14.581 Spring 2011 - Problem Set 2
The Ricardo-Viner and Heckscher-Ohlin Models
Solutions (Preliminary and Incomplete)

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1. *(15 marks) Bloom (Econometrica, 2009) uses firm-level data to estimate firm-level responses to an aggregate shock (the shock of interest to him is a shock to 'uncertainty', but clearly a shock of interest to trade economists would be different), and how these firm-level responses aggregate up to an aggregate-level response.*

(a) Discuss the elements of Bloom's microeconomic model that make it similar to the Specific Factors model, and those which do not. .

Solution: The Specific Factors model is typically specified as a model of price-taking firms that use two factors, one with zero adjustment costs (the 'mobile' factor) and one with infinite (and already sunk) adjustment costs (the 'specific' factor).

Bloom (2009) features a model with firms that face an iso-elastic demand curve (i.e. they are not price-taking in general, but are so if the firm's price elasticity goes to infinity) and that use two factors of production, both of which are subject to non-zero but finite adjustment costs.

Similarities: Bloom's model is a more general version of the SF model: to recover the SF model from Bloom's model, set the price elasticity of demand to infinity, the adjustment costs of one factor to zero, and the adjustment costs of the other factor to infinity.

Differences: Of course, the SF model and the Bloom (2009) model serve very different purposes. The SF model is a deliberately extreme special case that is designed to make a qualitative point, and the Bloom (2009) model is designed to contain realistic features of firm-level behavior. The firms face a downward-sloping demand curve, and face non-zero but finite costs of adjusting both factors—plus, these adjustment costs are not just convex but also 'lumpy'. Furthermore, Bloom (2009) model is dynamic as opposed to the SF model.

(b) Outline an empirical paper that could use (a slight extension or amendment of) Bloom's methodology to look at the response of an economy to a trade

liberalization (or perhaps exchange rate devaluation) shock. Describe the various steps that this exercise would entail.

Solution: Bloom (2009) uses firm-level (Compustat) data from the US to estimate the unknown parameters of his model. The key object of interest, for him, is to compute impulse-response plots for firm-level decisions (output, productivity, hiring/firing, investment) in response to a shock to the second moment of the firm's demand curve location (ie an "uncertainty shock"). Estimating this model is non-trivial, since there is no closed-form solution for firm-level decisions (due to the lumpiness of the adjustment costs) and hence the model's likelihood function at a given parameter value can't be specified. But in these settings, simulation-based methods (like the 'method of simulated moments' that Bloom uses) can be used to proceed.

In principle, one could apply a similar methodology to studying firm-level responses to a trade policy shock, such as a trade liberalization episode or a currency devaluation (either of which could effectively just shift the demand curve faced by a firm). The key attraction of such an approach would be that one could let the data trace out the exact nature of the real world we live in: somewhere between the stark adjustment costs in the SF model (infinite for one factor and zero for the other) and those in the H-O model (zero for both factors). From a policy perspective, it would also be useful to know something about the short-, medium- and long-run nature of adjustment to changes in trade policy. An 'impulse response function' to a trade policy change would deliver this. Das, Roberts and Tybout (Ecta, 2008) estimate this sort of function (in a different context).

(c) A hallmark of the field of International Trade is an attention to general equilibrium features generating interactions across markets. How does Bloom (2009) introduce GE forces into his empirical work? What complications arise?

Solution: First, Bloom (2009) is not a GE model due to computational constraints. The issue arises from the presence of both flexible prices and the adjustment costs he is considering. As such he pursues a partial equilibrium version in which all prices are taken as given. He tries to deal with this limitation in what he calls a pseudo-GE approach: (i) He first measures the reaction on the prices, the wages and the interest rate following a shock on the stock market volatility. (ii) He parametrize the model such that the VAR predictions from a volatility shock match the prices. (iii) He then explores the response of his model to a similar shock after adjusting for the VAR price. He concludes that the reaction incorporating the price adjustment is not large.

2. *(35 marks) This question asks you to work through a simple, analytic* $2 \times 2 \times 2$ *H-O model.*

(b) To start with, assume there is just one country (call it H*), which is endowed with* L *units of labor and* K *units of capital. There are two goods.*

Good 1 is produced with the production function $Y_1 = AL_1^{\alpha}K_1^{1-\alpha}$, good 2 is produced with the production function $Y_2 = BL_2^{\beta}K_2^{1-\beta}$, and $\alpha > \beta$. Production is perfectly competitive, in both goods and factor markets. The country has one representative consumer with Cobb-Douglas tastes: $U = C_1^{\mu}C_2^{1-\mu}$. Solve for the equilibrium goods prices (choose $p_1 = 1$ as the numeraire), factor prices, and production and consumption quantities.

Solution: From the household problem (FOC), the relative price is given by:

$$p = \frac{(1-\mu)\,Y_1}{\mu Y_2}$$

as the economy is closed and $C_1 = Y_1$ and $C_2 = Y_2$ in equilibrium.
The firm optimization problem requires that:

$$\begin{aligned} A\alpha L_1^{\alpha-1}K_1^{1-\alpha} &= w \text{ and } pB\beta L_2^{\beta-1}K_2^{1-\beta} = w \\ A(1-\alpha)L_1^{\alpha}K_1^{-\alpha} &= r \text{ and } pB(1-\beta)L_2^{\beta}K_2^{-\beta} = r \end{aligned}$$

Hence, taking the ratio for each factor FOC between the two firms, we have:

$$p = \frac{L_2\alpha Y_1}{L_1\beta Y_2} \text{ and } p = \frac{K_2(1-\alpha)Y_1}{K_1(1-\beta)Y_2}$$

Where I used the fact that $Y_1 = AL_1^{\alpha}K_1^{1-\alpha}$ and $Y_2 = BL_2^{\beta}K_2^{1-\beta}$. Notice that from the optimization condition of the consumer, we have:

$$L_1 = \frac{L_2\alpha}{\beta}\frac{\mu}{1-\mu} \text{ and } K_1 = \frac{K_2(1-\alpha)}{(1-\beta)}\frac{\mu}{1-\mu}$$

Where I substitute $\frac{Y_1}{Y_2} = p\frac{\mu}{1-\mu}$. Furthermore, from the equilibrium in the factor market we have, i.e. $L = L_1 + L_2$ and $K = K_1 + K_2$:

$$K_2\left(\frac{(1-\alpha)}{(1-\beta)}\frac{\mu}{1-\mu} + 1\right) = K \text{ and } L_2\left(\frac{\alpha}{\beta}\frac{\mu}{1-\mu} + 1\right) = L$$

Hence, we have:

$$\begin{aligned} K_2 &= K\kappa \text{ and } L_2 = L\lambda \\ K_1 &= K(1-\kappa) \text{ and } L_1 = L(1-\lambda) \end{aligned}$$

where $\kappa = \dfrac{(1-\beta)(1-\mu)}{((1-\alpha)\mu + (1-\beta)(1-\mu))}$; $\lambda = \dfrac{\beta(1-\mu)}{(\alpha\mu + \beta(1-\mu))}$. Substituting, one can recover the relative price:

$$p = \frac{(1-\mu)}{\mu}\frac{\tilde{A}L^{\alpha}K^{1-\alpha}}{\tilde{B}L^{\beta}K^{1-\beta}}$$

given $Y_1 = \tilde{A}L^{\alpha}K^{1-\alpha}$ and $Y_2 = \tilde{B}L^{\beta}K^{1-\beta}$., where $\tilde{A} = A(1-\lambda)^{\alpha}(1-\kappa)^{1-\alpha}$ and $\tilde{B} = B(\lambda)^{\beta}(\kappa)^{1-\beta}$

Finally, taking the ratios we have:

$$w = \alpha A\left(1-\lambda\right)^{\alpha-1} L^{\alpha-1}\left(1-\kappa\right)^{1-\alpha} K^{1-\alpha}$$

and

$$r = A(1-\alpha)\left(1-\lambda\right)^{\alpha} L^{\alpha}\left(1-\kappa\right)^{-\alpha} K^{-\alpha}$$

(b) Now suppose there are two countries (H and F). Country H is now endowed with ϕL units of labor and ψK units of capital, whereas country F is endowed with $(1-\phi)L$ units of labor and $(1-\psi)K$ units of capital (with $\phi \in (1/2, 1)$ and $\psi \in (0, 1/2)$). Explain the concept of the integrated equilibrium and solve for it (ie for all prices and quantities).

Solution: The integrated equilibrium is defined as the competitive equilibrium that would prevail if both goods and factors were freely traded. Given this, the integrated equilibrium is given by the equilibrium under autarky, as the preferences as well as the technologies are identical for the two countries and the sum of the endowments in each countries are given by the endowments in the previous exercise.

(c) Solve for the free trade equilibrium (ie for all prices and quantities) in this 2-country world under the restriction that both goods are produced by both countries (ie there is incomplete specialization) by working with all of the agents' first-order conditions. In factor space, draw an Edgeworth box (of dimensions L and K) for this 2-country world and illustrate the region of this Edgeworth box in which each country's endowment must lie (ie the values of ϕ and ψ) in order for the incomplete specialization equilibrium to be obtained. Which country contains the relatively richer workers and capitalists in this world?

Solution: Given that both goods are produced by both countries, we know that in each countries the factor prices are going to be identical from the supply side of the economy. The endowment in each country has to be in the factor price equalization region as we will show in the Edgeworth box. In order to solve for the free trade equilibrium, the following set of equations need to hold:

A. Good market equilibrium:

$$Y_1^H + Y_1^F = \mu(I^H + I^F)$$
$$Y_2^H + Y_2^F = (1-\mu)\frac{(I^H + I^F)}{p}$$

where $I^c = w^c L^c + r^c K^c$ for $c = H, F$. Notice that the demand function simply comes from the specific utility function and p is identical for both countries as a result of free trade. The two equations simply state that the markets for good

1 and 2 needs to be in equilibrium, i.e. Supply=Demand. As a consequence, we know that the price is defined by the world output as earlier:

$$p = \frac{Y_1(1-\mu)}{Y_2(\mu)}$$

B. Firms minimize their cost:

$$\begin{aligned} & \min w^c L_i^c + r^c K_i^c \\ \text{such that } C(L_i^c)^s (K_i^c)^{(1-s)} &= Y \end{aligned} \tag{1}$$

Where $i = 1, 2, c = H, F, C = A, B$ and $s = \alpha, \beta$.

From this problem, we have:

$$\begin{aligned} \lambda_1 A\alpha (L_1^c)^{\alpha-1}(K_1^c)^{1-\alpha} &= w^c \text{ and } \lambda_2 B\beta (L_2^c)^{\beta-1}(K_2^c)^{1-\beta} = w^c \qquad (2) \\ \lambda_1 A(1-\alpha)(L_1^c)^{\alpha}(K_1^c)^{-\alpha} &= r^c \text{ and } \lambda_2 B(1-\beta)(L_2^c)^{\beta}(K_2^c)^{-\beta} = r^c, \text{ for c=H,F} \end{aligned}$$

where the equations hold with equality as both countries produce both goods. Alternatively:

$$\begin{aligned} L_1^c = \lambda_1 \alpha Y_1^c / w_c \text{ and } K_1^c = \lambda_1 (1-\alpha) Y_1^c / r_c \qquad (3) \\ L_2^c = \lambda_2 \beta Y_2^c / w_c \text{ and } K_2^c = \lambda_2 (1-\beta) Y_2^c / r_c \end{aligned}$$

Hence, the cost function is given by: $C(w^c, r^c, Y_i^c) = w^c L_i^c + r^c K_i^c = \lambda_i Y_i$, where $\lambda_i = \frac{w^{c\,s} r^{c\,1-s}}{s^s(1-s^{1-s})C}$ and s=λ, β and C=A,B.

Notice that λ_i comes from substituting the factor demands from the FOC's above in the production function, we get:

$$\lambda_1 (\alpha / w^c)^{\alpha} (\alpha / r^c)^{1-\alpha} = 1 \tag{4}$$

$$\lambda_2 (\beta / w^c)^{\beta} (1 - \beta / r^c)^{1-\beta} = 1 \tag{5}$$

Hence, in equilibrium, from the firm level optimization problem we have:

B.1. The unit cost condition:

$$1 = \frac{w^{c(\alpha)} r^{c(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \tag{6}$$

$$p = \frac{w^{c(\beta)} r^{c(1-\beta)}}{\beta^{\beta}((1-\beta)^{1-\beta})} \tag{7}$$

B.2. The unit factor requirement matrix:

$$A(w^c, r^c) = \begin{bmatrix} \frac{\alpha r^{c(1-\alpha)}}{\tilde{A} w^{c(1-\alpha)}} & \frac{\beta r^{c(1-\beta)}}{\tilde{B} w^{c(1-\beta)}} \\ \frac{(1-\alpha) r^{c(-\alpha)}}{\tilde{A} w^{c(-\alpha)}} & \frac{(1-\beta) r^{c(-\beta)}}{\tilde{B} w^{c(-\beta)}} \end{bmatrix} \tag{8}$$

Which is the result of the Shephard's lemma. Hence, the factor market clearing conditions become:

C. Factor market equilibrium:

$$\begin{bmatrix} L^c \\ K^c \end{bmatrix} = A(w^c, r^c) \begin{bmatrix} Y_1^c \\ Y_2^c \end{bmatrix} \tag{8}$$

where it should hold for c=H,F.

From B.1., given that $p = \frac{\tilde{A}}{\tilde{B}}(\frac{r^c}{w^c})^{\alpha-\beta}$ for c=H,F and both countries face the same price p, the relative factor price is identical for both countries. From the factor market equilibrium, we know that, $A(w^c, r^c)$ is unique and identical for both countries. As such, we can add the condition for both countries and show that:

$$\begin{bmatrix} L \\ K \end{bmatrix} = A(w, r) \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \tag{9}$$

As such we can show that $\frac{Y_2}{Y_1}$ is a function of the relative factor price as well.

$$\begin{aligned} \frac{Y_2}{Y_1} &= \frac{\tilde{B}}{\tilde{A}}(\frac{r}{w})^{\beta-\alpha}\frac{-(1-\alpha)\frac{w}{r}L+\alpha}{(1-\beta)\frac{w}{r}L-\beta K} && (10)\\ &= p^{-1}\frac{-(1-\alpha)\frac{w}{r}L+\alpha}{(1-\beta)\frac{w}{r}L-\beta K} && (11)\\ & && (12) \end{aligned}$$

where I used $p = \frac{\tilde{A}}{\tilde{B}}(\frac{r}{w})^{\alpha-\beta}$. As such, we can show that:

$$\begin{aligned} \frac{Y_2}{Y_1}p &= \frac{-(1-\alpha)\frac{w}{r}L+\alpha}{(1-\beta)\frac{w}{r}L-\beta K} && (13)\\ \frac{1-\mu}{\mu} &= \frac{-(1-\alpha)\frac{w}{r}L+\alpha}{(1-\beta)\frac{w}{r}L-\beta K} && (14) \end{aligned}$$

Hence, solving for r/w:

$$\frac{r}{w} = \frac{L}{K}\gamma \tag{15}$$

where $\gamma = \frac{1-\beta+\mu(\beta-\alpha)}{\beta+\mu(\alpha-\beta)}$. From here, p and $\frac{Y_2}{Y_1}$ (the global ratio as well as its distribution for both countries) is given by the good market ratio and the factor market equilibrium. Finally, the distribution across the two countries is given as follows:

$$Y_1^H = L^\alpha K^{1-\alpha}\gamma^\alpha\frac{\tilde{A}}{\alpha-\beta}((1-\beta)\gamma^{-1}\phi - \beta\psi)$$

$$Y_1^F = L^\alpha K^{1-\alpha}\gamma^\alpha \frac{\tilde{A}}{\alpha-\beta}((1-\beta)\gamma^{-1}(1-\phi) - \beta(1-\psi))$$

$$Y_2^H = L^\beta K^{1-\beta}\gamma^\beta \frac{\tilde{B}}{\alpha-\beta}((1-\alpha)\gamma^{-1}\phi + \alpha\psi)$$

$$Y_2^F = L^\beta K^{1-\beta}\gamma^\beta \frac{\tilde{B}}{\alpha-\beta}((1-\alpha)\gamma^{-1}(1-\phi) - \alpha(1-\psi))$$

What are the conditions on the endowment in each countries under which there is incomplete specialization? We know from the class that the endowments need to be inside of the cone of diversification. (See Figure) Finally, notice that we know that if the price of the goods are equalized through free trade and both countries are producing both goods, then the factor prices equalize (FPE theorem). As such both groups (workers and capital owners) are paid the same per unit around the world.

(d) Solve for the amount of each good that each country is exporting/importing to/from the other country. Comment on which country is exporting which good.

Solution: The easiest way to find the amount each country export or import, we need to focus on one country and one good (as trade balance assures that if one country import a good, then it should export the other good for the same value).

Consider good 1 in country H consumption:

$$C_1^H = \mu(Y_1^H + pY_2^H)$$

Hence, the amount the country exchange with the other country of good 1 is given by:

$$Y_1^H - C_1 = Y_1^H - \mu(Y_1^H + pY_2^H))$$

Hence, the country will export or import depending on the sign of $(1-\mu(1+p\frac{Y_2^H}{Y_1^H}))$. First, notice that $pY_2^H = (\frac{\tilde{A}}{\tilde{B}}(\frac{K}{L}\gamma^{-1})^{\beta-\alpha})(L^\beta K^{1-\beta}\gamma^\beta \frac{\tilde{B}}{\alpha-\beta}((1-\alpha)\gamma^{-1}\phi + \alpha\psi))$. Hence,

$$\begin{aligned}
Y_1^H - C_1 &= Y_1^H - \mu(Y_1^H + pY_2^H)) && (16)\\
&= L^\alpha K^{1-\alpha}\gamma^\alpha \frac{\tilde{A}}{\alpha-\beta}((1-\beta)\gamma^{-1}\phi - \beta\psi) - \mu(L^\alpha K^{1-\alpha}\gamma^\alpha \tilde{A}(\gamma^{-1}\phi + \psi) && (17)\\
&= L^\alpha K^{1-\alpha}\gamma^\alpha \tilde{A}((\frac{1-\beta}{\alpha-\beta} - \mu)\gamma^{-1}\phi - (\frac{\beta}{\alpha-\beta} + \mu)\psi) && (18)\\
&= L^\alpha K^{1-\alpha}\gamma^\alpha \tilde{A}((\frac{\beta}{\alpha-\beta} + \mu)(\phi - \psi)) > 0 && (19)
\end{aligned}$$

which results from $\phi \geq 1/2$ and $\psi \leq 1/2$.

(e) Solve for the factor content of trade between each country.

Solution: Denote the share of home income over the global income:

$$s_H = \frac{w\phi L + r\psi K}{wL + rK} \tag{20}$$

$$s_H = \frac{\phi + \frac{r}{w}\frac{K}{L}\psi}{1 + \frac{r}{w}\frac{K}{L}} \tag{21}$$

$$s_H = \frac{\phi + \gamma\psi}{1 + \gamma} \tag{22}$$

The factor content of trade at home is is defined as:

$$FT_H = A(w, r)(Y_H - C_H)$$

where $FT_H = \begin{bmatrix} FT_H^1 & FT_H^2 \end{bmatrix}'$. From the demand of the factor of production specified as above and the definition of C_H:

$$FT_H = V^H - s_H V$$

where V's are the factor of production endowment at home and the global one. Hence, from the definition of s_H as given above, we have the factor content of trade:

$$FT_H = (\phi - \psi)\begin{bmatrix} \frac{\gamma}{1+\gamma}L \\ -\frac{1}{1+\gamma}K \end{bmatrix}$$

(f) Now suppose that country H is only producing good 1; find the restrictions on ϕ and ψ such that this is true. Hence sketch the output of good 1 by country H as a function of ϕ/ψ. What does this relationship imply about how one should approach the estimation of so-called 'Rybczinski regressions'?

Solution: Country H is only producing good 1 if $\frac{\phi}{\psi} \geq \frac{\alpha}{1-\alpha}\gamma$ from the equation of Y_2^H defined as if country H was producing both goods such that $Y_2^H \leq 0$. If H is only producing good 1, then:

$$Y_H^1 = A(\phi L)^\alpha(\psi K)^{1-\alpha}$$

As such, there is a change in the function such that Y_H^1 for (ϕ/ψ) ratios big enough, Y_H^1 is not defined as before anymore, but it is defined as the equation we just have shown. Hence, if one is looking at the empirical relation between output and endowment, the relation could move from being linear to non linear as a function of the relative factor endowment of the country. This can be seen from the drawing.

(g) Finally, suppose that there is a third good whose production function is $Y_3 = L_3^{\gamma} K_3^{1-\gamma}$. Describe and illustrate (in the Edgeworth box diagram) the restriction on ϕ and ψ such that both countries are producing all three goods. How much of each of the three goods will each country produce? How much of each of the three goods will they trade? Solve for the factor content of each country's net exports.

Solution: Assume that the new utility function for both countries is given by:

$$U(C_1, C_2, C_3) = C_1^{\mu_1} C_2^{\mu_2} C_3^{1-\mu_1-\mu_2}$$

Given that we consider the case where the two countries are producing the three goods, we have the factor unit requirement:

$$A(w, r) = \begin{bmatrix} \frac{\alpha}{\tilde{A}}(\frac{r}{w})^{1-\alpha} & \frac{\beta}{\tilde{B}}(\frac{r}{w})^{1-\beta} & \frac{\alpha}{\tilde{G}}(\frac{r}{w})^{1-\gamma} \\ \frac{1-\alpha}{\tilde{A}}(\frac{r}{w})^{-\alpha} & \frac{1-\beta}{\tilde{B}}(\frac{r}{w})^{-\beta} & \frac{\alpha}{1-\tilde{G}}(\frac{r}{w})^{-\gamma} \end{bmatrix} \tag{23}$$

where $\tilde{G} = \gamma^{\gamma}(1-\gamma)^1 - \gamma$. The free trade equilibrium is again defined through the zero profit condition, the factor market and good market equilibrium. Given the three goods are produced and they all face the world price, p, the factor prices are equalizing across countries. As such, we can show that as before:

$$\frac{r}{w} = \frac{L}{K}\tau \tag{23}$$

where $\tau = \frac{(\gamma-\alpha)\mu_1+(\gamma-\beta)\mu_2+(1-\gamma)}{(\gamma-\alpha)\mu_1+(\gamma-\beta)\mu_2+(\gamma)}$. Hence, we already know that the factor content of trade is going to be identical to earlier apart from the change from γ to τ. Overall, there is more than one distribution of the production across the two countries that can generate the world integrated equilibrium. However, as can be seen in the graph, all the endowments within the FPE set is satisfying both countries producing the three goods and having their prices equalized in equilibrium.

3. (10 marks) Consider a neoclassical economy with $G > 2$ goods (indexed by g) and $F > 2$ factors (indexed by f), with $G = F$.

(a) Is $G = F$ a reasonable assumption to make?

Solution: Clearly this is a difficult question to answer. Fundamentally, the answer depends on the substitutability of two goods/factors, and the units at which these goods/factors are measured (eg, are all CEOs equivalent to one another?). As discussed in the lectures, Bernstein and Weinstein (2003) provide a test based on production indeterminacy for $G > F$. A related idea appears in the labor literature (eg Welch (1975) and Rosen (1983)) on whether one can work with aggregates of workers like 'skilled workers' who might be doing different tasks. Another empirical approach to answering this question would perhaps be an attempt to estimate elasticities of substitution at higher levels and see whether, with a definition/convention that two factors/goods are substitutes if their elasticities of substitution go above some threshold (since they'd never be infinite), the number of G and F.

(b) Derive a relationship between the 'Stolper-Samuelson derivative' ($\frac{dw_f}{dp_g}$) and the 'Rybczinski derivative' ($\frac{dy_g}{dV_f}$). Comment on the intuition behind this relationship.

Solution: In this section, I will show that under the equality between the number of goods and the number of factors, the Stolper-Samuelson derivative and the Rybczinski derivative' are identical. First, notice that:

Let ω be the set of factor of production prices, $A(\omega)$ be the matrix of the unit of factor requirement from the cost minimization problem of the firm and p be the set of good price. From the zero profit conditions and the factor market clearing conditions:

$$\begin{bmatrix} \omega_1 \\ . \\ . \\ . \\ \omega_N \end{bmatrix} = A(\omega)^{-1} \begin{bmatrix} p_1 \\ . \\ . \\ . \\ p_N \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_N \end{bmatrix} = A(\omega)^{-1} \begin{bmatrix} \upsilon_1 \\ . \\ . \\ . \\ \upsilon_N \end{bmatrix}$$

Notice that the inverse is A exists due to the equality between the number of factor and the number of goods. Taking the derivative with respect to p and υ, from the envelope theorem, we have for each combination of good i and factor k:

$$\frac{d\omega_k}{dp_i} = (A(\omega))^{-1}_{i,k} = \frac{dy_i}{d\upsilon_k} \tag{23}$$

(c) Describe how you would design an empirical paper that would aim to test this relationship.

Solution: First, define goods/factors such that $G = F$. Then one needs to find a setting where we can observe prices, wages, output and endowments. A key challenge is isolationg exogenous variation in prices (perhaps using tariff changes, world price shocks to commodity-producing countries, or, as one student suggested, EU accession of Eastern European countries) and endowoments (such as immigration shocks, the fertility transition, or, as one student suggested, weather-induced changes in factors of production like water or soil.) Armed with these changes in prices, one could regress wage changes on price changes; and one could do the same for a regression of output changes on endowment changes. This would produce G^2 and F^2 regression coefficients respectively, which could be then compared statistically (or eg via a scatter plot).

4. (20 marks) The HO model without FPE.

(a) Factor prices are clearly not equal around the world. Discuss why this might be the case. Of the reasons you have just given, discuss which you think is most plausible (against the backdrop of the literature on Heckscher-Ohlin empirics).

Solution: The reasons for this are: endowments lie outside of the FPE set, the presence of factor intensity reversals, technological differences, more factors than goods, the existence of trade costs (so countries aren't facing the same goods prices). All of these are plausible reasons for non-FPE that find empirical support. For example, technology differences are Trefler (1995)'s preferred explanation for non-FPE—and indeed Trefler (1993) points out that (with a flexible enough version of 'technology differences') there exists a set of technology differences that can rationalize any departure from FPE we see in the world. Davis and Weinstein (2001)'s preferred explanation for the lack of FPE is that countries lie outside the FPE set. And finally, Anderson and van Wincoop (2004) survey a wide range of evidence for high international trade costs, which will (as soon as trade costs are non-zero) lead to non-FPE.

(b) Consider a country c with a vector of factor endowments V^c whose production can be characterized by a revenue function, $r^c(p^c, V^c)$, where p^c is the vector of goods prices in country c. Another country c' is exporting a vector of goods $T^{c'c}$ (in physical units) from c' to c. Consider the thought experiment that instead of country c' sending these goods to country c, country c' instead sent the factors that were needed to produce these goods when they were made in country c' (which we call the factor content of exports from c' to c, denoted by the vector $F^{c'c}$.) What can you say about the size of $T^{c'c}$ relative to $F^{c'c}$?

Solution: (NB: The solution here and in the next part follow Helpman (EJ 1984), Helpman and Krugman (1985) and Choi and Krishna (JPE 2004).) In equilibrium, we know that $r^c(p, V_c) = pQ_c$ where Q_c is the vector of country c production. If country c' is sending the factors that were needed to produce $T^{c'c}$, we know that country c could use these factor services (ie add them to its own factors, V_c) and produce at least $T^{c'c}$. Hence :

$$\begin{aligned} p\left(Q_c + T^{c'c}\right) &\leq r^c(p, V_c + F^{c'c}) \\ &\leq r^c\left(p, V_c\right) + r^c_V(p, V_c)F^{c'c} \\ &= pQ_c + w^c F^{c'c} \end{aligned}$$

where the results follow from the concavity of r^c in V and $r^c_V(p, V_c)$ is the vector of partial derivatives with respect to V. Hence, we have: $pT^{c'c} \leq w^c F^{c'c}$

(c) Now make some additional assumptions, of the sort that are commonly made in Heckscher-Ohlin settings, to derive the following bilateral relationship between factor prices in countries c and c' (call them vectors w^c and $w^{c'}$): $(w^{c'} - w^c).F^{c'c} \leq 0$. If there are N countries in the world, how many predictions does this theory make?

Solution: Assuming also that all countries face the same prices (ie zero trade costs), under perfect competition in country c' we must have $pT^{c'c} = w^{c'}F^{c'c}$:

$$\begin{aligned} w^{c'}F^{c'c} &\leq w^c F^{c'c} \\ &\Leftrightarrow \\ \left(w^{c'} - w^c\right)F^{c'c} &\leq 0 \end{aligned}$$

If there are N countries then this condition makes $N(N-1)$ predictions with empirical content.

(d) Can you make additional predictions about tri-lateral relationships between factor prices in countries A and B, and the factor content of exports from a third country C to either A or B? How many predictions does this theory make?

Solution: (NB: This follows Bernhofen (JIE 2009).) Suppose that there are three countries: c, c' and c''. Now consider a slightly different thought experiment from above: instead of c exporting to country c'', he decides to give the factor content of the trade to a third country, c'. In this case, applying similar arguments to that above we have:

$$\left(w^{c'} - w^c\right)F^{cc''} \leq 0$$

So if there are N countries in the world then this condition makes $N^2(N-1)^2/2$ predictions (for each cc' pair there are $N(N-1)$ different values of $F^{cc''}$ to test, and there are $N(N-1)/2$ different cc' pairs).

(e) Describe an empirical exercise that you could perform to test this set of predictions in the H-O model. What would be its attractions relative to other empirical HO approaches.

Solution: One could simply test the correlation stated above and apply a sign or rank test. This is what Choi and Krishna (2004) and Bernhofen (2009) do. The main advantage of this test relative to other branches of the empirical HO literature is that FPE does not need to hold (so countries can in different cones of specialization, and countries can have different technologies). Note however that if it is trade costs that are giving rise to non-FPE then this model is not robust to that (the model still assumes free trade). A few disadvantages of this test are: (a) we need data on the returns to *all* factors, unlike in the typical HOV equations case where the test operates one factor at a time; (b) the model doesn't predict anything other than the sign of a relationship (unlike the HOV equations which also predict magnitudes); and (c), comparable and high-quality data on factor prices around the world are hard to find (as Choi and Krishna (2004) explain).

5. (10 marks) Describe an extension of the model in Costinot, Donaldson and Komunjer (2010) that would add Heckscher-Ohlin features to it. Now outline an empirical paper that would use this extension to make as useful a contribution to the empirical H-O literature as possible. Be sure to state exactly what regression(s) or other empirical tests/exercises youre proposing, how they follow from the model, and what the estimates would tell us. State any attractive features of this approach you can think of, relative to existing empirical work on the H-O model.

Solution: It is very easy to add more than one factor to the Eaton and Kortum (2002) style model in CDK (2010): simply replace the wage rate w_i with a 'unit input bundle cost,' c_i^k. For example, if we were to assume that the technology for combining inputs were Cobb-Douglas in labor and capital then we have simply $c_i^k = w_i^{\alpha_k} r_i^{1-\alpha_k}$. The only complication this adds to the model is that now there is more than one factor market clearing condition to worry about when closing the model. But note that this isn't a particularly serious complication relative to standard applications of Eaton and Kortum (2002) style models. Most applications work with the gravity equation, which includes the endogenous w_i, or c_i^k, without solving these out in closed form. In fact, there is no closed form solution for these factor prices. If one needs or wants to solve for the full general equilibrium of the model one needs to solve for wi using the labor market clearing condition, which can only be done numerically using a computer. The same continues to be true of course when one needs to solve for, eg, both w_i and r_i. Armed with this extension, the CDK (2010) model becomes a HO-style model with trade costs, TFP differences across countries and industries (the z_i^k in CDK), and TFP differences across varieties within industries. In principle this is an important addition to the HO literature, which has not yet come to grips with the presence of either trade costs or intra-industry heterogeneity (in empirically applicable models). However, a key issue is what one does empirically with this model. One can certainly estimate gravity equations with c_i^k in them rather than w_i. Note two things about this, however. First, because CDK measure productivity as the inverse of producer (i.e. output) prices, CDK is already a generalized HO empirical application (since in any perfectly competitive environment, regardless of the number of factors, we will have that the output price is equal to the unit input cost). Second, in the current CDK implementation it is assumed that θ is the same in all industries k, which means that in the gravity equation wi enters as $-\theta ln(w_i)$, which means that the w_i term goes into an exporter fixed effect (so it is not estimated, but neither is its endogeneity a worry). In the proposed HO extension we now have that the input bundle cost c_i^k varies across both i and k so it would not be absorbed into a fixed effect. Instead, these terms would have to be included in the regression and instrumented for. An option would be to get data on w_i and r_i and instrument for them using the endowments of labor and capital respectively. Having estimated the models gravity equation, what would one want to do next? Finding support for an HO-style gravity equation would not be particularly interesting in its own right. To my mind the really interesting step would be to compare the models prediction on the net factor content of trade (NFCT) with the NFCT that we actually see in the data. This would be ambitious (youd need to solve for the full GE of the model and then compute the NFCT, and youd also need to take seriously the I-O structure of the economy by building intermediate input use into ci) but this would be a first serious step in the HO literature of looking at how the existence of trade costs (and intra-industry heterogeneity and trade) affect the empirical performance of the HO model.

6. (10 marks) Consider the sections of Costinot (Ecta 2009) that deal with a Heckscher-Ohlin-style model (ie Sections 5 and 6). Describe the best possible empirical paper you can imagine writing that would test this models predictions. What are the pros and cons of this approach to H-O empirics compared to other approaches we have studied?

Solution: One could test corollaries 2 and 3 of Costinot (2009). The corollaries predict an order between the ratio of the aggregate output, employment and revenues of two countries between different sectors. For instance, denote the ratio of skilled to unskilled labor $\gamma = \frac{S}{U}$, and order all the sectors in an economy with respect to the skilled labor intensity from the least to the highest intensity sector. Assume that country 1 is relatively more endowed with skilled labor. The corollaries 2 and 3 state that under some regularity conditions,

$$\frac{Q^{s_1 c_1}}{Q^{s_1 c_2}} \leq ... \leq \frac{Q^{s_N c_1}}{Q^{s_N c_2}} \tag{18}$$

$$\frac{E^{s_1 c_1}}{E^{s_1 c_2}} \leq ... \leq \frac{E^{s_N c_1}}{E^{s_N c_2}} \tag{19}$$

$$\frac{R^{s_1 c_1}}{R^{s_1 c_2}} \leq ... \leq \frac{R^{s_N c_1}}{R^{s_N c_2}} \tag{20}$$

One can use a cross sectional dataset of different countries aggregate output, employment and revenue per sector for the different countries trading with each other and test the qualitative relation for pairs of countries across all their sectors highlighting one dimension such as the skilled to unskilled labor ratio for instance. Pro: The predictions of the model are not specific to a given parametrization as they simply enumerate a set of ordinal relations. It is relatively easy to gather aggregate information at the sector level. The dataset is largely available and tractable. It allows one to set the H-O type of predictions unrestricted to the relation between the number of factors and the number of goods. As such, it allows one to make more general and realistic claims as well as to test them. Cons: In order to test the predictions, it is necessary to specify an order for the factors, sectors and countries. This could be non trivial. There is a need to internalize better non traded goods within the model. As much as it helps to generalize and give H-O type of predictions independent of the number of goods and factors, it could suffer from the same weakness of the H-O type of predictions: too general and not informative enough. In fact, one could fail to reject or reject the predictions and still the underlying theoretical mechanisms behind the results would remain unclear. One direction would be to make a more realistic as well specific model and move towards a structural methodology.

14.581 International Economics I
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