

14.581 Problem Set 4:  
Gravity Models, and the Estimation of Trade  
Costs

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Complete all questions (for a total of 100 marks). Due by May 2, 2011.

1. (15 marks) This question and the two that follow ask you to work through some of the results in Arkolakis, Costinot and Rodriguez-Clare (2011), henceforth ACRC. Consider first the Armington model. There are  $I$  countries  $i$ , each with fixed labor endowment  $L_i$ . Each good is produced with a production function  $Y_i = T_i L_i$ , where  $Y_i$  is the amount of output produced by country  $i$ . A large number of perfectly competitive firms in country  $i$  have access to this technology. Make the ‘Armington assumption’ on country technologies, which is to say that there are  $I$  different goods in the world and the only good that country  $i$  can make is ‘good  $i$ ’. Suppose all consumers in the world have the same preferences, which are CES preferences over each good with elasticity of substitution between any two goods equal to  $\sigma$ . Variable (iceberg) trade costs between any two country are  $\tau_{ij} \geq 1$  and  $\tau_{ii} = 1$ . Assume trade balance.
  - (a) Write down the ‘import demand system’ in this economy, ie  $\frac{\partial \ln(X_{ij}/X_{jj})}{\partial \ln \tau_{ik}}$ . Does it satisfy ‘R3’ in ACRC? Can you think of another assumption about preferences around the world that would satisfy R3?
  - (b) Describe the best possible empirical paper you could write that would test R3.
  - (c) Now consider any arbitrarily large change in trade costs around the world (except that domestic trade costs,  $\tau_{ii}$ , do not change). Show that the proportional change in welfare of consumers in country  $j$  can be written as  $\widehat{W}_j = \widehat{\lambda}_{jj}^{1/(1-\sigma)}$ , with  $\lambda_{ij} \equiv X_{ij}/X_j$ , and where we use the notation that for any variable  $v$ ,  $\widehat{v} \equiv v'/v$ , where  $v$  is the starting value and  $v'$  is the end value. Explain the intuition for this result as well as the intuition behind any intermediate steps you use in its derivation.
2. (25 marks) Now consider a more general Ricardian model than the particularly stark Ricardian model (the Armington model) assumed in Question

1 above. There is still one factor, labor. Now there is a fixed set of goods indexed by  $\omega$ , of measure  $N$ . All consumers have CES preferences (with elasticity of substitution  $\sigma$ ) over these goods. Country  $i$  requires  $\alpha_i(\omega)$  units of labor to produce one unit of good  $\omega$ . Assume there are many potential producers of each good  $\omega$  in each country  $i$ ; hence there is perfect competition. Let  $G(\alpha_1, \dots, \alpha_n)$  denote the share of goods  $\omega$  such that  $\alpha_i(\omega) \leq \alpha_i$  for all  $i$ , and let  $g(\alpha_1, \dots, \alpha_n)$  denote its associated density function.

- (a) Derive an expression for aggregate exports from country  $i$  to country  $j$  (denoted  $X_{ij}$ ) as a function of  $c_{ij} \equiv w_i \tau_{ij}$  and the function  $g_i(\alpha_i, c_{1j}, \dots, c_{nj})$  which is the density of goods with unit labor requirements  $\alpha_i$  in country  $i$  such that country  $i$  is the lowest cost supplier of these goods to country  $j$ .
- (b) Hence derive an expression for the import demand system in this model (ie  $\frac{\partial \ln(X_{ij}/X_{jj})}{\partial \ln \tau_{ik}}$ ). Feel free to use the notation

$$\gamma_{ij}^{i'} \equiv \partial \ln \left[ \int_0^\infty \alpha_i^{1-\sigma} g_i(\alpha_i, c_{1j}, \dots, c_{nj}) d\alpha_i \right] \partial \ln c_{ij}, \quad (1)$$

but be sure to explain and interpret this term if you do so.

- (c) Does the import demand system in this model necessarily satisfy R3 in ACRC? Does R3 imply perfect specialization? Does this model necessarily imply perfect specialization? Explain what would have to be true in this model if it were to satisfy R3 (be sure to explain both the math and the economics).
  - (d) Consider an arbitrarily large change in trade costs around the world (except that domestic trade costs,  $\tau_{ii}$ , do not change). Show that R3 in ACRC implies that  $\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}$  for some constant,  $\varepsilon$ .
  - (e) Now suppose that the particular Ricardian model we are working with is that in Eaton and Kortum (2002). Derive the density  $g(\alpha_1, \dots, \alpha_n)$  in this model. Hence show that this model satisfies R3. Explain why R3 is satisfied in both the Armington and Eaton and Kortum (2002) models. Can you explain what feature of the Frchet distribution in Eaton and Kortum (2002) allows the model to satisfy R3? Can you think of another distribution that would allow the model to satisfy R3 in general?
  - (f) Discuss the extent to which the Armington model and the Eaton and Kortum (2002) model are ‘isomorphic’ with respect to one another.
3. (20 marks) Finally, consider a similar model to that in Question 2 but where we now assume monopolistic competition. Again, there is one factor, labor. There is an infinitely large number of goods  $\omega$  that could potentially be produced. All consumers have CES preferences (with elasticity of substitution  $\sigma$ ) over these goods. The cost of a firm in country  $i$  producing  $q(\omega)_{ij}$  units of good  $\omega$  and selling them in country  $j$  is given

by:  $\alpha_i(\omega)q(\omega)_{ij}\tau_{ij}w_i + f_{ij}w_i^\mu w_j^{1-\mu}$ , where  $w_k$  is the wage in any country  $k$ . Once a firm starts producing a good  $\omega$  it obtains monopoly rights over that good, but otherwise there are no barriers to entry. Let  $G(\alpha_1, \dots, \alpha_n)$  again denote the share of goods  $\omega$  such that  $\alpha_i(\omega) \leq \alpha_i$  for all  $i$ , and let  $g(\alpha_1, \dots, \alpha_n)$  denote its associated density function.

- (a) Derive an expression for  $X_{ij}$  as a function of  $c_{ij}$ , the total number of varieties made by country  $i$  (denoted  $N_i$ ), and  $g_i(\alpha_i)$  which is the marginal distribution of  $\alpha_i$ .
  - (b) Hence derive an expression for the import demand system in this model. Feel free to use the notation  $\gamma_{ij} \equiv d \ln \int_0^{\alpha_{ij}^*} \alpha^{1-\sigma} g_i(\alpha) d\alpha / d \ln \alpha_{ij}^*$  but again explain and interpret this term if you do so.
  - (c) Does the IDS necessarily satisfy R3 in ACRC? Explain what would have to be true in this model if it were to satisfy R3 (be sure to explain both the math and the economics).
  - (d) Consider a small change in trade costs around the world (except that domestic trade costs,  $\tau_{ii}$ , do not change). Derive an expression for the resulting change in welfare in country  $j$ ,  $W_j$ . Explain in what respects this expression is similar to, and different from, that in the above perfectly competitive case in Questions 1 and 2 above.
  - (e) Is R3 sufficient to guarantee that, for an arbitrarily large change in trade costs around the world,  $\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}$  for some constant,  $\varepsilon$ ? If not, what other restrictions would guarantee this result?
  - (f) Suggest two prominent restrictions on  $g_i(\alpha_i)$  that would ensure that R3 is satisfied.
  - (g) Can you explain why Frechet-distributed productivities ensures R3 under perfect competition but not under monopolistic competition?
4. (20 marks) A large literature, surveyed in Anderson and van Wincoop (JEL 2004), uses estimates from the gravity model of trade to shed light on the nature of trade costs.
- (a) Explain this methodology precisely along with the assumptions that authors make when using it.
  - (b) Under these maintained assumptions from part (a) above: Are trade costs identified by this methodology? Are the effects of observable determinants of trade costs identified? Are the relative effects of observable determinants of trade costs (ie which determinants impede trade relatively more) identified?
  - (c) Write down a form of taste differences across countries that would not be separately identified from trade costs in the model developed by these authors.
  - (d) Do you believe the estimates that emerge from these studies? If not, explain which of the methodology's maintained assumptions you

find most troublesome. Can you suggest an empirical test for your confounding story?

- (e) Rossi-Hansberg (AER, 2004, “A Spatial Theory of Trade”) outlines one explanation for a ‘border effect’ in the data when one does not exist in the trade costs function. Explain the argument here and the intuition behind it.
  - (f) Discuss the implications of the findings in Bronnenberg, Dhar and Dube (JPE, 2009) for the existence, nature, and estimation of trade costs.
  - (g) How is it that empirical researchers employing this methodology are able to avoid bias due to general equilibrium ‘spillovers’ across their units of observation (ie the fact that export flow  $X_{ij}$  from country  $i$  to country  $j$  is likely to depend on both trade costs within this diad,  $\tau_{ij}$ , and on trade costs within any other diad,  $\tau_{lm}$ )? Which assumptions enable this?
  - (h) Is there any evidence from Anderson and van Wincoop (AER 2003) that these general equilibrium spillovers actually matter (ie that failing to control for them introduces significant econometric bias)?
5. (20 marks) Much of the attention to the estimation of trade costs (eg the entire content of Anderson and van Wincoop’s 2004 survey of ‘Trade Costs’) has been concerned with estimating *variable* trade costs. This question asks you to discuss approaches to estimating *fixed* trade (exporting) costs.
- (a) Explain what is meant by a fixed exporting cost (FEC). What is an example of such a cost?
  - (b) Why would the existence of FECs matter for trade theory and for policy?
  - (c) Discuss the implications of Chaney’s (AER 2008) theoretical work (on gravity models with FECs) for the method of estimating variable trade costs that Anderson and van Wincoop survey.
  - (d) Roberts and Tybout (AER 1997), Das, Roberts and Tybout (Ecta 2008), and Eaton, Kortum and Kramarz (2011) all provide estimates of FECs. Pick one of these papers and describe: how the authors estimate FECs, what assumptions are made in order to identify the FECs, the estimate of FECs that the authors arrive at, and the extent to which you believe the answer.
  - (e) Ciliberto and Tamer (Ecta 2009) develop new tools for estimating ‘entry games’—the interacting strategic decisions made by firms about whether to enter a market. Do these tools hold any promise for estimating FECs? What would be the attraction of applying these tools relative to the existing literature (eg the papers in part (d) above)?

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