14.581 Spring 2011—Problem Set 3: Increasing Returns to Scale and Heterogeneous Firms

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Complete all questions (for a total of 100 marks). Due by April 6 to Dave or Sahar.

1. (25 marks) The objective of this first exercise is to revisit Krugman's (1980) Home-market effect. Consider a world economy with two countries, Home and Foreign, each endowed with one factor of production, labor. L and L^* denote the endowments of labor in the two countries and w and w^* denote the associated wages.

   There are two sectors, agriculture (A) and manufacturing (M). The agricultural sector produces a homogeneous good one-for-one for labor under perfect competition, whereas the manufacturing sector produces a large number of varieties under increasing returns to scale and monopolistic competition. Production of $q(\omega)$ units of a given variety ω requires labor

   $$l(\omega) = f + q(\omega)$$

   where $f > 0$ is an overhead fixed cost.

   The preferences of a representative consumer can be represented by

   $$U = C_A^{1-\beta} C_M^{\beta}$$

   where $\beta \in (0,1)$ is the share of expenditure on manufacturing goods and the manufacturing aggregate is given by

   $$C_M = \left[\int_0^N [c(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega + \int_0^{N^*} [c^*(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

   where N and N^* are the endogenous number of Home and Foreign varieties and $\sigma > 1$ is the elasticity of substitution between varieties, respectively.

(a) For now, suppose that the agricultural good is freely traded, whereas manufacturing goods are subject to iceberg trade costs. In order to sell 1 unit of a given variety in the other country, domestic firms must ship $\tau > 1$ units.

(i) Show that

$$\frac{N}{N^*} = \frac{L/L^* - \tau^{1-\sigma}}{1 - (L/L^*)\,\tau^{1-\sigma}}$$

(ii) Using (i), show that Home is a net exporter of the manufacturing good if and only if $L > L^*$.

(b) As in question 1(a), we continue to assume that the agricultural good is freely traded, whereas manufacturing goods are subject to iceberg trade costs. However, now introduce bilaterally asymmetric manufacturing transport costs of the following form: we assume that in order to sell 1 unit in the Foreign country, Home firms must now ship $\tau^* > 1$ units. In order to sell 1 unit in the Home country, Foreign firms must still ship $\tau > 1$ units.

(i) Express $\frac{N}{N^*}$ as a function of τ and τ^*

(ii) Suppose that the Home government can choose the level of iceberg trade costs $\tau > 1$, perhaps by imposing product standards or other technical barriers to trade. What is the optimal level of $\tau > 1$ (i.e. the level that maximizes Home welfare) holding $\tau^* > 1$ constant?

(iii) Suppose now that the Home and Foreign governments choose τ and τ^* simultaneously in order to maximize welfare in their respective country. What is the volume of trade in Nash equilibrium?

(iv) Can you think of institutional arrangement(s) that would lead to Pareto improvements compared to the Nash equilibrium?

(c) Like in 1(a), we assume that $\tau = \tau^*$, but we now relax the assumption that the agricultural good is freely traded, whereas manufacturing goods are subject to iceberg trade costs. In order to sell 1 unit of the agricultural goods in the other country, domestic firms must now ship $\gamma > 1$.

(i) Show that if $\gamma = \tau$, then

$$\frac{N}{N^*} = \frac{L}{L^*}$$

(ii) What does that tell us about the Home-market effect? Can you think of necessary and sufficient conditions on τ and γ for the Home-market effect to occur?

2. (25 marks) The objective of this second exercise is to revisit Melitz and Ottaviano's (2008) results. Like in Exercise 1, we consider a world economy with two countries, Home and Foreign, each endowed with one factor of production, labor. L and L^* denote the endowments of labor in the two countries and w and w^* denote the associated wages.

Again, there are two sectors, agriculture (A) and manufacturing (M). The agricultural sector produces a homogeneous good one-for-one for labor under perfect competition, whereas the manufacturing sector produces a large number of varieties under increasing returns to scale and monopolistic competition. Firms are heterogeneous in terms of their productivity φ, which is randomly drawn from a Pareto distribution $G(\varphi) = 1 - \left(\frac{\underline{\varphi}}{\varphi}\right)^\theta$ for $\varphi \geq \underline{\varphi}$. In order to produce $q(\omega)$ units of a given variety ω, a firm with productivity φ requires labor:

$$l(\omega) = f_e + q(\omega)/\varphi$$

where $f_e > 0$ is an fixed entry cost paid *before* firms know their productivity φ.

The preferences of a representative consumer can be represented by

$$U = C_A + C_M$$

where the manufacturing aggregate is given by

$$C_M = \alpha \int_{\omega\in\Omega} c(\omega)\, d\omega - \frac{1}{2}\gamma \int_{\omega\in\Omega} [c(\omega)]^2\, d\omega - \frac{1}{2}\eta \left[\int_{\omega\in\Omega} c(\omega)\, d\omega\right]^2$$

(a) We start by analyzing the Home country under autarky.

(i) Show that total demand for a given variety is given by

$$q(\omega) = \frac{\alpha L}{\eta N + \gamma} - \frac{L}{\gamma} p(\omega) + \frac{\eta N}{\eta N + \gamma} \frac{L}{\gamma} \overline{p}$$

where N is the measure of consumed varieties and $\overline{p} \equiv \frac{1}{N} \int_{\omega\in\Omega} p(\omega)\, d\omega$ is the average price

(ii) Let $\varphi^* = \inf\left\{\varphi \geq \underline{\varphi} | \pi(\varphi) \geq 0\right\}$ where $\pi(\varphi)$ are the profits of a firm with productivity φ. Show that the mark-up $m(\varphi) \equiv \frac{p(\varphi) - w/\varphi}{p(\varphi)}$ of a firm with productivity $\varphi \geq \varphi^*$ satisfies

$$m(\varphi) = \frac{1/\varphi^* - 1/\varphi}{1/\varphi^* + 1/\varphi}$$

(iii) Let $\overline{m}$ denote the average mark-up. What is the relationship between $\overline{m}$ and L? Explain.

(b) Suppose now that all goods can be freely traded between Home and Foreign.

(i) Show that the utility of the representative agent in country c can be written as

$$U^c = 1 + \frac{1}{2}\left(\eta + \frac{\gamma}{N^c}\right)^{-1}(\alpha - \bar{p}^c)^2 + \frac{1}{2}\frac{N^c}{\gamma}\left(\sigma_p^c\right)^2,$$

where $\left(\sigma_p^c\right)^2 = \frac{1}{N^c}\int_{\omega\in\Omega}\left[p\left(\omega\right) - \bar{p}^c\right]^2 d\omega$

(ii) Show that the measure of consumed varieties in country c is given by

$$N^c = \frac{2(\theta+1)\left(\alpha\varphi_c^* - 1\right)\gamma}{\eta}$$

(iii) Using your results in (i) and (ii), show that the utility of the representative agent can be written as

$$U^c = 1 + \frac{1}{2\eta}\left(\alpha - \frac{1}{\varphi_c^*}\right)\left(\alpha - \frac{\theta+1}{\theta+2}\frac{1}{\varphi_c^*}\right)$$

3. (25 marks) This question concerns the empirical implications of the 'home market effect' (HME), and ways in which it can be estimated.

(a) Describe what is meant by the HME and the intuition behind it.

(b) Davis and Weinstein (JIE 2003) argues that the HME is empirically powerful: it is a prediction made by increasing returns to scale models of trade, but not made by neoclassical (comparative advantage) models of trade. Explain this argument.

(c) Davis and Weinstein (2003) implements a test for the HME. Explain how they do this. What do they find? Critically assess the extent to which the findings of Davis and Weinstein (2003) speak in favor or against the existence of the HME.

(d) Can you think of a 'natural experiment' that would enable you to test the HME more directly?

(e) Suppose you had access to a consumer-level 'scanner' dataset (like that used by Broda and Weinstein (2008, "Understanding International Price Differences Using Barcode Data"), Gopinath, Gourinchas, Hseih and Li (2010), or Burstein and Jaimovich (2008)) that contains the prices of identical goods (identified with their barcode or UPC) at various points in space. (Or alternatively, consider any dataset you can dream up that contains very high quality price data across regions or countries.) Is there a way to use this dataset to test for the HME?

4. (25 marks) Bernard, Redding and Schott (ReStud, 2007) describe a 2-by-2-by-2 Heckscher-Ohlin model with increasing returns to scale (a la Helpman

and Krugman, 1995), and intra-industry heterogeneity with fixed costs of exporting (a la Melitz, 2003). This question asks you to consider some of the implications of this framework.

(a) Explain the intuition behind Proposition 4 in BRS (2007) and the reason why it is different from Proposition 2.

(b) Proposition 11 of BRS (2007) states the implications of this model for the HOV equations (ie the factor content of trade) in this model relative to those in the baseline Helpman-Krugman model. Explain the intuition behind this result. Does this proposition rationalize any of the empirical failures of the HOV predictions that we have seen in this course? Describe an empirical paper that you could write that would build on this model to explore how the presence of intra-industry heterogeneity (and fixed trade costs) alters our understanding of how factor endowments affect trade.

(c) Consider the BRS (2007) model with costly trade. Write down the problem of a social planner who wishes to maximize the value of total output in the economy subject to the economy's resource constraints, and while holding fixed the same variables that monopolistically competitive firms (with a continuum of firms) are assumed to take as given. (When a small, monopolistically competitive firm from country H is active in industry i of the domestic market, call the variable that is the composite of all the variables the firm takes as given, A_{id}; call the equivalent in the export market A_{ix}.) Show that the solution of this problem is identical to the equilibrium conditions in the monopolistically competitive economy. Hence show that the economy admits a revenue function (of sorts) and characterize the properties of this function. (The revenue function can be written as $R(A_{1d}, A_{1x}, A_{2d}, A_{2x}; V)$, where V is the vector of factor endowments.)

(d) Now assume that firms in industry i draw their productivities from a Pareto distribution whose CDF is: $G_i(\varphi_i) = 1 - \varphi_i^{-\theta_i}$, with $\theta_i > \sigma - 1$. Under this restriction, derive a simplified form for the revenue function's dependence on A_{1d}, A_{1x}, A_{2d} and A_{2x}.

(e) Can you think of any empirical implications of the results you've derived in parts (c) and (d)?

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