods sold (Compustat quarterly item COGSQ) divided by average accounts payable (Compustat quarterly item APQ) over the current and previous quarters. Finally, gross and net margins are measures of profitability. Gross margin, *gmarg*, is defined as gross profit (Compustat quarterly item REVTQ minus COGSQ) divided by revenue (Compustat quarterly item REVTQ). Similarly, net margin, *nmarg*, is defined as operating profit (Compustat quarterly item OIBDPQ) divided by revenue (Compustat quarterly item REVTQ).

In months in which quarterly earnings are reported, we define these accounting ratios using the newly updated data. In other months, we define them based on the most recent accounting data, and they are allowed to be stale for a maximum of five months. Finally, we define changes in these accounting variables as a year-over-year change, current value minus the value twelve months ago. The reason for defining annual, rather than quarterly, changes is because quarterly accounting data displays significant seasonality. We note changes in the accounting ratios by Δ preceeding the accounting ratio, so for example, Δroa denotes the year-over-year change in roa of a firm. We will often refer to these accounting ratios and their changes as "the levels" and "the changes", respectively. Table 1 summarizes the data definitions.

In the empirical analyses, we will form a number of long-short portfolios by sorting on various sorting variables. For each sorting variable, we take the following steps in constructing the long-short portfolio, in the spirit of Fama and French (1993). At the beginning of each month t, we use the median market equity of NYSE firms and split all stocks into two groups, small and big. Independently, we define breakpoints for the sorting variable at 30th and 70th percentiles using all firms, and assign firms to three groups low, middle, and high. We then form six portfolios by taking the intersection and compute their value-weighted returns for month t. Finally, return on the long-short portfolio is defined as the simple average of return on the high portfolio minus return on the low portfolio, across the two size groups. As a matter of notation, we denote the time series of factor returns on the long-short portfolio by upper case letters corresponding to the sorting variable in lower case letters. For example, ROA and ΔROA are factor returns on portfolios formed by sorting on roa and Δroa, respectively. We will often refer to portfolios formed by sorting on the levels and the changes in the accounting ratios as "factors formed by levels" and "factors formed on changes", respectively. In addition, we denote by UMD, the standard momentum factor constructed by sorting on cumulative returns from month $t-12$ to month $t-1$ following Jegadeesh and Titman (1993).

Table 2 reports summary statistics of accounting ratios and their changes. We make three main observations. First, while changes in the accounting ratios are close to zero on average, their standard deviations are quite large. Second, correlations between levels and changes of the accounting ratios are generally large and positive. Combined with the first observation, this suggests that a substantial fraction of cross-sectional dispersion in levels of these accounting variables is driven by recent changes in them. Third, we note that some of these accounting ratios have significant correlations among themselves, both in terms of levels and changes. Most notably, *roa* is highly correlated with *roe*, and to a lesser extent, *gmarg* and *nmarg*. This is mechanical because the numerators of these ratios are closely related to one another[2]. This pattern of correlations mostly holds in terms of changes as well. The two turnover ratios, however, show much smaller correlations with other ratios, both in terms of levels and changes.

3 Changes or Levels?

There is a large number of papers documenting abnormal returns associated with both levels and changes in some of the accounting variables that we are currently studying. First, there is an extensive literature that documents abnormal returns of portfolios formed by sorting on *roa* and *roe* (e.g., Fama and French (2006), Chen, Novy-Marx, and Zhang (2010), and Hou, Xue, and Zhang (2012) to name a few). Also, in a recent paper, Novy-Marx (2012) studies the profit margin premium, and finds that portfolios sorted by profit margins produce a large spread in returns. Moreover, originating in the accounting literature, many authors have documented abnormal returns associated with changes in earnings. In particular, Ball and Brown (1968), Chan, Jegadeesh, and Lakonishok (1996), and Chordia and Shivakumar (2006) report significant abnormal returns of the SUE portfolio which closely resembles our ΔROA and ΔROE portfolios.

Given these previous findings, we begin our discussion by studying the average returns and portfolio α of factors formed on levels and changes in the accounting variables. The top panel of Table 3 reports return performances of factors formed by sorting on levels of the accounting ratios. Sorting stocks by *roa*, *roe*, and *arturn* produce large return spreads of 0.60%, 0.65%, and 0.38% per month, significant at 1% level. Moreover, factors ROA, ROE, and $ARTURN$ have large and highly significant α with respect to both Fama-French three-factor and Carhart four-factor models. Moreover, although sorting stocks by profit margins *gmarg* and *nmarg* do not produce large return spreads, factors $GMARG$ and $NMARG$ have highly significant positive α against the two benchmark models. However, $APTURN$ does not show significant return spread or α with respect to the two models.

The bottom panel of Table 3 reports summary statistics of factors formed by sorting on changes in the accounting ratios. We can see that return spreads generated by sorting

[2]Gross profits minus operating expenses equals operating profits. Operating profits minus taxes and interests equals income before extraordinary iterms.

on changes in the accounting raios are greater than or around similar magnitudes as those generated by sorting on levels of the accounting ratios. Moreover, all of these factors formed on the changes have highly significant and positive average returns and α with respect to Fama-French three-factor and Carhart four-factor models. Another notable observation is that R^2 of factor regressions in the four-factor model are significantly higher than in the three-factor model, suggesting that these factors formed on the changes are highly correlated with the momentum factor UMD[3]. On the other hand, factors formed by sorting the levels do not show a large increase R^2 when UMD is included in the regressions, with $ARTURN$, $APTURN$, $GMARG$, and $NMARG$ showing negligible increases. Finally, we observe significant correlations between factors formed on levels and on changes for the ratios roa, roe, and $gmarg$. As we will discuss shortly, a large part of this correlation is mechanical. As we have shown in Table 1, the levels and changes in these accounting ratios have correlations between 40 and 50 percent. Therefore, it is not surpring that factors formed on significantly correlated sorting variables show large return correlations.

In summary, we can see, for the most part, that the factors formed by sorting on levels and changes in the accounting ratios both have large and positive average returns α with respect to both Fama-French three-factor and Carhart four-factor models. These results are in broad agreement with the aforementioned papers that have documented large return spreads and abnormal returns associated with factors formed on both levels and changes in the accounting ratios. Given this observation, we examine the question of whether levels or changes are more important in the cross-section of stock returns. We first note that we cannot directly compare factors based on levels and changes in answering this question. The reason is that the current level of an accounting ratio could be decomposed into its lagged value a year ago and its year-over-year change. Therefore, if it were the case that changes in the accounting ratios drive spreads in returns, factors formed on current levels could mechanically inherit these return spreads. In order to control for this mechanical component, we examine effects of one-year-lagged levels and year-over-year changes in these accounting ratios in the cross-section of stock returns[4]. We tackle this problem in two directions, first by using Fama-MacBeth (1973) procedure and second by implementing double-sorts.

In the first approach, we use the methodology of Fama and MacBeth (1973). For each accounting ratio, we determine the relative strengths of level and change in this variable as follows: In each month t, we regress the cross-section of stock returns on both levels and changes in the accounting ratio, in addition to log market capitalizations, log book-to-market

[3]We provide direct evidence for this in Table 10.

[4]If the accounting ratios followed a random walk, lagged levels and changes would be uncorrelated.

ratios, and cumulative past returns from month $t-12$ to month $t-1$, denoted by *umd*. In other words, we are interested in the predictive powers of level and change in the particular accounting ratio, controlling for characteristics that are well-known to be related to expected returns. Moreover, in each month t, we normalize both levels and changes by their respective cross-sectional standard deviations in month t prior to running the regression, so that their slope estimates could directly be compared to each other. In the final step, we aggregate the slope estimates into a time series and determine their average estimates and statistical significance. Table 4 reports the results.

For *roa* and *roe*, we see very strong results that changes in these accounting ratios are much stronger than levels in explaining the cross-section of stock returns, with magnitudes 15 to 30 times greater[5]. Moreover, though both levels and changes are significant, levels show much higher statistical significance. These results are true regardless of whether past returns *umd* are controlled for. The results are similar for the margin measures, *gmarg* and *nmarg*. Lagged levels are hardly significant, yet changes are strongly significant with magnitudes of slope estimates around 10 times greater. For the two turnover ratios *arturn* and *apturn*, the magnitudes of slope coefficients for levels and changes are approximately equal, but the result is much more statistically significant for changes than levels. In summary, the cross-sectional regression results are all highly significant for changes in the accouting ratios, with magnitudes often exceeding those of levels by more than 10 times. These results clearly suggest that changes in the accounting ratios are stronger determinants in the cross-section of returns.

In the second approach, we make use of the double-sorting methods to see if recent changes in the accounting ratios produce return spreads, after controlling for the current levels. The reason for this specification is as follows: From Table 4, we see that both levels and changes are significant in the cross-section of returns, but changes being stronger of the two. We are interested in a relative comparison of the two, noting that both are positively correlated with average returns. We achieve this horse race between levels and changes via double-sorting. We first sort stocks into five quintiles by the current level of an accounting ratio. Within each quintile, we are effectively controlling for the current level, and we then sort stocks in this quintile into further quintiles based on changes. In this manner, we produce 25 portfolios, (dependently) double-sorted first by current level and then by change. Note that the second step of sorting by changes also corresponds to sorting on lagged levels in the reverse order, as we are already controlling for current levels. Therefore, increasing patterns of returns across the change quintiles would suggest that recent changes are stronger than

[5]We can directly compare their magnitudes because we have normalized both levels and changes prior to the regression.

past levels in the cross-section of stock returns.

Tables 5, 6, and 7 report our results from this double-sorting exercise. Looking across rows, we see that sorting by changes produce mostly positive return spreads, after having controlled for current levels. Moreover, around half of these results are statistically significant. Again, these results serve as solid corroborating evidence to the results from the Fama-MacBeth regressions. Therefore, we conclude that changes in the accounting ratios are more significant determinants of stock returns than levels.

4 Fundamental Momentum

The previous section has illustrated that factors formed by sorting on changes in the accounting variables produce large and positive average returns and α, and that they are stronger represent strong phenomena than factors formed by sorting on levels of the accounting ratios. In this section, we take this finding a step further by documenting various properties of the factors formed on changes in accounting ratios.

We observed in Table 3 that these factors have highly significant and positive average returns and α with respect to both Fama-French three-factor and Carhart four-factor models. In Table 8, we elaborate on this finding by reporting the full time-series regression statistics of these factors. There are three main observations. First, all of the factors formed by sorting on changes in the accounting ratios produce large, positive, and highly significant α with respect to both Fama-French and Carhart models. In particular, magnitudes of α are quite large, ranging from 0.5% to 0.75% per month for ΔROA and ΔROE. With the exception of $\Delta APTURN$, other factors also exhibit large α ranging from 0.25% to 0.5% per month. Second, we can see that all of these factors have significant factor loadings on the momentum factor UMD, and R^2 of time-series regressions increase dramatically once UMD is included. This close relationship between factors formed by sorting on changes in the accounting ratios and price momentum is a strong and important one and we will investigate this in more detail in the following section. Finally, we note that these factors do not exhibit significant factor loadings on the Fama-French factors, especially after taking into account the momentum factor UMD. Though often statistically insignificant, these factors in general have negative loadings on the HML factor.

In Table 9, we check robustness of our results on the factors formed by sorting on changes in the accounting ratios. We regress our factor returns on Fama-French factors as well as the momentum factor UMD, Pastor-Stambaugh (2003) liquidity factor PS, credit spread DEF[6], and NBER Recession Indicator REC. We can see that the factors continue to have

[6]Motivated by Chen, Roll, and Ross (1986)

large, significant α after controlling for the additional factors. They also consistently have positive loadings on the momentum factor. One noteworthy observation is that many of the factors formed on changes in the accounting ratios exhibit negative, and often statistically significant, loadings on credit spread, DEF.

Table 10 reports pairwise correlations of these factors formed on changes in the accounting ratios and the momentum factor UMD. Similar to our results in Table 1, factors ΔROA and ΔROE are highly correlated with each other, while these two are also significantly and positively correlated with $\Delta GMARG$ and $\Delta NMARG$. $\Delta ARTURN$ and $\Delta APTURN$ show weaker correlations with other factors, but the correlations are all positive. Finally, we note that all factors except for $\Delta ARTURN$ and $\Delta APTURN$ show high correlations with UMD, with magnitudes above 45%.

Based on these results, we refer to our results on the factors formed by sorting on changes in the accounting variables as "fundamental momentum". First, "fundamental" refers to the fact that these factors are formed by sorting on accounting data, rather than stock market data. Second, "momentum" refers to the cross-correlation between changes in the accounting variables and subsequent changes in prices. In a similar spirit, Chordia and Shivakumar (2006) refer to their results as "earnings momentum".

5 Relationship between Price and Earnings Momentum

In this section, we first examine whether the momentum profits can be explained by the factors formed on changes. Table 11 reports regressions of the momentum factor UMD on the factors formed on levels and changes in the accounting ratios in addition to the Fama-French factors. There are two main observations. First, UMD has highly positive and significant factor loadings on factors formed on changes. Moreover, ΔROA, ΔROE, $\Delta GMARG$, and $\Delta NMARG$ eliminate statistical significance of momentum α, with the first two reducing the magnitudes of momentum α to 0.01% per month. Second, factors formed on levels, on contrast, cannot explain the momentum profits. Though controlling for ROA and ROE marginally reduces statistical significance of momentum α, but momentum remains largely profitable.

Chordia and Shivakumar (2006) makes a similar finding where they use the SUE portfolio to explain the momentum profits. Based on this evidence, they conclude "our results support the argument that price momentum is primarily subsumed by the systematic component of earnings momentum and that price momentum is merely a manifestation of the earnings momentum." If this were true, this would be a very significant statement. Price momentum still remains elusive despite much efforts, and if earnings momentum captured the essential elements of price momentum, it would greatly help our quest for price momentum. Alas, we find that price momentum is neither fully explained nor subsumed by earnings momentum.

We make this point by implementing double-sorts. If it were true that price momentum is fully subsumed by earnings momentum, then sorting by past returns *umd* should not produce large return spreads, after controlling for changes in earnings by either Δroa or Δroe. Tables 12 and 13 report average returns and α against Fama-French three-factors of 25 double-sorted portfolios for Δroa and Δroe, respectively. In these tables, we first sort by Δroa or Δroe, and then sort by *umd*. In Table 12, we see that return spreads across quintiles sorted by *umd* are all positive and moreover most of the Δroa quintiles show significantly positive α on long-short portfolios sorted by *umd* within the Δroa quintile. A similar pattern occurs for Δroe in Table 13. These results provide strong evidence that change in earnings, measured by Δroa or Δroe, does not subsume large return spreads associated with past returns *umd*.

As a robustness check, we perform double-sorts in the reverse order, first sorting by *umd*, and then sorting by Δroa or Δroe. Here, we see that sorting by Δroa or Δroe produce return spreads, after controlling for *umd*. Therefore, it seems that both past returns and changes in earnings are strong predictors of subsequent stock returns and neither one subsumes the other. If anything, the results for return spreads associated with *umd* controlling for Δroa or Δroe (Tables 12 and 13) are stronger than return spreads associated with Δroa or Δroe controlling for *umd* (Tables 14 and 15).

Therefore, we disagree with the conclusion of Chordia and Shivakumar (2006) that "price momentum is merely a manifestation of the earnings momentum." Though our factors formed on changes in the accounting ratios are able to *explain* momentum profits in the sense of insignificant portfolio α, we observe that both price changes and earnings changes produce large and significant spreads in stock returns. We conclude that though price and earnings momentum seem to share a common systematic component, residual components in them are still significant in the cross-section of stock returns.

6 Conclusion

We study cross-sectional pattern in stock returns associated with accounting ratios and their changes. We consider measures of book returns, operating efficiency, and profit margins. We find that levels and changes in many of these accounting ratios produce large and significant return spreads. In further investigating their properties, we make three main findings. First, changes in these accounting ratios, rather than their levels, are important and strong determinants of stock returns in the cross-section. Second, portfolios formed by sorting on changes in the accounting ratios have very large and significant α with respect to Fama-French and Carhart factor models, and this is robust to inclusion of additional risk factors. We call our results, "fundamental momentum". Third, we document that momentum profits can be explained away by these factors formed on changes. However, we demonstrate that this regression result does not mean that earnings momentum fully explains price momentum. In particular, we find that past returns generate large and positive return spreads after controlling for changes in earnings, and vice versa. This leads us to conclude that residual components in price and earnings momentum after accounting for their common component still contain much explanatory power in the cross-section of stock returns.

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Table 1: Summary of Accounting Ratio Definitions

Variable	Description	Definition
$roa\,(t)$	Return on assets	$IBQ\,(t)\,/ATQ\,(t-3)$
$roe\,(t)$	Return on equity	$IBQ\,(t)\,/CEQQ\,(t-3)$
$arturn\,(t)$	Acconts receivable turnover	$REVTQ\,(t)\,/\frac{1}{2}\,(RECTQ\,(t-3)+RECTQ\,(t))$
$apturn\,(t)$	Accounts payable turnover	$COGSQ\,(t)\,/\frac{1}{2}\,(APQ\,(t-3)+APQ\,(t))$
$gmarg\,(t)$	Gross profit margin	$(REVTQ\,(t)-COGSQ\,(t))\,/REVTQ\,(t)$
$nmarg\,(t)$	Net profit margin	$OIBDPQ\,(t)\,/REVTQ\,(t)$

Compustat item at month t is defined as the most recent data entry if there has been at least one reported quarterly earnings during the past five months, and undefined otherwise. Changes in these accounting ratios, denoted with a preceeding Δ, are defined as year-over-year changes, i.e.,$\Delta x\,(t) = x\,(t) - x\,(t-12)$.

Table 2: Summary Statistics on Accounting Ratios and Their Changes

Levels

Variable	Mean	Std. Dev.	Corr				
			roe	*arturn*	*apturn*	*gmarg*	*nmarg*
roa	0.01411	0.0448	0.7821	0.0513	0.0782	0.3217	0.3518
roe	0.03037	0.1018		0.0539	0.0349	0.2167	0.2788
arturn	3.32528	6.5013			0.1375	-0.0390	-0.0350
apturn	3.10638	2.8238				-0.2066	-0.0889
gmarg	0.25196	1.6959					0.6810
nmarg	0.01949	2.7173					

Changes

Variable	Mean	Std. Dev.	Corr(·, Δ·)
Δ*roa*	0.0006	0.0455	0.4154
Δ*roe*	-0.0003	0.0969	0.5148
Δ*arturn*	-0.0056	2.6819	0.1965
Δ*apturn*	-0.0093	1.6581	0.2580
Δ*gmarg*	0.0107	1.2987	0.3560
Δ*nmarg*	0.0070	2.1800	0.3809

Changes

Variable	Corr				
	Δ*roe*	Δ*arturn*	Δ*apturn*	Δ*gmarg*	Δ*nmarg*
Δ*roa*	0.7755	0.0551	0.0215	0.1848	0.2916
Δ*roe*		0.0418	0.0078	0.1409	0.2246
Δ*arturn*			0.1052	0.0246	0.0363
Δ*apturn*				-0.1644	0.0027
Δ*gmarg*					0.5686
Δ*nmarg*					

Corr(x, Δx) denotes correlation between accounting ratio x and its year-over-year change Δx.

Table 3: Summary Statistics of Factors Based on Accounting Ratios

Factors Formed by Sorting on Accounting Ratios

Factor	Mean	Std. Dev.	FF3 α	FF3 R^2	FF3+UMD α	FF3+UMD R^2
ROA	0.0058***	0.0268	0.0075***	11.4%	0.0062***	17.3%
	[4.27]		[5.49]		[4.4]	
ROE	0.0065***	0.0283	0.0077***	14.7%	0.0061***	22.8%
	[4.62]		[5.61]		[4.46]	
$ARTURN$	0.0038***	0.0219	0.0042***	37.8%	0.0039***	38.1%
	[3.11]		[4.44]		[4.1]	
$APTURN$	0.0016*	0.0157	0.0009	14.4%	0.0005	15.8%
	[1.86]		[1.18]		[0.65]	
$GMARG$	0.0009	0.0194	0.0026***	48.1%	0.0027***	48.3%
	[0.8]		[3.07]		[3.21]	
$NMARG$	0.0020*	0.0248	0.0036***	39.6%	0.0034***	39.8%
	[1.73]		[3.55]		[3.07]	

Factors Formed by Sorting on Changes in Accounting Ratios

Factor	Mean	Std. Dev.	FF3 α	FF3 R^2	FF3+UMD α	FF3+UMD R^2	Corr(·, Δ·)
ΔROA	0.0065***	0.0202	0.0075***	8.0%	0.0055***	32.6%	0.4317
	[6.08]		[7.79]		[6.3]		
ΔROE	0.0063***	0.0190	0.0069***	3.1%	0.0049***	31.6%	0.4236
	[6.44]		[7.9]		[6.14]		
$\Delta ARTURN$	0.0047***	0.0166	0.0046***	0.4%	0.0037***	8.0%	0.0425
	[6.24]		[5.96]		[5.07]		
$\Delta APTURN$	0.0018***	0.0112	0.0017***	0.1%	0.0014**	2.1%	0.0534
	[3.36]		[3.01]		[2.46]		
$\Delta GMARG$	0.0037***	0.0160	0.0039***	6.1%	0.0025***	25.4%	0.2789
	[4.81]		[5.3]		[3.34]		
$\Delta NMARG$	0.0046***	0.0198	0.0051***	12.0%	0.0033***	31.1%	-0.0200
	[4.88]		[6.13]		[3.96]		

Corr(X, ΔX) denotes return correlation between factors X and ΔX. Columns FF3 and FF3+UMD report α and R^2 of regressions of factors on Fama-French three-factor and Carhart four-factor models. Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 4: Fama-MacBeth Regressions

	Without Controlling for *umd*		Controlling for *umd*	
Variable	Lagged Level	Change	Lagged Level	Change
roa	0.0061**	0.1703***	0.0054*	0.1401***
	[2.11]	[6.77]	[1.94]	[6.30]
roe	0.0036***	0.0545***	0.0032***	0.0456***
	[3.61]	[7.64]	[3.43]	[7.26]
arturn	0.0005	0.0005***	0.0004	0.0041***
	[1.04]	[6.81]	[0.87]	[6.40]
apturn	0.0009***	0.0006***	0.0008**	0.0006***
	[2.71]	[5.27]	[2.54]	[5.55]
gmarg	0.0011*	0.0078***	0.0009	0.0060***
	[1.74]	[5.22]	[1.52]	[4.86]
nmarg	0.0012	0.0121***	0.0007	0.0094***
	[1.64]	[4.28]	[1.08]	[3.88]

This table reports average slopes and their standard errors in Fama-MacBeth (1973) regressions of returns on lagged levels and changes of accounting ratios, controlling for log market capitalization, log book-to-market ratio, and cumulative return from month $t-12$ to month $t-1$ denoted by *umd*. The two regressors of interest, lagged levels and changes of accounting ratios are standarized by their cross-sectional standard deviations in each month t in the Fama-MacBeth regressions. Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 5: Double-Sorted Portfolios on Levels and Changes, *roa* and *roe*

FF3+UMD α of Portfolios Double Sorted by *roa* and Δroa

		Δroa 1	2	3	4	5	5-1
roa	1	-0.0069***	-0.0036*	-0.0061***	-0.0045*	-0.0014	0.0055**
		[-3.49]	[-1.94]	[-3.05]	[-1.96]	[-0.51]	[2.13]
	2	-0.0007	-0.0021	-0.0002	0.0018	-0.0012	-0.0005
		[-0.39]	[-1.59]	[-0.14]	[1.35]	[-0.9]	[-0.27]
	3	0.0007	-0.0010	0.0021**	0.0004	0.0033**	0.0027
		[0.5]	[-0.77]	[2.22]	[0.34]	[2.33]	[1.3]
	4	-0.0003	0.0000	0.0027**	0.0019	0.0046***	0.0049**
		[-0.2]	[-0.02]	[2.31]	[1.44]	[2.97]	[2.35]
	5	0.0018	0.0033***	0.0032***	0.005***	0.0068***	0.0050*
		[1.3]	[2.71]	[2.61]	[3.18]	[3.09]	[1.95]
	5-1	0.0087***	0.0069***	0.0093***	0.0095***	0.0081***	
		[3.65]	[2.84]	[3.94]	[3.28]	[2.73]	

FF3+UMD α of Portfolios Double Sorted by *roe* and Δroe

		Δroe 1	2	3	4	5	5-1
roe	1	-0.0060***	-0.0042**	-0.0033*	-0.0028	-0.0047*	0.0012
		[-3.23]	[-2.25]	[-1.81]	[-1.33]	[-1.96]	[0.48]
	2	-0.002	-0.0024*	-0.001	-0.0008	-0.0004	0.0016
		[-1.28]	[-1.69]	[-0.73]	[-0.72]	[-0.23]	[0.73]
	3	-0.0001	-0.0014	0.0000	0.0011	0.0028*	0.0029
		[-0.06]	[-1.2]	[-0.01]	[0.99]	[1.79]	[1.38]
	4	-0.0009	0.0008	0.0020*	0.0026**	0.0028*	0.0038*
		[-0.62]	[0.7]	[1.75]	[2.40]	[1.65]	[1.71]
	5	0.0018	0.004***	0.0036***	0.0027*	0.0068***	0.0050**
		[1.23]	[3.27]	[3.22]	[1.83]	[3.94]	[2.34]
	5-1	0.0077***	0.0081***	0.0069***	0.0055*	0.0115***	
		[3.18]	[3.75]	[2.99]	[1.89]	[3.87]	

This table reports α of 25 double-sorted portfolios formed by first sorting on accounting ratio x, then by sorting on change in the ratio Δx within each bin (dependent double sort; capturing return spread associated with different levels of Δx controlling for the level x). Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 6: Double-Sorted Portfolios on Levels and Changes, *arturn* and *apturn*

FF3+UMD α of Portfolios Double Sorted by *arturn* and $\Delta arturn$

		$\Delta arturn$ 1	2	3	4	5	5-1
arturn	1	-0.0087***	-0.0032**	-0.0027**	0.0015	0.0001	0.0088***
		[-4.93]	[-2.14]	[-1.98]	[1.09]	[0.07]	[3.57]
	2	-0.0023	-0.0019	0.0012	-0.0002	0.004**	0.0062**
		[-1.53]	[-1.46]	[1.07]	[-0.18]	[2.18]	[2.47]
	3	-0.0013	-0.0001	0.0029**	0.0009	0.0049***	0.0062***
		[-0.9]	[-0.06]	[2.12]	[0.68]	[3.14]	[2.67]
	4	-0.0016	0.0017*	0.0013	0.0046***	0.0012	0.0028
		[-1.03]	[1.75]	[1.13]	[3.26]	[0.76]	[1.27]
	5	0.0024*	0.0017	0.0016	0.0046***	0.003*	0.0006
		[1.69]	[1.04]	[1.37]	[3.32]	[1.92]	[0.36]
	5-1	0.0110***	0.0049**	0.0044**	0.0031	0.0029	
		[4.96]	[2.29]	[2.31]	[1.56]	[1.32]	

FF3+UMD α of Portfolios Double Sorted by *apturn* and $\Delta apturn$

		$\Delta apturn$ 1	2	3	4	5	5-1
apturn	1	-0.0016	0.0031**	0.0005	0.0002	0.0022	0.0038*
		[-1.21]	[2.2]	[0.42]	[0.19]	[1.4]	[1.92]
	2	-0.0025	0.0024*	0.0012	0.0026**	0.0006	0.0031
		[-1.63]	[1.8]	[1.21]	[2.01]	[0.44]	[1.59]
	3	0.0001	-0.0001	0.0026**	0.0007	0.001	0.0009
		[0.09]	[-0.12]	[2.13]	[0.56]	[0.62]	[0.48]
	4	-0.0001	0.0009	0.0011	0.0014	0.0015	0.0016
		[-0.11]	[0.85]	[0.75]	[1.11]	[1.04]	[0.82]
	5	-0.0011	-0.0006	0.0004	0.0015	0.0026	0.0037
		[-0.66]	[-0.44]	[0.39]	[1.24]	[1.32]	[1.34]
	5-1	0.0005	-0.0037*	-0.0001	0.0013	0.0004	
		[0.24]	[-1.85]	[-0.04]	[0.69]	[0.16]	

This table reports α of 25 double-sorted portfolios formed by first sorting on accounting ratio x, then by sorting on change in the ratio Δx within each bin (dependent double sort; capturing return spread associated with different levels of Δx controlling for the level x). Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 7: Double-Sorted Portfolios on Levels and Changes, *gmarg* and *nmarg*

FF3+UMD α of Portfolios Double Sorted by *gmarg* and $\Delta gmarg$

		$\Delta gmarg$					
		1	2	3	4	5	5-1
	1	-0.0039***	-0.0014	0.0014	0.0014	-0.0002	0.0037*
		[-2.7]	[-0.97]	[1.13]	[1.05]	[-0.13]	[1.84]
	2	-0.0022	0.0008	0.0024	0.0002	0.0014	0.0036**
		[-1.59]	[0.63]	[1.64]	[0.14]	[1.17]	[2.07]
	3	-0.0022*	0.0009	0.0000	0.0022*	-0.0006	0.0017
gmarg		[-1.69]	[0.63]	[-0.01]	[1.84]	[-0.32]	[0.97]
	4	0.0001	0.0009	0.0021*	0.0033***	0.0009	0.0008
		[0.07]	[0.74]	[1.70]	[2.89]	[0.50]	[0.39]
	5	0.0010	0.0019	0.0036***	0.0041***	0.0040*	0.0030
		[0.58]	[1.37]	[2.69]	[2.86]	[1.86]	[1.09]
	5-1	0.0049**	0.0033	0.0022	0.0026	0.0042*	
		[2.17]	[1.62]	[1.14]	[1.27]	[1.75]	

FF3+UMD α of Portfolios Double Sorted by *nmarg* and $\Delta nmarg$

		$\Delta nmarg$					
		1	2	3	4	5	5-1
	1	-0.0059**	-0.0075***	-0.0015	0.0027	0.0020	0.0080***
		[-2.57]	[-3.4]	[-0.74]	[1.36]	[0.74]	[3.11]
	2	-0.0015	0.0007	-0.0006	0.0035**	0.0023	0.0038
		[-0.85]	[0.46]	[-0.44]	[2.46]	[1.27]	[1.52]
	3	-0.0012	0.0021*	-0.0001	0.0005	0.0028	0.0040
nmarg		[-0.81]	[1.68]	[-0.1]	[0.37]	[1.29]	[1.54]
	4	-0.0009	0.0000	0.0033***	0.0046***	-0.0014	-0.0005
		[-0.58]	[0.03]	[2.67]	[2.8]	[-0.78]	[-0.24]
	5	-0.0006	0.0004	0.0011	0.0047***	0.0025	0.0031
		[-0.41]	[0.40]	[0.89]	[3.79]	[1.41]	[1.44]
	5-1	0.0053*	0.0079***	0.0026	0.0021	0.0005	
		[1.85]	[3.10]	[1.10]	[0.82]	[0.16]	

This table reports α of 25 double-sorted portfolios formed by first sorting on accounting ratio x, then by sorting on change in the ratio Δx within each bin (dependent double sort; capturing return spread associated with different levels of Δx controlling for the level x). Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 8: Regression of Accounting Factor Returns on FF3 and FF3+UMD

Factor	α	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	R^2
ΔROA	0.0075***	0.00	-0.06	-0.19***		8.0%
	[7.79]	[-0.02]	[-1.26]	[-3.12]		
	0.0055***	0.04	-0.08	-0.11**	0.23***	32.6%
	[6.3]	[1.58]	[-1.54]	[-2.13]	[6.55]	
ΔROE	0.0069***	0.00	-0.06	-0.11**		3.1%
	[7.9]	[-0.05]	[-1.63]	[-2.36]		
	0.0049***	0.04**	-0.07*	-0.03	0.23***	31.6%
	[6.14]	[2.06]	[-1.96]	[-0.78]	[7.97]	
$\Delta ARTURN$	0.0046***	0.00	0.02	0.03		0.4%
	[5.96]	[-0.16]	[0.44]	[0.74]		
	0.0037***	0.01	0.01	0.06	0.10***	8.0%
	[5.07]	[0.45]	[0.18]	[1.64]	[5.39]	
$\Delta APTURN$	0.0017***	0.00	0.01	0.01		0.1%
	[3.01]	[0.08]	[0.35]	[0.44]		
	0.0014**	0.01	0.00	0.02	0.04**	2.1%
	[2.46]	[0.56]	[0.18]	[0.95]	[2.42]	
$\Delta GMARG$	0.0039***	0.03	0.02	-0.10**		6.1%
	[5.3]	[1.26]	[0.52]	[-2.3]		
	0.0025***	0.06***	0.00	-0.04	0.16***	25.3%
	[3.34]	[2.97]	[0.15]	[-1.31]	[6.00]	
$\Delta NMARG$	0.0051***	0.01	0.05	-0.19***		12.0%
	[6.13]	[0.46]	[1.27]	[-4.22]		
	0.0033***	0.05**	0.04	-0.13***	0.20***	31.1%
	[3.96]	[2.13]	[1.22]	[-3.12]	[6.70]	

This table reports time-series regression results of accounting factor returns on Fama-French three factors and Carhart four factors. Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 9: Regression of Factor Returns on Additional Risk Factors

Factor	α	β_{UMD}	β_{PS}	β_{DEF}	β_{REC}
ΔROA	0.0042***	0.2173***	-0.0028	-0.1243***	0.0031
	[4.38]	[7.35]	[-0.09]	[-3.06]	[1.40]
ΔROE	0.0045***	0.2195***	0.0133	-0.1065*	0.0021
	[5.39]	[8.21]	[0.47]	[-1.87]	[0.93]
$\Delta ARTURN$	0.0044***	0.0900***	0.0301**	-0.0643	0.0002
	[5.46]	[4.36]	[2.17]	[-1.75]	[0.08]
$\Delta APTURN$	0.0014***	0.0469***	-0.0108	0.0181	0.0008
	[2.59]	[2.71]	[-0.66]	[0.58]	[0.50]
$\Delta GMARG$	0.0023***	0.1342***	0.0016	-0.0768**	-0.0007
	[3.32]	[5.34]	[0.06]	[2.11]	[-0.35]
$\Delta NMARG$	0.0032***	0.1845***	-0.0076	-0.1024***	-0.0005
	[3.90]	[6.55]	[-0.21]	[-2.68]	[-0.23]

This table reports time-series regression results of accounting factor returns on an extensive set of risk factors including the Fama-French three factors (market, size, and value), momentum UMD, Pastor-Stambaugh liquidity factor PS, excess return on the Dow Jones Corporate Bond Return Index DEF, and NBER Recession Indicator REC. The Pastor-Stambaugh factor is from Pastor and Stambaugh (2003). The use of credit spread DEF is based on Chen, Roll, and Ross (1986). Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 10: Correlations among Accounting Factors

Correlations

	ΔROE	$\Delta ARTURN$	$\Delta APTURN$	$\Delta GMARG$	$\Delta NMARG$	UMD
ΔROA	0.8997	0.2946	0.1253	0.6177	0.7055	0.5295
ΔROE		0.2621	0.1356	0.6197	0.6643	0.5436
$\Delta ARTURN$			0.2595	0.3526	0.2757	0.2610
$\Delta APTURN$				-0.1309	0.0741	0.1319
$\Delta GMARG$					0.7438	0.4522
$\Delta NMARG$						0.4819

Table 11: Regression of Momentum Returns on Accounting Factors

Factors Formed by Sorting on Changes in Accounting Ratios

Factor	α	β_{MKT}	β_{SMB}	β_{HML}	β_{FACTOR}
None	0.0085***	-0.1800*	0.0850	-0.3384*	
	[4.50]	[-1.93]	[0.13]	[-1.91]	
ΔROA	-0.0001	-0.1864**	0.1513	-0.1238	1.1795***
	[-0.03]	[-2.37]	[1.16]	[-0.77]	[6.65]
ΔROE	-0.0001	-0.1855***	0.1499	-0.2112	1.2821***
	[-0.02]	[-2.62]	[1.2]	[-1.41]	[6.95]
$\Delta ARTURN$	0.0054**	-0.1655*	0.0939	-0.3362**	0.7320**
	[2.20]	[-1.76]	[0.72]	[-1.98]	[2.42]
$\Delta APTURN$	0.0080***	-0.1869*	0.0936	-0.3501*	0.5577**
	[3.93]	[-1.83]	[0.70]	[-1.91]	[2.48]
$\Delta GMARG$	0.0038	-0.2288***	0.0563	-0.2209	1.2868***
	[1.64]	[-2.72]	[0.48]	[-1.53]	[5.05]
$\Delta NMARG$	0.0032	-0.2034**	0.0180	-0.1336	1.1089***
	[1.44]	[-2.52]	[0.16]	[-0.86]	[6.66]

Factors Formed by Sorting on Accounting Ratios

Factor	α	β_{MKT}	β_{SMB}	β_{HML}	β_{FACTOR}
ROA	0.0054**	-0.1767**	0.1995	-0.2685	0.4525**
	[2.04]	[-2.02]	[1.48]	[-1.4]	[2.36]
ROE	0.0048*	-0.1799**	0.2551*	-0.3404*	0.5197**
	[1.67]	[-2.12]	[1.95]	[-1.95]	[2.57]
$ARTURN$	0.0080***	-0.1659*	0.1097	-0.3934**	0.1812
	[3.29]	[-1.84]	[0.83]	[-2.1]	[0.76]
$APTURN$	0.0083***	-0.1609*	0.0884	-0.3871**	0.3904
	[4.12]	[-1.78]	[0.61]	[-2.26]	[1.47]
$GMARG$	0.0091***	-0.1881**	0.0769	-0.4127**	-0.1441
	[4.6]	[-1.99]	[0.57]	[-2.24]	[-0.57]
$NMARG$	0.0083***	-0.1746*	0.1226	-0.3603**	0.1172
	[3.88]	[-1.78]	[1.13]	[-2.04]	[0.6]

This table reports regression statistics of UMD on factors formed by sorting on changes and levels of the accounting variables, in addition to the Fama-French three-factors. Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 12: Double sorts on price and earnings changes

Average Returns of Portfolios Double-Sorted on Δroa and *umd*

		umd 1	2	3	4	5	5-1
	1	0.0017	0.0025	0.0026	0.0024	0.008**	0.0063*
		[0.42]	[0.75]	[0.89]	[0.8]	[2.12]	[1.73]
	2	0.0034	0.0044	0.0007	0.0048**	0.0081***	0.0047
		[0.96]	[1.58]	[0.27]	[2.01]	[2.78]	[1.41]
	3	0.007**	0.0081***	0.0051**	0.0071***	0.0088***	0.0018
Δroa		[2.32]	[3.66]	[2.03]	[3.03]	[3.15]	[0.61]
	4	0.004	0.0046*	0.008***	0.0073**	0.0091***	0.0051*
		[1.35]	[1.89]	[3.55]	[2.5]	[2.7]	[1.65]
	5	0.0055*	0.0071**	0.0079**	0.0106***	0.0159***	0.0104**
		[1.71]	[2.43]	[2.39]	[2.68]	[3.13]	[2.38]
	5-1	0.0038*	0.0047**	0.0053**	0.0083***	0.0079***	
		[1.66]	[2.07]	[2.07]	[2.65]	[2.98]	

FF3 α of Portfolios Double-Sorted on Δroa and *umd*

		umd 1	2	3	4	5	5-1
	1	-0.0055**	-0.0038*	-0.0033**	-0.0023	0.0035	0.0090**
		[-2.25]	[-1.94]	[-2.26]	[-1.46]	[1.65]	[2.44]
	2	-0.0034	-0.0018	-0.0053***	-0.0003	0.0032**	0.0066**
		[-1.52]	[-1.16]	[-3.82]	[-0.25]	[2.26]	[1.98]
	3	0.0003	0.003**	-0.0002	0.0022**	0.0040***	0.0037
Δroa		[0.17]	[2.35]	[-0.15]	[2.04]	[2.71]	[1.23]
	4	-0.0022	-0.0007	0.0028**	0.0025	0.0038**	0.0060**
		[-1.25]	[-0.56]	[2.39]	[1.62]	[2.12]	[2.09]
	5	-0.0004	0.0021	0.0031	0.0054**	0.0112***	0.0116***
		[-0.21]	[1.24]	[1.63]	[2.26]	[3.89]	[2.84]
	5-1	0.0050**	0.0059***	0.0065***	0.0078**	0.0077***	
		[2.38]	[2.62]	[2.61]	[2.48]	[3.14]	

This table reports average returns and Fama-French α of 25 double-sorted portfolios formed by first sorting on Δroa, then by sorting cumulative past return *umd* within each bin (dependent double sort; capturing return spread associated with *umd* controlling for Δroa. Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 13: Double sorts on price and earnings changes

Average Returns of Portfolios Double-Sorted on Δroe and *umd*

		umd 1	2	3	4	5	5-1
	1	0.0012	0.0034	0.0036	0.0022	0.007**	0.0058
		[0.29]	[1.01]	[1.24]	[0.76]	[2.01]	[1.62]
	2	0.0039	0.0047*	0.0025	0.0062**	0.0076**	0.0037
		[1.08]	[1.65]	[1.04]	[2.47]	[2.57]	[1.28]
	3	0.0056*	0.0061***	0.0036	0.0067***	0.0076**	0.0020
Δroe		[1.91]	[2.67]	[1.40]	[2.93]	[2.55]	[0.67]
	4	0.0057**	0.005**	0.0086***	0.0076**	0.011***	0.0054*
		[2.11]	[2.2]	[3.5]	[2.47]	[3.29]	[1.71]
	5	0.0053	0.0073***	0.0073**	0.009***	0.016***	0.0106**
		[1.63]	[2.82]	[2.44]	[2.62]	[3.06]	[2.24]
	5-1	0.0042	0.0039*	0.0037*	0.0068**	0.0090***	
		[1.45]	[1.80]	[1.71]	[2.52]	[2.97]	

FF3 α of Portfolios Double-Sorted on Δroe and *umd*

		umd 1	2	3	4	5	5-1
	1	-0.0058**	-0.0030	-0.0027*	-0.0028*	0.0025	0.0083**
		[-2.32]	[-1.58]	[-1.94]	[-1.88]	[1.30]	[2.30]
	2	-0.0032	-0.0014	-0.0032***	0.0012	0.0027*	0.0060**
		[-1.57]	[-0.96]	[-2.72]	[0.99]	[1.95]	[2.04]
	3	-0.0003	0.0009	-0.0017	0.0019	0.0029**	0.0032
Δroe		[-0.17]	[0.64]	[-1.31]	[1.55]	[2.15]	[1.12]
	4	0.0001	0.0001	0.0033***	0.0031*	0.0062***	0.0061*
		[0.05]	[0.08]	[2.69]	[1.83]	[3.44]	[1.95]
	5	-0.0014	0.0018	0.0022	0.0035*	0.0111***	0.0125***
		[-0.66]	[1.27]	[1.4]	[1.76]	[3.34]	[2.87]
	5-1	0.0044*	0.0048**	0.0050**	0.0063**	0.0086***	
		[1.66]	[2.23]	[2.37]	[2.39]	[3.04]	

This table reports average returns and Fama-French α of 25 double-sorted portfolios formed by first sorting on Δroe, then by sorting cumulative past return *umd* within each bin (dependent double sort; capturing return spread associated with *umd* controlling for Δroe. Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 14: Double sorts on price and earnings changes

Average Returns of Portfolios Double-Sorted on *umd* and Δroa

		Δroa 1	2	3	4	5	5-1
	1	-0.0010	0.0041	0.0046	0.0041	0.0050	0.0060***
		[-0.27]	[1.13]	[1.44]	[1.23]	[1.43]	[2.70]
	2	0.0037	0.0022	0.0048*	0.0073***	0.0053*	0.0017
		[1.27]	[0.80]	[1.89]	[2.96]	[1.88]	[0.83]
	3	0.0022	0.0029	0.0055**	0.0059***	0.0062**	0.0040**
umd		[0.82]	[1.22]	[2.42]	[2.61]	[2.28]	[2.13]
	4	0.0076***	0.0069***	0.0061**	0.0099***	0.0064**	-0.0012
		[2.85]	[2.72]	[2.19]	[3.78]	[2.00]	[-0.59]
	5	0.0071*	0.0110***	0.0104***	0.0131***	0.0136***	0.0066**
		[1.72]	[3.13]	[3.08]	[3.27]	[2.67]	[2.51]
	5-1	0.0081**	0.0069*	0.0058*	0.009**	0.0087*	
		[2.12]	[1.88]	[1.70]	[2.32]	[1.84]	

FF3 α of Portfolios Double-Sorted on *umd* and Δroa

		Δroa 1	2	3	4	5	5-1
	1	-0.0076***	-0.0027	-0.0022	-0.0023	-0.0018	0.0057***
		[-3.25]	[-1.33]	[-1.1]	[-1]	[-0.75]	[2.77]
	2	-0.0021	-0.0039***	-0.0012	0.0024	0.0000	0.0021
		[-1.29]	[-2.91]	[-0.93]	[1.59]	[0.00]	[1.02]
	3	-0.0028*	-0.0026**	0.0005	0.0006	0.0007	0.0035*
umd		[-1.94]	[-2.42]	[0.46]	[0.54]	[0.5]	[1.83]
	4	0.0025*	0.0020	0.0014	0.0051***	0.0012	-0.0013
		[1.78]	[1.49]	[1.09]	[3.40]	[0.68]	[-0.64]
	5	0.0024	0.0066***	0.0052**	0.0079***	0.0090***	0.0065***
		[1.10]	[3.21]	[2.52]	[3.42]	[2.96]	[2.61]
	5-1	0.0100***	0.0093***	0.0074**	0.0102***	0.0108**	
		[2.60]	[2.61]	[2.26]	[2.81]	[2.38]	

This table reports average returns and Fama-French α of 25 double-sorted portfolios formed by first sorting on cumulative past return *umd*, then by sorting by Δroa within each bin (dependent double sort; capturing return spread associated with Δroa controlling for *umd*. Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.

Table 15: Double sorts on price and earnings changes

Average Returns of Portfolios Double-Sorted on *umd* and Δroe

		Δroe					
		1	2	3	4	5	5-1
	1	-0.0031	0.0051	0.0044	0.0052	0.0035	0.0065***
		[-0.81]	[1.45]	[1.31]	[1.58]	[1.04]	[2.63]
	2	0.0033	0.0044	0.0039	0.0059**	0.0055**	0.0021
		[1.10]	[1.63]	[1.57]	[2.37]	[2.05]	[1.09]
	3	0.002	0.0033	0.0054**	0.0052**	0.0067***	0.0048***
umd		[0.72]	[1.29]	[2.27]	[2.33]	[2.66]	[2.94]
	4	0.0059**	0.0080***	0.0067***	0.0095***	0.0070**	0.0011
		[2.13]	[3.33]	[2.64]	[3.28]	[2.37]	[0.67]
	5	0.0067*	0.0102***	0.0116***	0.0129***	0.0138***	0.0071***
		[1.74]	[2.69]	[3.36]	[3.42]	[3.01]	[2.83]
	5-1	0.0097***	0.0051	0.0072**	0.0077**	0.0103**	
		[2.81]	[1.42]	[1.97]	[2.07]	[2.33]	

FF3 α of Portfolios Double-Sorted on *umd* and Δroe

		Δroe					
		1	2	3	4	5	5-1
	1	-0.0097***	-0.0021	-0.0025	-0.0011	-0.0033	0.0065***
		[-4.04]	[-1.13]	[-1.14]	[-0.47]	[-1.33]	[2.67]
	2	-0.0028*	-0.0016	-0.0019*	0.0005	0.0003	0.0031
		[-1.66]	[-1.14]	[-1.67]	[0.34]	[0.19]	[1.57]
	3	-0.0036***	-0.0017	0.0003	0.0002	0.0011	0.0047***
umd		[-2.77]	[-1.37]	[0.29]	[0.13]	[0.80]	[2.68]
	4	0.0009	0.0033**	0.0020	0.0050***	0.0018	0.0010
		[0.63]	[2.36]	[1.63]	[3.42]	[1.08]	[0.58]
	5	0.0022	0.0055**	0.0068***	0.0077***	0.0086***	0.0064***
		[1.10]	[2.47]	[3.45]	[3.52]	[3.27]	[2.60]
	5-1	0.0119***	0.0076**	0.0092***	0.0087**	0.0119***	
		[3.32]	[2.33]	[2.63]	[2.40]	[2.86]	

This table reports average returns and Fama-French α of 25 double-sorted portfolios formed by first sorting on cumulative past return *umd*, then by sorting by Δroe within each bin (dependent double sort; capturing return spread associated with Δroe controlling for *umd*. Numbers in square brackets are t-statistics and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels.