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India Project
C/60-5
FEBRUARY 1960.

THE USE OF SHADOW PRICES IN PROGRAMME EVALUATION[1]

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I. Introduction

In recent years, occasional mention has been made of the use[2] that should be made of shadow prices in determining the priorities in an investment program. This applies particularly to the shadow prices of two of the most important productive factors in an underdeveloped economy, i.e. capital and foreign exchange.

Stated explicitly, the rationale of using shadow prices is derived from considering the perfect competition general equilibrium model as an analogue computing device, where parametrically treated prices serve as signalling devices, resulting in an efficient allocation of resources. Since the observed market prices in an underdeveloped economy reflect mainly a situation of imperfect competition and severe structural disproportionalities, it is thought that the prices that should be regarded as pointers in planning investment are not those obtaining on the market but the prices that would result from an ideal experiment obeying the perfect competition rules of the game. In

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1. I am greatly indebted to Professor P. N. Rosenstein-Rodan for the suggestion of the problem and his stimulating comments. I am also very much grateful to Professors R. M. Solow, R. S. Eckaus and I. M. D. Little for several penetrating discussions. None of them should, however, be held responsible for any of the views expressed here.

2. Tinbergen, The Design of Development; also in Economic Policy "Principles and Designs"; Chenery, Economic Bulletin for Latin America, 1958, and in other places.

programming language, these are the Lagrange multipliers of the constrained maximization problem.

It should be obvious that in an intertemporal planning problem, such as is involved in the problem of capital accumulation, the relevant endowments of the primary factors are continually changing and their scarcity aspects are therefore shifting. Hence, what we need is not merely the shadow price relating to one point of time, but the development of shadow prices over a period of time, i.e. the time-path of shadow prices.

The shadow prices may greatly simplify the task of properly assigning priorities in an investment program. Thus, once they are known, the "social benefit-cost" ratios (e.g. the costs and benefits of a project which take into account the direct and indirect effects and using 'shadow prices' of primary factors instead of their market prices), of the projects may be calculated and those projects chosen which give the maximum social benefit, for a prescribed amount of cost. In this way, the method of program evaluation involving the use of shadow prices serves as a substitute for the full-fledged programming procedure. The usual programming procedure consists in determining all the decision variables, namely, the investments in each project or constellation of projects, as the result of a constrained maximization problem. It is true that if the solution to the over-all programming problem is known, the shadow prices are known at the same time, since the two sets of variables, prices and quantities are related in a dual fashion. But even when it is not possible to solve the full dynamic program--and that is often the case--an approximate estimate may still be possible. Certain suggestions are made here in this connection which yield approximations that constitute improvements on the current methods of decision making.

The chief advantage of approximations to shadow prices developed below over the centralized programming procedure discussed in an earlier paper[1] lies in the relative simplicity with which (in an operational context) the approximations to the shadow prices of the two most important primary factors, capital and foreign exchange, may be calculated. Yet this is done in a way that does some rough justice to the interdependences which exist in the economy over time as well as at a particular point of time.

Since the present practice in development programming is based almost exclusively on the use of current market prices of primary factors which are heavily out of line with their "intrinsic" values, the use of shadow prices will represent qualitatively a move in the right direction. This holds good even if our knowledge of the subject is not exact and we have to use only approximate results. Further, in a mixed economy, the use of shadow prices makes it possible for relatively simple decision rules to be laid down with respect to private investment. This raises the problem of administration which should be discussed separately.

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1. S. Chakravarty, "An Outline of a Method of Programme Evaluation," C/60-1, Center For International Studies, Massachusetts Institute of Technology.

II. The Shadow Price of Foreign Exchange

It is a well-known observation that the shadow price of foreign exchange in many underdeveloped countries suffering from chronic balance of payments difficulties is substantially higher than the official rate of exchange. The reason for such maintained prices of foreign currency is that price elasticity of the exports and imports being quite low, the mechanism of letting price find its own level by equating the total demand for foreign currency to the total supply of foreign currency either does not work or works at the expense of income growth. Further, there is a widespread opinion that balance of payments difficulties of newly developing countries are transitional in character, so that once certain structural changes have been well under way, excessive demands for imports or diversion of exports to home uses may cease, thus making it possible to approximate closely the equilibrium rate of exchange.[1]

Thus while it is necessary to maintain an official rate of exchange different from the shadow rate, the shadow rate will still be the appropriate one to use in order to discriminate between alternative programs or, in marginal cases, between alternative projects. Since sectors as well as the processes within any sector differ remarkably with respect to foreign exchange requirements, direct and cumulative, such discrimination is essential in order to satisfy the constraint relating to balance of payments equilibrium. If these constraints

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1. One may, however, argue for a devaluation of the home currency instead of letting the exchange rate seek its own level. This, however, runs into problems that are not entirely economic in character. Further, too frequent devaluations, depending on the variations in the import composition of the successive plans, will introduce nearly the same type of destabilizing influence as the method of floating exchange rates.

refer to different points of time, a time-path of the shadow rate of exchange will be involved, rather than a single rate of exchange to be applied indefinitely. We suggest the following method to determine the shadow price of foreign currency. Essentially the method consists in equating demand to supply of foreign exchange. What we elaborate is how all the components of demand and supply may be taken into account. The following notations are employed in the formula for determining the shadow rate of exchange:

$\{e\}$ - Column vector of exports.

$\{e'\}$ - is the corresponding row vector.

$\{w\}$ - Column vector of investment delivered by the sectors.

$\{\bar{w}\}$ - Column vector of investment received by the sectors.

$\{c\}$ - Row vector of final consumption.

$\bar{p}$ - Price level of goods produced at home.

$\{p\}$ - Vector of domestic prices.

p_m - Price of imports, here assumed to be homogeneous for simplicity.

k - The shadow rate of exchange.

m_1 - The quantity of raw materials imported.

m_2 - The quantity of investment goods imported.

m_3 - Import of consumer goods.

Coefficients:

$\{a\}$ - Leontief's matrix of flow coefficients.

$\{v_1\}$ - Row vector of imports per unit of gross output. These may also be called noncompetitive import requirements per unit of output.

$\{v_2\}$ - Row vector of imports per unit of investment received. This gives the import composition of the investment program.

v_3 - The functional dependence of imports of consumer goods on home consumption and the relative prices at home and abroad.

M - Total value of imports (measured in domestic prices.)

E - Total value of exports (measured in domestic prices).

D - Permissible balance of payments deficit. This need not be a single number, but may only indicate a range within which the deficits should lie.

The problem then consists in determining the value or values of 'k' so that the balance of payments deficits are confined to a certain preassigned range determined by possibilities regarding foreign aid. Assuming that the plan specifies a set of values of $\{e\}$, $\{w\}$, and $\{c\}$, and the coefficients are inflexible, then 'k' is the only variable to adapt itself to such predetermined magnitudes. It will, however, be desirable to determine the sensitivity of 'k' to adjustment in some of the physical magnitudes which are subject to some degree of control, e.g. $\{w\}$ which gives the import composition of investment or $\{c\}$, the import of consumer goods. We have the following final equation for this purpose:

$$\bar{D} = M - E$$

$$= kp_m m - e'p$$

$$= kp_m\ (m_1 + m_2 + m_3) - e'p$$

$$\bar{D} = kp_m\left\{\{v_1\}'(I - a)^{-1}(e + w + c) + v_2'\{\bar{w}\} + v_3(c,\bar{p} - p_m)\right\} - \left\{p_1e_1 + p_2e_2 + \cdot\ \cdot\ \cdot + p_ne_n\right\}$$

We give 'n' export quantities for generality, but some of these will be identically equal to zero, since we have sectors which do not export anything, like services for example. The dimensionalities in matrix multiplication are also properly observed in as much as $\{v_1\}'$ is (1 x n), $(I - a)^{-1}$ is (n x n), (e + w + c) is (n x 1). Thus the whole expression is (1 x 1) and may be multiplied by 'p_m' to get the value in foreign currency of the required amount of imports of raw materials.

$\{\bar{w}\}$ and $\{w\}$ are connected by the following matrix equation:

$\{w\} = [\ w\ ]\ \{\bar{w}\}$ where $[w]$ is the matrix of investment coefficients.[1]

Each 'p_n' may be written in the following way: (2) $p_i = A_{oi}kp_m$ $i = 1 \ldots n$ + other terms, indicatingthe influence of whatever other primary factors are assumed to be important. Thus we have (n + 1) equations to determine the (n + 1) unknowns, the shadow rate of exchange, 'k' and 'n' domestic prices. This circularity arises because the production of domestic goods needs imports, and as such prices of domestic goods are dependent on prices of imports as expressed in domestic currency.

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1. For a discussion of this matrix, see S. Chakravarty, The Logic of Investment Planning, Chap. V, North Holland Publishing Co.

The above analysis may be easily extended to take into account the heterogeneity of imports, and thus we need not assume only one composite type of imports which is capable of being used for various functional purposes. The extension is of merely algebraic nature and is thus relegated to an appendix.

It should be apparent from the above discussion that exports for this purpose have been assumed to be exogenously prescribed. This is a simplification, although of a nature that is not difficult to justify, especially when price elasticity of exports is very low or low in relation to the other factors involved. These other factors involve the level of world demand as determined by rising world incomes, as well as the domestic expansion of demand for export commodities. If the price elasticities are assumed to be significant, then this may also be taken account of by a further complication in analysis.

Maximization of income over an interval of time is considered here as the primary target, while the balance of payments condition is a side constraint to be satisfied by any optimal program. Thus our problem may be recast in the language of linear or nonlinear programming in a very easy way. What is suggested above is merely an iterative procedure to the solution of the programming problem. It further differs from the usual programming problem in operating on an 'n'-dimensional Euclidean space, where 'n' is the number of industries rather than in the Cartesian product space of dimension $(m_1 \times m_2 \times \ldots \times m_n)$ where 'm_i' is the number of techniques corresponding to the i^{th} industry. We might have considered other kinds of preferences, namely, an inverted one where balance of payments becomes the primary criterion and income growth is merely a derived one.[1]

Section III: The Shadow Rate of Interest

The shadow rate of interest is commonly regarded as a concept more difficult than the shadow rate of foreign exchange. This may be for two reasons, e.g. valuation of terminal capital stocks is not easy to ascertain conceptually and also, because of the greater degree of uncertainty that attaches to this question. Here we assume away the second, but we refrain from making any assumption regarding the terminal capital stock. In the case of foreign exchange, we are concerned with flow magnitudes; so much imports representing a flow demand for foreign currency and so much exports representing a flow supply of foreign currency. The shadow rate of exchange equilibrates the demand and supply of foreign currency. With the shadow rate of interest, we want to find out the hypothetical equilibrium rate of return on a stock, which is by no means uniform, but consists of different types of capital goods. The problem necessarily raises

1. The more general approach including balance of payments deficit (or surplus), as well as the rate of growth of income in the social welfare function cannot be implemented unless we have some method of numerically estimating the relative rates of substitution between the different policy objectives. No very convenient method exists in this connection, not withstanding the contribution of Frisch. R. Frisch, "The Numerical Determination of the Coefficients of a Preference function," Oslo, (mimeographed).

questions relating to certain important intertemporal linkages in this connection, which cannot be got rid of by numbering our variables according to the time period involved.

In spite of these difficulties, it is very useful to have some ideas regarding the shadow rate of interest, if the policy maker is concerned with rationing out scarce capital amongst a number of competing projects. True enough that if we know the solution to a full-fledged dynamic programming problem, we know at the same time the shadow rates of interest, because the optimum program of capital accumulation determines the shadow rates of interest. In that context, they may be used to decentralize decision making by permitting simple decision rules to be specified. But when that is not feasible, we still need a kind of computational shorthand in order to rank projects. Whatever approximations we may devise for computing the shadow rate of interest, even though they are correct in only a qualitative sense, will be more useful than relying on the observed market rate of interest.

In the subsequent paragraphs, certain methods of approximation are discussed under the following sets of assumptions.

a) Where capital stocks are growing at the same proportionate rate and the production functions are linear and homogeneous;

b) Where the relative rates of growth of the capital stocks are different, but we still maintain the linear homogeneity assumption;

c) Where the production functions are no longer assumed to satisfy the linear homogeneity conditions, and the equi-proportionate rate of growth of all the sectors does not hold.

We shall discuss these various cases in the order presented above.

a) The situation (a) may be further subdivided into the following two cases: (i) where there is no final demand; and (ii) where the system admits of final demand, i.e. not all the net product is reinvested. An illustration of case (i) is the closed dynamic model enunciated by Von Neumann in the early thirties. The specific setup of the Von Neumann model is well known and does not require any repetition. Von Neumann stated as the main conclusion of his investigation the now famous equality between the rate of interest and the maximum rate of balanced growth that the system can perform. As recent work by Samuelson and Solow has demonstrated, the Von Neumann path in the closed case has important normative significance in as much as it satisfies all the intertemporal conditions of efficiency. Thus the equilibrium rate of interest in known as soon as the maximum rate of steady growth is determined.

The Von Neumann model of a closed expanding economy has been generalized by Solow and Malinvaud, who relax the assumption that all the net product is reinvested. In other words, they assume the savings coefficient to be less than unity. Despite differences in presentation, the relationship between the rate of interest and the rate of growth given by the above authors is the same.

The following expression of the relationship is due to Solow who considers both the capitalists and the wage earners to be saving constant proportions of their incomes:

$$\rho = \frac{g}{\sigma_R + \frac{1-D}{D}\sigma_W}$$

where: ρ is the rate of interest

g is the rate of growth

σ_R is the savings coefficient for profit receivers.

σ_W is the savings coefficient for wage earners

D is the share of profit income in total income

It is evident that the $\rho \gtreqless g$ according as the denominator is $\lesseqgtr 1$. Now the denominator may be written as follows:

$$\frac{D\sigma_R + (1-D)\,\sigma_W}{D}$$

The expression $D\,\sigma_R + (1-D)\,\sigma_W$ is nothing other than the weighted average of the two savings coefficients or the savings coefficient for the economy as a whole. Thus we may write $\rho = \frac{g}{s/D}$ where 's' is the global savings ratio. That this relationship is merely a generalization of the Von Neumann result may be seen easily. On the specific Von Neumann assumption that $\sigma_R = 1$ and $\sigma_W = 0$, the above formula indicates $\rho = g$. When σ_W is allowed to assume positive values, there are other constellations of the coefficients for which equality holds. Although the formula indicates the theoretical possibility that the rate of interest may be lower than the rate of growth, whatever empirical evidence we have rules out this as a realistic case. Thus we may be justified to consider the equality as the limiting case.

From the data given by S. J. Patel, (Indian Economic Review, February, 1956) it appears that 's/D' in India may lie somewhere between .5 and .3 depending on how one classifies income in the household sectors. Thus, if we assume the steady rate of growth of 4% in income, the rate of interest lies between 8% and 12%. It is obvious that with a larger rate of growth, the equilibrium value of the rate of interest goes up, or with a higher rate of savings, it falls.

There are two points that one should particularly remember in this context: (a) When the system is no longer closed, Malinvaud has demonstrated that the maximal rate of balanced growth is not necessarily an efficient one. In the closed case, efficiency in balanced growth entails maximality, while this need not be the case in an open system. But since we are concerned with efficiency,[1] rather than maximality, we should interpret the rate of growth appearing in the numerator as referring to an efficient program of capital accumulation. (b) The rate of interest as deduced from the Solow formula is different from the pure rate of time discount. It takes into account both productivity and thrift. The influence of productivity is taken into account in the numerator, while the savings coefficient subsumes the influence of thrift. Behind thrift lies the factor of time preference. The rate of pure time discount that is involved may be estimated if we assume that the observed savings rate is the result of an operational decision to maximize the sum of discounted values of consumption over a period of time.

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1. The word 'efficiency' is used here as the equivalent of Pareto-optimality in the dynamic context, while 'maximality' has the usual meaning.

This is similar to the famous Ramsay model of capital accumulation. The difference consists in introducing a nonzero rate of time discount as well as in reducing the 'path maximmm' problem to a point maximum problem. The period of time may be finite or infinite, depending on the planner's point of view. In the finite case, there should be a provision for terminal equipment. Then, for every savings rate, we can find the underlying rate of time preference.

This problem has been investigated by Tinbergen.[1] He gives a number of equilibrium relations involving the rate of time discount, the savings rate, and the capital coefficient, each based on a specific hypothesis relating to the utility function. The utility function underlying the simplest problem is in his case a logarithmic one. It should, however, be noted that our problem here is the logical inverse to Tinbergen's problem. He is interested in finding out the optimum rate of savings corresponding to any given values of the capital-coefficient, and time preference. In our case, we want to know the underlying time preference, assuming that the savings rate is already an optimal one, other parameters remaining the same.

The Tinbergen result can be generalized by introducing more general types of production functions and utility functions other than the lobarithmic or hyperbolic ones considered by him. There is scope for further investigation along these lines.

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1. J. Tinbergen, "The Optimum Rate of Savings", Economic Journal, 1956.

b) We now consider the situation when all the sectors are not assumed to grow at the same proportionate rate, but all the relevant production functions have the needed convexity properties.

In this case, the relative prices and the interest rate are no longer constant. Further, since the rate of growth is not a unique number characterizing the entire process, we have to deal with constantly changing moving equilibria, as it were, and the relation in which the growth rate stands to the rate of interest would therefore be continually shifting. It appears then that we could say very little on the question without going the whole hog of solving a problem in dynamic programming. In principle, this is always possible in case (b). But to do that we have to specify first the appropriate terminal conditions, the initial stocks and the time profile of consumption over the entire period. Having done that, we have to apply the usual techniques of maximization over time. Such problems have been considered in the earlier paper entitled "A Complete Model of Program Evaluation." For a general reference, see Dorfman, Samuelson and Solow, Linear Programming and Economic Analysis, Chapter 12.

In practice, the whole procedure outlined above will be difficult to apply for at least some time to come. In the meantime, we may consider if there is any kind of approximation that we may try here. If we are concerned with a kind of over-all accuracy, this may be quite feasible.

Assume first a situation where all the sectors are growing at a proportionate rate of 'r' per cent. This is the situation discussed in (a). Now consider that one group of sectors is moving at the rate of $(r + \epsilon)$ per cent, as against the rest. The over-all rate of growth is given by the expression $(r + \lambda\epsilon)$ per cent. But since 'λ' is a variable magnitude indicating the proportion of total capital stock invested in the sectors growing at the rate of $(r + \epsilon)$ per cent, it appears therefore that $(r + \lambda\epsilon)$ represents an ever changing sequence of moving equilibria. Now we may ask ourselves how much error do we commit if we assume the whole system to be growing at the rate 'r' when in reality it is growing according to the rate $(r + \lambda\epsilon)$. Obviously, over a long period of time, the error would be very considerable indeed even though 'ϵ' is small.[1] But suppose we are interested only in a period of five to ten years, is it possible to say how large the error would be? The answer to this is 'yes', subject to an index number ambiguity that arises whenever the prices are changing at different rates. Leaving this complication out for the time being, we may calculate the difference $(\lambda\epsilon)$ in the following way:

$$\lambda(t) = \frac{K_1(t)}{K(t)}$$ where $K_1(t)$ is the capital stock of the sector growing at the rate $(r + \epsilon)$. $K(t)$ = total capital stock, assumed to grow at r per cent.

$$= \frac{K_1^o\{1 + (r + \epsilon)\}^t}{K^o\,(1 + r)^t}$$

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1. As a matter of fact, the system would asymptotically be growing at the rate of $(r + \epsilon)$ per cent, since it is the largest root that dominates. Thus the above discussion is meaningful only if we are interested in a small segment of time.

$$= \frac{\lambda^o\{1 + (r + \epsilon)\}^t}{(1 + r)^t}$$

$$= \frac{\lambda^o\left\{1 + t(r + \epsilon) + \frac{t(t-1)}{2!}(r + \epsilon)^2 + \ldots\right\}}{\left\{1 + tr + \frac{t(t-1)}{2!} r^2 + \ldots\right\}}$$

We may take a linear approximation, since terms $(r + \epsilon)^2$ will be of the second order of smalls. Then we have

$$\lambda(t) = \lambda^o\left\{1 + \frac{\epsilon t}{1 + r t}\right\}$$

$$\text{therefore: } \epsilon\lambda(t) = \epsilon\lambda^o + \frac{\lambda^o \epsilon^2 t}{1 + r t}$$

Now, if we consider the following numerical situations, we may get some idea of the error that we make when we take the over-all situation to be increasing at a steady rate of r per cent, while in reality it is not.

Assuming $\lambda^o = .20$, $\epsilon = .04$, $r = .04$ and $t = 5$, we find that

$$\epsilon\lambda(5) = 0.008 + \frac{.0016 \times r \times .2}{1.2} = .009$$

In other words, the error that we commit for the fifth year is of the order of .009 on the level of approximation we have chosen. The error for the whole period will be somewhat less, approximately, than half the above amount. If necessary, more precise relations for this purpose can be worked out. To put it simply, the above procedure understates the rate of growth by approximately 10 per cent. All this, of course, makes sense only if the relative prices are not altogether different.

The above example is in many ways an extreme example. We have assumed a very important segment of the economy to be growing twice as fast as the rest of the economy. In more realistic cases, the errors would be even less.

Thus, roughly speaking, over a small period of time we do not make a significant error when we assume the system to be growing at a steady rate, even though it is not exactly so. Once this is accepted, the Solow formula connecting the rate of growth with the rate of interest may be applied to give us an approximation to the shadow rate of interest.

In spite of its inaccurate nature, the approximation suggested above is very important because in the real world examples of strict balanced expansion are very rare. Thus the Solow formula we recommend will in this case give the lower limit to the rate of interest.

c) This is logically the most difficult case. We may consider the following sub-cases:

(i) Where the individuals production functions show only local nonconvexities, but they are convex in the large;

(ii) Where some of the individual production functions are non-convex throughout, but the aggregate production function is convex;

(iii) Where the relevant functions can be approximated by piecewise linear functions.

We may also consider an extreme case where the aggregate production function is also nonconvex. This, however, does not seem to be a realistic situation. In case (i), where nonconvexities are merely local, the shadow price device which consists in maximizing net present value with parametrically treated prices and interest rates still works. The reason of course is that the decision maker having some foresight will expand production till he reaches the convex segment. The case (ii) deserves some special consideration. In this case, since individual sectors have nonconvex production functions, the parametization device breaks down even though the over-all maximization process is a determinate one. This means that the coordinated decision making of the central planner, which maximizes a preference function taking into account all the interdependencies, will yield an optimal pattern of investment which, however, cannot be built up from piecemeal choices, each being profitable on given interest rates and prices. Thus investment in sectors like social overhead capital will either not be made or, if made, they will be made on an insufficient scale. Thus the use of the shadow price criterion breaks down for this problem. In case (iii) the procedure works provided we have knowledge about the nodal points. What we do is to use a succession of interest rates, corresponding to the succession of linear facets. In empirical work, this may be a useful simplification.

But even in case (ii), the choice of alternative techniques for a specified time shape of output will involve a minimization problem that should employ the shadow rates for primary factors rather than the observed market rates.[1] We shall discuss this aspect of the question in greater detail in the following section.

Section IV

In this section we consider the method of calculating priorities in an investment program by using shadow prices. We must bear in mind that while we calculate the benefit-cost ratios for a single project, we do it as of a given program, and not for the project in isolation. This follows out of the fact that the projects are necessarily interlinked, and imply certain assumptions about the rest of the economy. Thus one project may be chosen from a set of competing projects, if the rest of the programs may be assumed to be relatively unaffected by this choice.

We may also consider a more generalized situation where there is a technically nonseparable collection of projects which can be singled out for piecemeal decision making. Now in this case this whole collection has to be treated as one unit and the benefit-cost calculations have to be calculated for this one unit as a whole.

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1. K. J. Arrow and A. C. Enthoven discuss the possibilities of extending the theorem on 'efficient ' production to situations where the production functions show 'quasi-concavity', ("Quasi, Concave Programming," The Rand Corporation, p. 1847.) Quasi-concavity is defined as the situation where increasing returns prevail to scale, but there are diminishing returns to each particular input. Their statement (p. 30) that under these conditions, efficient combinations of inputs may be determined, given preassigned output and factor prices, although the device of profit maximization at parametrically treated prices breaks down which agrees with our observations on page 19.

The word 'technical nonseparability' is important in this connection. For if the relative weights of the different components are variable depending on economic calculations, there is an unavoidable element of a jigsaw puzzle involved that cannot be solved by the shadow price device if the assumption of linear homogeneity is abandoned.

The advantage of the shadow price technique becomes considerably greater if the complex of planning problems may be assumed to be decomposable into the following stages:

a) How much to invest in total over a number of years;

b) How to distribute the total investment resources among different sectors of the economy;

c) How to choose the best method of utilizing the resources allocated to a sector.

If the stages are strictly consecutive, we may think that the decision on level (b) is reached on the basis of maximizing income over a period of time subject to all the interdependences in production, investment and consumption. This would roughly indicate how much to invest in each sector. If there are sectors like social overhead capital where investment is made on grounds independent of any maximization process, then we should consider the remaining sub-set of sectors for our decision purposes.

The decision on stage (c) can be reached on the basis of utilizing a shadow rate of interest and for a given time profile of production, on the requirement that the costs are minimized.

In theory as well as practice, the stages may not be that distinct, in which case decisions on (b) and (c) may have to be reached simultaneously. The shadow rate technique should then be replaced by the general methods of dynamic programming.

Now let us consider the problem quantitatively. We use the following notations:

$W_i(t)$ - The investment in the project per unit time.

$F_i(t)$ - The foreign component of investment per unit time. $F_i = aW_i$ where $0 \leq a \leq 1$.

g - The length of the gestation period.

n - The length of the operating period.

r - The shadow rate of interest.

k - The shadow rate of exchange.

D(t) - The current operating expenses of a project.

Then the cost of a project may be calculated as follows:

We have $F_i = aW_i$

Therefore $H_i = (1-a) W_i$ where H_i is the domestic component of investment. Since we value the foreign investment component at the shadow exchange rate, we have:

$$kaW_i + (1 - a)W_i = W_i (ka + 1 - a)$$

$$= W_i \left\{1 - a (1 - k)\right\}$$

Let us assume that we know the timeshape on construction effort: W(t). Then the cost of investment in the project may be calculated as:

$$C = \sum_{t=g}^{0} W(t)\left\{1 - a(1 - k)\right\} (1 + r)^{-t} + \sum_{0}^{n} D(t) \ (1 + r)^{-t}$$

The first term on the left-hand side indicates the investment that is made during the gestation period of the project and the second part indicates the cost that is incurred during the exploitation period. Now the decision rule consists in minimizing "C" for a given time profile of 'output.' To put it differently the projects to be compared are those which give the same time profile of output, given by the over-all planning problem. Out of these projects, the one will be chosen which minimizes total cost, over the combined gestation and exploitation period of the project.

Section V

In this section we may briefly review the conclusions reached in the earlier sections and indicate the relevance of the shadow price concept with respect to a few practical problems encountered in Indian planning.

Briefly stated, our discussion has clearly indicated that the technique of using shadow prices serves as a useful computational shorthand in devising a relatively "efficient" system of program evaluation. The qualification on "efficiency" arises because in the presence of nonconvexities in the production processes of certain sectors, the shadow price device does not enable one to reach the "efficient" constellation of the system. This holds good even though the shadow prices we use are not exact, but merely approximations, although it is important that they should be in the right direction. Given the data, the calculation of the shadow rate of exchange does not raise great difficulties. The simplified procedure indicated in this paper, or the more elaborate linear programming method discussed by Chenery may be usefully employed. With respect to the shadow rate of interest, the conceptual difficulties are greater. But if we use the approximation procedure outlined earlier in this paper, we get a range of 8 per cent to 12 per cent for the shadow rate of interest under Indian conditions. The exact shadow rate of interest may be higher than this, but it is unlikely that this would be lower than given by this range. This already gives us a basis for how to judge projects which are economic only if the rate of interest is 4 per cent or 4½ per cent.

The relevance of the shadow prices to practical problems may be understood if we take into account the problem of choosing between importing fertilizer, or setting up a fertilizer plant, or a machinery for manufacturing fertilizer producing equipment. In the simple Austrian models, where choice is confined to a pair of alternatives, the cost of one is the opportunity foregone with the other projects. This is difficult to apply if there exists a manifold of possibilities for each unit of investment. Under such conditions, the opportunity cost of a unit of investment is measured by its shadow rate of interest. Similarly, the cost of a unit of import should be valued at the shadow rate of exchange, rather than at the official rate. Now, if we take, for example, a shadow rate of exchange of Rs. 6 to a dollar and a rate of interest lying between 8 per cent and 12 per cent, we may calculate the cost of each type of project, over the gestation period, given the time shape of the construction effort. Further, with a given time profile of 'output,' in this case agricultural production, we can calculate the total costs for each project, e.g. investment costs and operating costs. Naturally, with other things remaining the same, the project with the lowest cost should be chosen.

The same line of reasoning may be applied to other problems such as the choice between various types of power stations. An interesting contribution in this regard is the paper of Professor P. N. Rosenstein-Rodan on the contribution of atomic energy to India's development program.[1]

All this is to suggest the fruitfulness of the shadow price method in practical policy making.

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1. P. N. Rosenstein-Rodan, Contribution of Atomic Energy to a Power Program, C/59-15.

Appendix 1: The Shadow Rate of Exchange: The General Case.

This appendix deals with the case of how to determine the shadow rate of exchange where imports consist of different types of goods.

The price of each domestic commodity in domestic currency is given by the following equation:

$$p_i = k\,(A_{n+1,i}\,p_{n+1} + A_{n+2,i}\,p_{m_{n+2}} + \ldots + A_{n+j,i}\,p_{m_{n+j}})\,(i = 1, 2 \ldots n)$$

+ contribution of other primary factors.

Here $A_{n+1,i}$ is the cumulative coefficient of the first import commodity in the production of i^{th} domestic commodity. We have 'n' such equations for 'n' domestic commodities.

In addition we have the equation relating to the permissible balance of payments deficit:

$$C = k\Big[\Big\{(p_{n+j})' [v_1][I-a]^{-1}\Big\}\,(e + w + c) + (p_{n+j})' [v_2]\{w\} + (p_{n+j})'\{v_3\,(c, \{p_{n+j}\}', \{p_i\}')\}\Big] - (p)'\,(e)$$

Thus we have (n + 1) equations to determine (n + 1) prices, 'n' domestic prices and one shadow rate of exchange.

The dimensionalities of above matrices and column vectors are as follows:

(i) $(p_{n+j})'$ is a row vector of the dimension (1 x j).

(ii) $[v_1]$ is a matrix of dimensions (j x n).

(iii) $[I-a]^{-1}$ is a matrix of dimension of (n x n). Thus the product has dimension (1 x n), hence a row vector.

(iv) (e + w + c) is a column vector of dimensions (n x 1). Thus the first term in brackets is a scalar, indicating the total amount spent on imports of raw materials.

(v) $[v_2]$ is a matrix of dimensions (j x n).

(vi) $\{w\}$ is a column vector of dimensions (n x 1).

(vii) The second term in brackets is (1 x 1), also a scalar, indicating the amount spent on imports of investment goods.

(viii) v_3 $(c, p_{n+j})'$, $(p_i)'$ is a column vector of dimensions (j x 1).

The third term is also a scalar, indicating the amount spent on imports of consumer goods.

(ix) $(p)'$ (e) is also a scalar since (p') is (1 x n) and (e) is (n x 1).

In this case, exports have been exogenously determined. We may also consider the more general case, where exports are determined from within the above set of calculations. This, however, requires a more complicated approach.