14.581 MIT PhD International Trade
— Lecture 17: Trade and Growth (Theory) —

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Spring 2011

Open economy versions of canonical growth models:

- Neoclassical growth model

- Learning-by-doing models

- Endogenous growth models

Overview

- We will consider three types of growth models:
  1. Neoclassical growth model [Factor accumulation]
  2. Learning-by-doing models [Accidental technological progress]
  3. Endogenous growth models [Profit-motivated technological progress]

- **Questions:**
  1. How does openness to trade affect predictions of closed-economy growth models?
  2. Does openness to trade have positive or negative effects on growth?

- **Theoretical Answer:**

  It depends on the details of the model...

Today's Plan

Open economy versions of canonical growth models:

- **Neoclassical growth model**

- Learning-by-doing models

- Endogenous growth models

- In a closed economy, neoclassical growth model predicts that:

  - If there are diminishing marginal returns to capital, then different capital labor ratios across countries lead to different growth rates along transition path.

  - If there are constant marginal returns to capital (AK model), then different discount factors across countries lead to different growth rates in steady state.

- In an open economy, both predictions can be overturned.

- For simplicity, we will assume throughout this lecture that:
  - No population growth: $l(t) = 1$ for all t.
  - No depreciation of capital.

- Representative household at $t = 0$ has log-preferences

$$U = \int_0^{+\infty} \exp\left(-\rho t\right) \ln c\left(t\right) dt \tag{1}$$

- Final consumption good is produced according to

$$y(t) = aF\left(k\left(t\right), l\left(t\right)\right) = af\left(k\left(t\right)\right)$$

  where output (per capita) f satisfies:

$$f' > 0 \text{ and } f'' \leq 0$$

Neoclassical Growth Model
Perfect competition, law of motion for capital, and no Ponzi condition

- Firms maximize profits taking factor prices $w(t)$ and $r(t)$ as given:

$$r(t) = af'(k(t)) \tag{2}$$
$$w(t) = af(k(t)) - k(t)af'(k(t)) \tag{3}$$

- Law of motion for capital is given by

$$\dot{k}(t) = r(t)k(t) + w(t) - c(t) \tag{4}$$

- No Ponzi-condition:

$$\lim_{t \to +\infty} \left[k(t) \exp\left(-\int_0^t r(s)\,ds \right) \right] \geq 0 \tag{5}$$

- **Definition** *Competitive equilibrium of neoclassical growth model consists in (c, k, r, w) such that representative household maximizes (1) subject to (4) and (5) and factor prices satisfy (2) and (3).*

- **Proposition 1** *In any competitive equilibrium, consumption and capital follow the laws of motion given by*

$$\begin{aligned}
\frac{\dot{c}(t)}{c(t)} &= af'(k(t)) - \rho \\
\dot{k}(t) &= f(k(t)) - c(t)
\end{aligned}$$

- Suppose first that $f'' < 0$.

- In this case, Proposition 1 implies that:

  - Growth rates of consumption is decreasing with k.

  - There is no long-run growth without exogenous technological progress.

  - Starting from $k(0) > 0$, there exists a unique equilibrium converging monotonically to (c^*, k^*) such that

$$af'(k^*) = \rho$$
$$c^* = f(k^*)$$

- Now suppose that $f'' = 0$. This corresponds to

$$af(k) = ak$$

- In this case, Proposition 1 implies the existence of a unique equilibrium path in which c and k all grow at the same rate

$$g^* = a - \rho$$

- We will now illustrate how trade integration—through its effects on factor prices—may transform a model with diminishing marginal returns into an AK model and vice versa

- Neoclassical growth model with multiple countries indexed by j
  - No differences in population size: $l_j(t) = 1$ for all j
  - No differences in discount rates: $\rho_j = \rho$ for all j
  - *Diminishing marginal returns: $f'' < 0$*
- Capital and labor *services* are freely traded across countries
  - No trade in assets, so trade is balanced period by period.
- **Notation:**
  - $x_j^l(t)$, $x_j^k(t) \equiv$ labor and capital services used in production of final good in country j

$$y_j(t) = aF\left(x_j^k(t), x_j^l(t)\right) = ax_j^l(t)\, f\left(x_j^k(t)/x_j^l(t)\right)$$

  - $l_j(t) - x_j^l(t)$ and $k_j(t) - x_j^l(t) \equiv$ net exports of factor services

- Free trade equilibrium reproduces the integrated equilibrium.

- In each period:

  1. Free trade in factor services implies FPE:

  $$
  \begin{aligned}
  r_j(t) &= r(t) \\
  w_j(t) &= w(t)
  \end{aligned}
  $$

  2. FPE further implies identical capital-labor ratios:

  $$
  \frac{x_j^k(t)}{x_j^l(t)} = \frac{x^k(t)}{x^l(t)} = \frac{\sum_j k_j(t)}{\sum_j l_j(t)} = \frac{k^w(t)}{l^w(t)}
  $$

- Like in static HO model, countries with $k_j(t)/l_j(t) > k^w(t)/l^w(t)$ export capital and import labor services.

- Let $c(t) \equiv \sum_j c_j(t) / l^w(t)$ and $k(t) \equiv \sum_j k_j(t) / l^w(t)$
- Not surprisingly, world consumption and capital per capita satisfy

$$\frac{\dot{c}(t)}{c(t)} = af'(k(t)) - \rho$$
$$\dot{k}(t) = f(k(t)) - c(t)$$

- For each country, however, we have

$$\frac{\dot{c}_j(t)}{c_j(t)} = af'(k(t)) - \rho \qquad (6)$$
$$\dot{k}_j(t) = f'(k(t)) k_j(t) - c_j(t) \qquad (7)$$

- If $k(t)$ is fixed, Equations (6) and (7) imply that everything is *as if* countries were facing an *AK* technology.

- Ventura (1997) hence shows that trade may help countries avoid the curse of diminishing marginal returns:
  - As long as country j is "small" relative to the rest of the world, $k_j(t) \ll k(t)$, the return to capital is independent of $k_j(t)$.
  - This is really just an application of the 'factor price insensitivity' result we saw when we studied the small open economy (or partial equilibrium version of a large economy) H-O model.

- This insight may help explain growth miracles in East Asia:
  - Asian economies, which were more open than many developing countries, accumulated capital more rapidly but without rising interest rates or diminishing returns.
  - These economies were also heavily industrializing along their development path. H-O mechanism requires this. Country accumulates capital and shifts into capital-intensive goods, exporting that which is in excess supply.

- Now we go in the opposite direction.

- *AK* model with multiple countries indexed by j.
  - No differences in population size: $l_j(t) = 1$ for all j.
  - Constant marginal returns: $f'' = 0$.

- Like in an "Armington" model, capital services are differentiated by country of origin.

- Capital services are freely traded and combined into a unique final good—either for consumption or investment—according to:

$$c_j(t) = \left[\sum_{j'} x_{jj'}^c(t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

$$i_j(t) = \left[\sum_{j'} x_{jj'}^i(t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- **Lemma** *In each period, $c_j(t) = \rho_j k_j(t)$.*
- **Proof:**
  - Euler equation implies:

  $$\frac{\dot{c}_j(t)}{c_j(t)} = r_j(t) - \rho_j.$$

  - Budget constraint at time t requires:

  $$\dot{k}_j(t) = r_j(t) k_j(t) - c_j(t).$$

  - Combining these two expressions, we obtain:

  $$\left[k_j(t) / c_j(t)\right]^{\cdot} = \rho_j \left[k_j(t) / c_j(t)\right] - 1.$$

  - 3 + no-Ponzi condition implies:

  $$k_j(t) / c_j(t) = 1/\rho_j.$$

- **Proposition 2** *In steady-state equilibrium, we must have:*

$$\frac{\dot{k}_j(t)}{k_j(t)} = \frac{\dot{c}_j(t)}{c_j(t)} = g^*.$$

- **Proof:**
  - In steady state, by definition, we have $r_j(t) = r_j^*$.
  - Lemma + Euler equation $\Rightarrow \frac{\dot{k}_j(t)}{k_j(t)} = r_j(t) - \rho_j$.
  - 1 + 2 $\Rightarrow \frac{\dot{k}_j(t)}{k_j(t)} = g_j^*$.
  - Market clearing implies:

    $$r_j(t) k_j(t) = r_j^{1-\sigma}(t) \sum_{j'} r_{j'}(t) k_{j'}(t), \text{ for all } j.$$

  - Differentiating the previous expression, we get $g_j^* = g^*$.
  - 5 + Lemma $\Rightarrow \frac{\dot{c}_j(t)}{c_j(t)} = g^*$.

Acemoglu and Ventura (2002)
Summary

- Under autarky, AK model predicts that countries with different discount rates ρ_j should grow at different rates.

- Under free trade, Proposition 2 shows that all countries grow at the same rate.

- Because of terms of trade effects, everything is *as if* we were back to a model with diminishing marginal returns.

- From a theoretical standpoint, Acemoglu and Ventura (2002) is the mirror image of Ventura (1997)

Today's Plan

Open economy versions of canonical growth models:

- Neoclassical growth model

- **Learning-by-doing models**

- Endogenous growth models

- In neoclassical growth models, technology is exogenously given.
    - So trade may only affect growth rates through factor accumulation.

- **Question:**
  How may trade affect growth rates through technological changes?

- **Learning-by-doing models:**
    - Technological progress $\equiv$ 'accidental' by-product of production activities.
    - So, patterns of specialization also affect TFP growth.

Learning-by-Doing Models

- Consider an economy with two intermediate goods, $i = 1, 2$, and one factor of production, labor ($l_j = 1$).

- Intermediate goods are aggregated into a unique final good:

$$y_j(t) = \left[y_j^1(t)^{\frac{\sigma-1}{\sigma}} + y_j^2(t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \ \sigma > 1.$$

- Intermediate goods are produced according to:

$$y_j^i(t) = a_j^i(t) \, l_j^i(t).$$

- Knowledge spillovers are sector-and-country specific:

$$\frac{\dot{a}_j^i(t)}{a_j^i(t)} = \eta^i l_j^i(t). \tag{8}$$

- For simplicity, there are no knowledge spillovers in sector 2: $\eta^2 = 0$.

- Incomplete specialization (which we assume under autarky) requires:

$$\frac{p_j^1(t)}{p_j^2(t)} = \frac{a_j^2(t)}{a_j^1(t)} \tag{9}$$

- Profit maximization by final good producers requires:

$$\frac{y_j^1(t)}{y_j^2(t)} = \left(\frac{p_j^1(t)}{p_j^2(t)}\right)^{-\sigma} \tag{10}$$

- Finally, labor market clearing implies:

$$\frac{y_j^1(t)}{y_j^2(t)} = \frac{a_j^1(t)\, l_j^1(t)}{a_j^2(t)\left(1 - l_j^1(t)\right)} \tag{11}$$

- **Proposition** *Under autarky, the allocation of labor and growth rates satisfy* $\lim_{t \to +\infty} l_j^1(t) = 1$ *and* $\lim_{t \to +\infty} \frac{\dot{y}_j(t)}{y_j(t)} = \eta^1$.

- **Proof:**

  1. Equations (9)-(11) imply:

     $$\frac{l_j^1(t)}{1 - l_j^1(t)} = \left(\frac{a_j^2(t)}{a_j^1(t)} \right)^{1-\sigma}.$$

  2. With incomplete specialization at every date, Equation (8) implies:

     $$\lim_{t \to +\infty} \left(\frac{a_j^2(t)}{a_j^1(t)} \right) = 0.$$

  3. $1 + 2 \Rightarrow \lim_{t \to +\infty} l_j^1(t) = 1$.

  4. $3 \Rightarrow \lim_{t \to +\infty} y_j(t) = a_j^1(t) \Rightarrow \lim_{t \to +\infty} \frac{\dot{y}_j(t)}{y_j(t)} = \eta^1$.

- Suppose that country 1 has CA in good 1 at date 0:

$$\frac{a_1^1(0)}{a_1^2(0)} > \frac{a_2^1(0)}{a_2^2(0)}. \tag{12}$$

- **Proposition** *Under free trade*, $\lim_{t \to +\infty} y_1(t)/y_2(t) = +\infty$.
- **Proof:**
  - Equation (8) and Inequality (12) imply:

    $$\frac{a_1^1(t)}{a_1^2(t)} > \frac{a_2^1(t)}{a_2^2(t)} \text{ for all } t.$$

  - $1 \Rightarrow l_1^1(t) = 1$ and $l_2^1(t) = 0$ for all t.
  - $2 \Rightarrow y_1(t)/y_2(t) = a_1^1(t)/a_2^2(t)$.
  - $3 + \lim_{t \to +\infty} a_j^1(t) = +\infty \Rightarrow \lim_{t \to +\infty} y_1(t)/y_2(t) = +\infty$.

- World still grows at rate η^1, but small country does not.

- Learning-by-doing models illustrate how trade may hinder growth if you specialize in the "wrong" sector.
  - This is an old argument in favor of trade protection (see e.g. Graham 1923, Ethier 1982).

- Country-specific spillovers tend to generate "locked in" effects.
  - If a country has CA in good 1 at some date t, then it has CA in this good at all subsequent dates.

- History matters in learning-by-doing models:
  - Short-run policy may have long-run effects (Krugman 1987).

Open economy versions of canonical growth models:

- Neoclassical growth model

- Learning-by-doing models

- **Endogenous growth models**

- In endogenous growth models, technological progress results from deliberate investment in R&D.

- In this case, economic integration may affect growth rates by changing incentives to invest in R&D through:
  - Knowledge spillovers.
  - Market size effect.
  - Competition effect.

- Two canonical endogenous growth models are:
  - Expanding Variety Model: Romer (1990).
  - Quality-Ladder Model: Grossman and Helpman (1991) and Aghion and Howitt (1992).

- We will focus on expanding variety model

- Labor is the only factor of production $(l = 1)$.
- Final good is produced under perfect competition according to:

$$c(t) = \left(\int_0^{n(t)} x(\omega, t)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}}, \, \sigma > 1.$$

- Inputs ω are produced under monopolistic competition according to:

$$x(\omega, t) = l(\omega, t).$$

- New inputs can be invented with the production function given by:

$$\frac{\dot{n}(t)}{n(t)} = \eta l^r(t). \tag{13}$$

- Similar to learning-by-doing model, but applied to innovation.

- Euler equation implies:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho. \tag{14}$$

- Monopolistic competition implies:

$$p(\omega, t) = \frac{\sigma w(t)}{\sigma - 1}.$$

- Accordingly, instantaneous profits are equal to:

$$\pi(\omega, t) = [p(\omega, t) - w(t)] l(\omega, t) = \frac{1}{\sigma - 1} \frac{w(t) l^e(t)}{n(t)}. \tag{15}$$

  where $l^e(t) \equiv \int_0^{n(t)} l(\omega, t) \, d\omega$ is total employment in production

- Because of symmetry, we drop index ω from now on.

Expanding Variety Model
Closed economy

- The value of a typical input producer at date t is:

$$v(t) = \int_t^{+\infty} \exp\left(-\int_t^s r(s')ds'\right) \pi(s)\, ds.$$

- Asset market equilibrium requires:

$$r(t)\, v(t) = \pi(t) + \dot{v}(t). \tag{16}$$

- Free entry of input producers requires:

$$\eta n(t) v(t) = w(t). \tag{17}$$

- Finally, labor market clearing requires:

$$l^r(t) + l^e(t) = 1. \tag{18}$$

- **Proposition** *In BGP equilibrium, aggregate consumption grows at a constant rate $g^* \equiv \frac{\eta - (\sigma - 1)\rho}{\sigma(\sigma - 1)}$.*

- **Proof:**
  1. In BGP equilibrium: $r(t) = r^*$, $l^e(t) = l^{e*}$, and $l^r(t) = l^{r*}$.
  2. From Euler equation, (14), we know that $g^* \equiv \frac{\dot{c}(t)}{c(t)} = r^* - \rho$.
  3. From asset market clearing, (16), we also know that

$$r^* = \frac{\pi(t)}{v(t)} + \frac{\dot{v}(t)}{v(t)} = \frac{\eta(1 - l^{r*})}{\sigma - 1} + \frac{\dot{w}(t)}{w(t)} - \frac{\dot{n}(t)}{n(t)}$$

     where the second equality derives from (15), (17), and (18).
  4. By our choice of numeraire, $\frac{\dot{w}(t)}{w(t)} = \frac{\dot{c}(t)}{c(t)} = g^*$. Thus 3 + (13) imply:

$$r^* = \frac{\eta(1 - l^{r*})}{\sigma - 1} + g^* - \eta l^{r*}.$$

  5. Using 2 and 4, we can solve for l^{r*}, and in turn, r^* and g^*.

- In expanding variety model, aggregate consumption is given by:

$$c\left(t\right) = n^{\frac{\sigma}{\sigma-1}}\left(t\right) x\left(t\right) = n^{\frac{1}{\sigma-1}}\left(t\right) l^{e}\left(t\right).$$
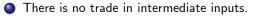
- In BGP equilibrium, we therefore have:

$$\frac{\dot{c}(t)}{c\left(t\right)} = \left(\frac{1}{\sigma - 1}\right) \times \left(\frac{\dot{n}(t)}{n\left(t\right)}\right).$$

- Predictions regarding $\dot{n}(t)/n\left(t\right)$, of course, rely heavily on innovation PPF. If $\dot{n}(t)/n\left(t\right) = \eta\phi\left(n\left(t\right)\right) l^{r}\left(t\right)$, then:
  - $\lim_{n\to+\infty} \phi\left(n\right) = +\infty \Rightarrow$ unbounded long-run growth.
  - $\lim_{n\to+\infty} \phi\left(n\right) = 0 \Rightarrow$ no long-run growth.

- Now suppose that there are two countries indexed by $j = 1, 2$.

- In order to distinguish the effects of trade from those of technological diffusion, we start from a situation in which:

  - There is no trade in intermediate inputs.

  - There are knowledge spillovers across countries:

  $$\frac{\dot{n}_j(t)}{n_j(t) + \Psi n_{-j}(t)} = \eta l_j^r(t)$$

  where $1 - \Psi \in [0, 1] \equiv$ share of inputs produced in both countries.

- Because of knowledge spillovers across countries, it is easy to show that growth rate is now given by

$$g_j^* = \frac{\eta(1 + \Psi) - (\sigma - 1)\rho}{\sigma(\sigma - 1)} > g_{autarky}^*$$

- **Question:**
  What happens when two countries start trading intermediate inputs?

- **Answer:**

  1. Trade eliminates redundancy in R&D ($\Psi \to 1$), which $\nearrow$ growth rates. Producers now have incentive to not duplicate effort.

  2. However, trade has *no further effect* on growth rates.

- Intuitively, when the two countries start trading:

  1. Spending $\nearrow$, which $\nearrow$ profits, and so, incentives to invest in R&D.

  2. But competition from Foreign suppliers $\searrow$ CES price index, which $\searrow$ profits, and so, incentives to invest in R&D.

  3. With CES preferences, 1 and 2 exactly cancel out.

- This neutrality result heavily relies on CES (related to predictions on number of varieties per country in Krugman 1980).

- Not hard to design endogenous growth models in which trade has a positive impact on growth rates (beyond R&D redundancy):

  - Start from same expanding variety model, but drop CES, and assume

    $$c(t) = n^{\alpha} \left(\int_0^{n(t)} x(\omega, t)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

    If $\alpha > 0$, market size effect dominates. (If $\alpha < 0$, it's the contrary.)

  - Start from a lab-equipment model in which final good rather than labor is used to produce new inputs.

Concluding Remarks

- Previous models suggest that trade integration may have a profound impact on the predictions of closed-economy growth models.

  - But they do not suggest a systematic relationship between trade integration and growth.

- *Ultimately, whether trade has positive or negative effects on growth is an empirical question.*

- In this lecture, we have abstracted from issues related to firm-level heterogeneity and growth (e.g. learning by exporting, technology adoption at the firm-level).

  - For more on these issues, see, eg, Atkeson and Burstein (2010), Bustos (2010), and Constantini and Melitz (2007).

14.581 International Economics I

Spring 2011