14.581 Spring 2011 - Problem Set 1
Gains from Trade and the Ricardian Model
Solutions

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1. *(10 marks) Consider a Ricardian model with a continuum of goods, indexed by $z \in [0, +\infty)$, and two countries, indexed by $i = N, S$, each endowed with L_c units of labor. w_i denotes the wage in each country. Constant unit labor requirements in country c and industry z are given by:*

$$a_i(z) = \alpha_i e^{\beta_i z}, \tag{1}$$

where $\alpha_S > \alpha_N > 0$ and $\beta_S > \beta_N > 0$. Hence North (N) has an absolute advantage in all goods and a comparative advantage in high-z goods.

Households have identical Cobb-Douglas preferences in both countries:

$$U_i = \int_0^{+\infty} \ln c_i(z)\, dz. \tag{2}$$

(a) Solve implicitly for the relative wage, w^N/w^S, and the "cut-off" good, $\widetilde{z}$.

Solution: Note that this problem is inspired by Krugman (1986) "A 'Technology Gap' Model of International Trade". We denote $A(z) \equiv \dfrac{a_S(z)}{a_N(z)}$, which is a increasing and continuous function of z. From the production side of the economy, we know that

$$p(z) = \min\{a_N(z)\, w_N, a_S(z)\, w_S\}$$

Furthermore, given the functional form, there exists a marginal good $\tilde{z} \in \mathbb{R}^+$, the switching point in the chain of comparative advantage, such that North will produce only the goods $z \geq \tilde{z}$ and South will produce only the goods $z \leq \tilde{z}$. $\tilde{z}$ must satisfy in equilibrium:

$$\frac{\alpha_S}{\alpha_N} e^{(\beta_S - \beta_N)\tilde{z}} = w^N/w^S \tag{3}$$

Conditional on wages, goods should be produced in the country where it is cheaper to do so.

In equilibrium, we also know that the value of imports must be equal to the value of exports, which implies:

$$\frac{(1-\tilde{z})\,L_S}{\tilde{z}L_N} = w^N/w^S \tag{4}$$

Equations (3) and (4) implicitly define ω and $\tilde{z}$.

(b) Study the welfare implications in both countries of a decrease in α_S.

Solution: A decrease in α_S corresponds to uniform technological catch-up in the South. Let $\tilde{z}'$ and $\left(w^N/w^S\right)'$ denote the cut-off good and the relative wage in the new equilibrium. In this case, we have $\tilde{z}' \geq \tilde{z}$ and $\left(w^N/w^S\right)' \leq w^N/w^S$. At a given relative wage, the technological advance generates $\tilde{z}$ to increase and South has a trade surplus which will pressure the $\left(w^N/w^S\right)'$ down. North and South both gain from trade.

Consider the South first. The change in the indirect utility function is given by

$$\begin{aligned}
\Delta U_S &= \int_0^{\tilde{z}} \left[\ln\left(\frac{1}{a'_S(z)}\right) - \ln\left(\frac{1}{a_S(z)}\right)\right] dz \\
&\quad + \int_{\tilde{z}}^{\tilde{z}'} \left[\ln\left(\frac{1}{a'_S(z)}\right) - \ln\left(\frac{1}{a_N(z)\left(w^N/w^S\right)}\right)\right] dz \\
&\quad + \int_{\tilde{z}'}^{1} \left[\ln\left(\frac{w'_S}{a_N(z)\,w'_N}\right) - \ln\left(\frac{w_S}{a_N(z)\,w_N}\right)\right] dz
\end{aligned}$$

There are three regions: (i) $z \in [0,\tilde{z}]$, i.e. goods produced by South before and after, (ii) $z \in [\tilde{z},\tilde{z}']$, i.e. goods produced by South after but by North before and (iii) $z \in [\tilde{z}',1]$, i.e. goods produced by North before and after. The first term is positive because $\alpha'_S < \alpha_S$. The second term is positive because international specialization requires $a_N(z)\,w_N \geq a'_S(z)\,w'_S$, $\forall z \in z \in [\tilde{z},\tilde{z}']$, and hence $1/a'_S(z) \geq 1/\left(w^N/w^S\right)a_N(z)$ if $\left(w^N/w^S\right)' \leq w^N/w^S$. The third terms is positive because $\left(w^N/w^S\right)' \leq w^N/w^S$. So the South gains from trade.

Now consider the North. The change in the indirect utility function is given by

$$\begin{aligned}
\Delta U_N &= \int_0^{\tilde{z}} \left[\ln\left(\frac{\left(w^N/w^S\right)'}{a'_S}\right) - \ln\left(\frac{\left(w^N/w^S\right)}{a_S}\right)\right] dz \\
&\quad + \int_{\tilde{z}}^{\tilde{z}'} \left[\ln\left(\frac{\left(w^N/w^S\right)'}{a'_S(z)}\right) - \ln\left(\frac{1}{a_N(z)}\right)\right] dz \\
&\quad + \int_{\tilde{z}'}^{1} \left[\ln\left(\frac{1}{a_N(z)}\right) - \ln\left(\frac{1}{a_N(z)}\right)\right] dz
\end{aligned}$$

Let $\omega = \left(w^N/w^S\right)$, the first term is positive because $\omega/\omega' = A\left(\tilde{z}\right)/A'\left(\tilde{z}'\right) = \alpha_S e^{(\beta_N - \beta_S)\left(\tilde{z}' - \tilde{z}\right)}/\alpha'_S$. Hence $\beta_N < \beta_S$ and $\tilde{z}' > \tilde{z}$ implies $a'_S/\omega' < a_S/\omega$. The second term is positive for the same reason as in the South. The third term is trivially equal to zero. So the North gains from trade as well.

(c) Study the welfare implications in both countries of an decrease in β_S. [Maintain the assumption that $T_N > T_S$.] Explain the difference between the setting here and that in 1(b).

Solution: First notice that an increase in β_S to β'_S is such that $a_S\left(z\right) \geq a'_S\left(z\right)$ for all z, i.e. the South experiences a technological advance. It requires less labor units to produce one unit of the good z. This results in $\tilde{z} \leq \tilde{z}'$ and $\omega \geq \omega'$. At given relative wages, the technological loss causes $\tilde{z}$ to decrease and the South has a trade deficit which will pressure the ω down. Consider the South first. The change in the indirect utility function is given by

$$\begin{aligned}
\Delta U_S &= \int_0^{\tilde{z}} \left[\ln\left(\frac{1}{a'_S\left(z\right)}\right) - \ln\left(\frac{1}{a_S\left(z\right)}\right)\right] dz \\
&+ \int_{\tilde{z}}^{\tilde{z}'} \left[\ln\left(\frac{1}{a'_S\left(z\right)}\right) - \ln\left(\frac{1}{a_N\left(z\right) w^N/w^S}\right)\right] dz \\
&+ \int_{\tilde{z}'}^{1} \left[\ln\left(\frac{w'_S}{a_N\left(z\right) w'_N}\right) - \ln\left(\frac{w_S}{a_N\left(z\right) w_N}\right)\right] dz
\end{aligned}$$

There are again three regions: (i) $z \in \left[0, \tilde{z}\right]$, i.e. goods produced by South before and after, (ii) $z \in \left[\tilde{z}, \tilde{z}'\right]$, i.e. goods produced by South after but by North before and (iii) $z \in \left[\tilde{z}', 1\right]$, i.e. goods produced by North before and after. The first term is positive because $a'_S\left(z\right) < a_S\left(z\right)$, i.e. $e^{\left(\beta'_S - \beta_S\right)\tilde{z}} < 1$. The second term is positive because international specialization requires $a'_S\left(z\right)/a_N\left(z\right) \leq \left(w_N/w_S\right)' \leq w^N/w^S$, $\forall z \in z \in \left[\tilde{z}, \tilde{z}'\right]$, and hence $1/a'_S\left(z\right) \geq 1/\left(w^N/w^S\right) a_N\left(z\right)$. The third terms is positive because $\left(w^N/w^S\right)' \leq w^N/w^S$. So the South gains from trade.

Now consider the North. The change in the indirect utility function is given by

$$\begin{aligned}
\Delta U_N &= \int_0^{\tilde{z}} \left[\ln\left(\frac{\left(w^N/w^S\right)'}{a'_S}\right) - \ln\left(\frac{\left(w^N/w^S\right)}{a_S}\right)\right] dz \\
&+ \int_{\tilde{z}}^{\tilde{z}'} \left[\ln\left(\frac{\left(w^N/w^S\right)'}{a'_S\left(z\right)}\right) - \ln\left(\frac{1}{a_N\left(z\right)}\right)\right] dz \\
&+ \int_{\tilde{z}'}^{1} \left[\ln\left(\frac{1}{a_N\left(z\right)}\right) - \ln\left(\frac{1}{a_N\left(z\right)}\right)\right] dz
\end{aligned}$$

Let $\omega = \left(w^N/w^S\right)$. The third term is trivially equal to zero. $\forall z \in z \in \left[\tilde{z}, \tilde{z}'\right]$, international specialization requires $w^{N'} a_N\left(z\right) > w^{S'} a'_S\left(z\right)$. The first term is positive iff $\dfrac{\left(w^N/w^S\right)'}{\left(w^N/w^S\right)} = \dfrac{a'_S\left(\tilde{z}'\right)}{a_S\left(\tilde{z}\right)} \geq \dfrac{a'_S\left(z\right)}{a_S\left(z\right)}$, $\forall z \in [0,\tilde{z}]$, i.e. $e^{\beta'_S\tilde{z}'-\beta_S\tilde{z}} \geq e^{\left(\beta'_S-\beta_S\right)z}$.

2. (10 marks) Continue with the environment in Question 1 but now assume that $z \in [0,+\infty)$ and that households have non-homothetic preferences in both countries:

$$U_i = \int_0^{+\infty} c_i\left(z\right)dz, \tag{5}$$

with $c_i\left(z\right) = 0$ or 1 for all z. In addition, we assume that all households are endowed with one unit of labor in both countries.

(a) Compute $c_i\left(z\right)$ as a function of w_i in both countries.

Solution: Note that this entire exercise is inspired by Matsuyama (JPE, 2000). First, notice that an increase in utility is created by an increase in diversity, not an increase in the consumption of the same good. Furthermore, given the functional form we are considering, we know that the order in which the consumers purchase goods is such that they consume first the lower index goods and then the higher index good as long as it can afford it. This is because $p\left(z\right) = \min\left\{w_N a_N\left(z\right), w_S a_S\left(z\right)\right\}$ is increasing in z given $a\left(.\right)$ are increasing and continuous in z. Since the household purchases all the lower-indexed goods and expands its range of consumption upward as far as it can afford,

$$c_i\left(z\right) = \begin{cases} 1 & \text{if } z \in \left[0,\bar{z}\left(w_i\right)\right] \\ 0 & \text{if } z \in \left(\bar{z}\left(w_i\right),\infty\right) \end{cases}$$

for $i = N, S$, where $\bar{z}\left(w_i\right)$ is implicitly defined as

$$\int_0^{\bar{z}(w_i)} \min\left(a_N\left(z\right)w_N, a_S\left(z\right)w_S\right)dz = w_i$$

Notice that the highest good the consumer consumes, $\bar{z}\left(w_i\right)$, can also be viewed as the utility level of the consumer with income w_i as

$$U\left(\bar{z}\left(w_i\right)\right) = \int_0^{\bar{z}(w_i)} 1\cdot dz = \bar{z}\left(w_i\right)$$

(b) Solve implicitly for the relative wage, w^N/w^S, and the "cut-off" good, $\widetilde{z}$.

Solution: w^N/w^S Like in Question 1, efficient international specialization requires

$$\frac{\alpha_S}{\alpha_N}e^{\left(\beta_S-\beta_N\right)\tilde{z}} = \frac{w_N}{w_S} \tag{6}$$

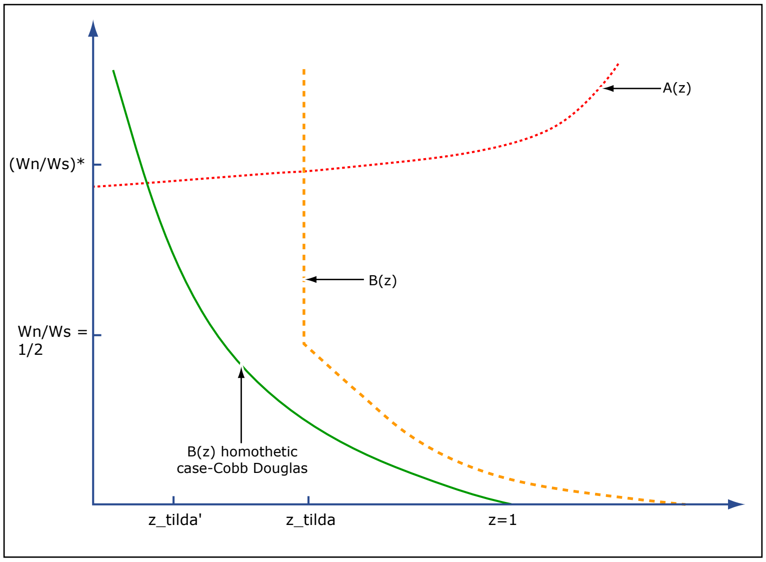
The difference between Questions 1 and 2 comes from the trade balance condition. Given that South will produce all the goods z such that $z \leq \tilde{z}$, $p\left(z\right) =$

$w_S a_S(z)$, the value of South imports is $\max\left\{w_S - \int_0^{\tilde{z}} w_S a_S(z)\,dz, 0\right\}$, i.e. either the South will export the remainder of its income after consuming all the lower goods produced in the South or it won't have any income left and won't export. South will spend the remainder of its income on exports if something remains. The value of their exports is $\min\left\{w_N, \int_0^{\tilde{z}} w_S a_S(z)\,dz\right\}$, i.e. the North will either spend all of its income on the lower range of goods produced by South by importing or purchasing all the South produced goods. Hence, if $w_N \leq \int_0^{\tilde{z}} w_S a_S(z)\,dz$, the trade balance condition is $1 - \int_0^{\tilde{z}} a_S(z)\,dz = w_N/w_S$ and if $w_N > \int_0^{\tilde{z}} w_S a_S(z)\,dz$, the trade balance condition becomes $1 - \int_0^{\tilde{z}} a_S(z)\,dz = \int_0^{\tilde{z}} a_S(z)\,dz$. It can be rearranged and summarized as

$$\begin{aligned}\int_0^{\tilde{z}} a_S(z)\,dz &= \frac{1}{2} \text{ if } w_N/w_S \geq 1/2 \\ \int_0^{\tilde{z}} a_S(z)\,dz &= 1 - \frac{w_N}{w_S} \text{ if } w_N/w_S < 1/2\end{aligned}$$

Given the North has an absolute advantage in all goods, i.e. $a_N(z) < a_S(z)\ \forall z$, we know that $w_N/w_S = a_S(\tilde{z})/a_N(\tilde{z}) > 1$. Hence, there exists an equilibrium where $\widetilde{z}$ solves $\int_0^{\tilde{z}} a_S(z)\,dz = \dfrac{1}{2}$ from the trade balance condition. w_N/w_S solves the $A(z)$ schedule given the solution to $\widetilde{z}$. However, notice that given $a_N(z) < a_S(z)\ \forall z$, $\omega < 1$ and $B(z) = \int_0^{\tilde{z}} a_N(z)\,dz = \dfrac{1}{2}$ which defines the $\tilde{z}$.

There are several differences from the previous case which all come directly from the non-homotheticity of preferences. The unit labor requirements do not appear in the trade balance condition because of the Cobb-Douglas preferences. In this case, a change in $a(.)$ affects the labor market equilibrium through the prices as it will shift the demand towards the North goods when $a_S(z)$ decreases. To restore the balance of trade, the North will expand the range of good it produces. Furthermore, there is an asymmetry between $a_S(.)$ and $a_N(.)$. A decrease in $a_N(.)$ will only affect the price of North goods and the consumer will increase the range of North goods keeping unchanged the range of South goods it consumes. There is no transfer from the goods produced from the South to the North goods as we had earlier in the Cobb Douglas case.



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(c). Study the welfare implications in both countries of a decrease in α_N. Explain the difference between the settings here and that in Question 1 part (b).

Solution: Suppose that α_N decreases, i.e. North experiences (uniform) technological progress. In this case, $w_S/w_N \searrow$ and $\tilde{z}$ remains the same as we remain in the region where $\tilde{z}$ is defined by $\int_0^{\tilde{z}} a_S(z)\,dz = \frac{1}{2}$.

As we have already mentioned, the utility of the agent in country i is equal to $\bar{z}(w_i)$. In order to show that there has been a welfare improvement, one therefore needs to show that the range of goods consumed in a particular country increases.

Consider the North first. The budget constraint of a Northern agent before technological change is given by:

$$w_S \int_0^{\tilde{z}} a_S(z)\,dz + \int_{\tilde{z}}^{\bar{z}(1)} a_N(z)\,dz = 1$$

The budget constraint of a Northern agent after technological change is given by:

$$w_S' \int_0^{\tilde{z}} a_S(z)\,dz + \int_{\tilde{z}}^{\bar{z}'(1)} a_N'(z)\,dz = 1$$

It is easy to argue from the two equations above that given w_S' is smaller and $a_N'(z)$ is smaller too for all z, the range of goods the North is consuming increases, i.e. $\bar{z}(1)' \geq \bar{z}(1)$. So the North gains.

Now consider the South. The budget constraint of a Southern agent before technological change is given by:

$$\int_0^{\tilde{z}} a_S(z)\,dz + \int_{\tilde{z}}^{\bar{z}(w_S)} \frac{a_N(z)}{w_S}dz = 1$$

The budget constraint of a Southern agent after technological change is given by:

$$\int_0^{\tilde{z}} a_S(z)\,dz + \int_{\tilde{z}}^{\bar{z}'(w_S)} \frac{a'_N(z)}{w'_S} dz = 1$$

If $\dfrac{a_N(z)}{w_S} = \dfrac{a'_N(z)}{w'_S}$ for all $z \geq \tilde{z}$, then $\bar{z}'(w_S) = \bar{z}(w_S)$. Given that the technological change is uniform, we can show that $\dfrac{a_N(z)}{w_S} = \dfrac{a'_N(z)}{w'_S}$. Hence, the technological advance does not have any welfare implication for the South. From the A schedule we know that in equilibrium:

$$\frac{a_N(z)}{w_S} = \frac{a'_N(z)}{w'_S} = \alpha_S e^{\beta_N(z-\bar{z})+\beta_S\bar{z}}$$

So welfare does not change in the South. A decrease in α_N leads to an increase in demand only for northern goods as the income of the northern group increases. This differs from the homothetic preferences case with a uniform technology improvement. A change in productivity will have an impact on the range of goods produced in the South and as a consequence a technological advance in the North will have spillover effects on the South.

3. *(10 marks) Continue with the environment in Question 1 (i.e. return to the assumptions that* $z \in [0,1]$ *and that households have preferences given by (1).) However, technology is now characterized by local external economies of scale*

$$a_i(z) = \frac{\alpha e^{\beta_i z}}{A[q_i(z)]}, \tag{7}$$

where $\alpha > 0$ is a constant, $q_i(z) \geq 0$ *is the output of good* z *in country* i*, and* $A(.)$ *is strictly increasing concave, and has an elasticity smaller than one.*

(a) Show that there exist multiple free trade equilibria under perfect competition.

Solution: Note that this question and Question 4 are inspired by Grossman and Rossi-Hansberg (QJE 2009). Suppose that $A(0) = 0$. In this case, the multiplicity of equilibrium is trivially shown as a country producing a good has an infinite productivity advantage over a country not producing it. Hence, any pattern of trade could be an equilibrium.

(b) Show that one country can be worse off under free trade than under autarky.

Solution: Suppose that $A(\cdot) = q^{1/2}$. Then under autarky, we have:

$$q_N(z) = L_c w^a / w^a \left[q_N(z)\right]^{-1/2} e^{\beta_N z}$$

As a consequence, the North consumption of good z, is given by $q_N(z) = \left(L_c/e^{\beta_N z}\right)^{2/3}$

$$U_{Na} = \int_0^1 \ln\left(\left(L_c/e^{\beta_N z}\right)^{2/3}\right) dz$$

Under free trade, consider an equilibrium where the North produces the goods $z \le \bar{z}$, and the South produces the goods $z \ge \bar{z}$. Hence, North aggregate consumption of good z is given by

$$q_N(z) = \begin{cases} L_c/\left[q_S(z) + q_N(z)\right]^{1/2} e^{\beta_N z} & \text{if } z \le \bar{z} \\ L_c w_N/w_S \left[q_S(z) + q_N(z)\right]^{1/2} e^{\beta_S z} & \text{if } z \ge \bar{z} \end{cases}$$

where $q_S(z) + q_N(z) = \left(L_c\left(1 + w_S/w_N\right)/e^{\beta_N z}\right)^{2/3}$, if $z \le \bar{z}$ and $q_S(z) + q_N(z) = \left(L_c\left((w_N/w_S) + 1\right)/e^{\beta_S z}\right)^{2/3}$, if $z > \bar{z}$.

$$U_{NT} = \int_0^{\bar{z}} \ln\left(\left(L_c\left(1 + w_S/w_N\right)/e^{\beta_N z}\right)^{2/3}\right) dz + \int_{\bar{z}}^1 \ln\left(\left(L_c\left((w_N/w_S) + 1\right)/e^{\beta_S z}\right)^{2/3}\right) dz$$

Hence, we have:

$$\begin{aligned} \Delta U_N &= \int_0^{\bar{z}} \ln\left(\left(L_c\left(1 + w_S/w_N\right)/e^{\beta_N z}\right)^{2/3}\right) - \ln\left(\left(L_c/e^{\beta_N z}\right)^{2/3}\right) dz \\ &+ \int_{\bar{z}}^1 \ln\left(\left(L_c\left((w_N/w_S) + 1\right)/e^{\beta_S z}\right)^{2/3}\right) - \ln\left(\left(L_c/e^{\beta_N z}\right)^{2/3}\right) dz \\ &= \frac{2}{3}\bar{z}\ln\frac{1}{1-\bar{z}} + \frac{2}{3}\ln\left(1/\bar{z}\right)\left(1-\bar{z}\right) + \left(\beta_N - \beta_S\right)\frac{1-\bar{z}^2}{3} \end{aligned}$$

The second line follows from the trade balance condition which should hold: $w_N/w_S = \dfrac{(1-\bar{z})}{\bar{z}}$. The third term is negative. The North could lose under free trade if for instance, for given $\bar{z}$, $\beta_S \ggg \beta_N$.

$$2\frac{\bar{z}\left(\ln\frac{\bar{z}}{1-\bar{z}}\right) - \ln\left(\bar{z}\right)}{1-\bar{z}^2} < \left(\beta_S - \beta_N\right)$$

Suppose $\bar{z} = 1/2$, $\frac{8\ln(2)}{3} < \beta_S - \beta_N$.

4. (10 marks) Continue with the environment in Question 3 (ie this is the economy in Question 1 but with external economies of scale given by (7)*.) However, now let there be, in each country i and industry z, $n_{iz} > 2$ firms competing a la Bertrand.*

(a) Show that there exists a unique free trade equilibrium under Bertrand competition.

Solution: Bertrand competition between the producers in an industry in each country will drive the price for every good z down to the average cost,

$\frac{\alpha e^{\beta_i z}}{A[q_i(z)]} w_i$. Furthermore, producers in the two countries, as they also compete a la Bertrand, will drive the international price in an industry to the minimum average cost, i.e. $p(z) = \min\left\{\frac{\alpha e^{\beta_N z}}{A[\bar{q}(z)]} w_N, \frac{\alpha e^{\beta_S z}}{A[\bar{q}(z)]} w_S\right\}$ where $\bar{q}(z)$ is the world demand for good z produced by country i. Suppose that the Northern firm in industry z has set the price of good z at $p(z) = \frac{\alpha e^{\beta_N z}}{A[\bar{q}(z)]} w_N$, then a firm in the country z could enter the market and set the price at $p(z)' = \frac{\alpha e^{\beta_N z}}{A[\bar{q}(z)]} w_N - \varepsilon$, $\forall \varepsilon > 0$, and capture the entire demand $\bar{q}(z)$. This strategy will be profitable for the Southern producer in industry z if its per unit cost is less than the price, i.e. $p(z)' \geq \frac{\alpha e^{\beta_S z}}{A[\bar{q}(z)]} w_S$. Hence, an industry is localized in a country i if the average cost of producing the total demand in country i is smaller than the average cost of producing the total demand in country j. This is then exactly the DFS case explored in class.

(b) Show that free trade is always Pareto superior to autarky.

Solution: One could rank the goods $z \in [0,1]$ such that $\widetilde{A}(z) \equiv e^{(\beta_S - \beta_N)z}$ is increasing in z. Hence, the pattern of trade and comparative advantage will be identical to the previous case where there is a good $\tilde{z}$ where North produces all the goods above and foreign produces all the goods below. In both countries, the price of good z under autarky is given by $p_i^a(z) = \frac{\alpha e^{\beta_i z}}{A[q_i(z)]} w_i$, for all $i = N, S$ and all $z \in [0,1]$, where w_i^a and $q_i^a(z)$ are the wage and the output of good z, respectively, in country i under autarky. Hence, the utility under autarky is given by

$$U_{ia} = \int_0^1 \ln\left(\frac{A[q_i^a(z)]}{\alpha e^{\beta_i z}}\right) dz$$

Under free trade, the utility level is given by

$$U_{iT} = \int_0^1 \ln\left(\frac{w_i A[\bar{q}(z)]}{\alpha \min\{w_N e^{\beta_N z}, w_S e^{\beta_S z}\}}\right) dz$$

Hence, we have again

$$\begin{aligned} \Delta U_S &= U_{ST} - U_{Sa} \\ &= \int_0^{\bar{z}} \ln\left(\frac{A[\bar{q}(z)]}{A[q_S^a(z)]}\right) dz + \int_{\bar{z}}^1 \ln\left(\frac{w_S e^{\beta_S z} A[\bar{q}(z)]}{w_N e^{\beta_N z} A[q_S^a(z)]}\right) dz \end{aligned}$$

First, notice that $\forall z < \bar{z}$, $\bar{q}(z) \geq q_S^a(z)$ because $\bar{q}(z) = q_S(z) + q_N(z)$ is such that $\bar{q}(z) = L\frac{A[\bar{q}(z)]}{\alpha e^{\beta_S z}}\left(1 + \frac{w_N}{w_S}\right)$ and $\forall z \geq \bar{z}$, $\bar{q}(z) \geq q_S^a(z)$ because $\bar{q}(z) = q_S(z) + q_N(z)$ is such that $\bar{q}(z) = L\frac{A[\bar{q}(z)]}{\alpha e^{\beta_N z}}\left(1 + \frac{w_S}{w_N}\right)$. Furthermore, we know that $e^{(\beta_S - \beta_N)(z - \bar{z})} \geq 1$, $\forall z \geq \bar{z}$. Hence, $\frac{w_S e^{\beta_S z} A[\bar{q}(z)]}{w_N e^{\beta_N z} A[q_S^a(z)]} \geq 1$. The same argument applies for North. Here in both cases there are two effects that enter:

(i) External economies of scale which drives $A[\bar{q}(z)]$ up as a country produces for the world now and (ii) the usual terms of trade effect. Both effects play a role in driving the welfare gains from trade.

5. (25 marks) Consider the model in (the extremely influential work of) Eaton and Kortum (2002). This question will ask you to work through and comment on the key results in this paper. For simplicity, throughout this question set the unit cost of production c_i equal to simply wi (that is, no intermediate goods are used in production).

(a) Explain what the parameter θ in the EK2002 model captures. Given this understanding, explain how you expect the interdependence of countries in this model to scale with θ. Discuss further how you expect concepts like the size of the gains from trade, the extent to which trade flows rise as trade costs fall, and the extent to which foreign technology shocks affect economic outcomes at home, all to vary with θ.

Solution: In the EK 2002 model, let $z_i(j)$ be defined as the efficiency in producing good $j \in [0,1]$ in county j, i.e. the amount of input needed to produce a unit of the good. They assume that z_i is randomly drawn from a Frechet distribution independently for each j :

$$\begin{aligned} F_i(\bar{z}) &= P(z_i \leq \bar{z}) \\ &= e^{-T_i z^{-\theta}} \end{aligned}$$

where $T_i > 0$ captures the location of the good. In other words, it will capture the absolute advantage of a country in producing all the goods as a bigger T_i implies that a high efficiency draw for any good j is more likely in country i. On the other hand, the parameter $\theta > 1$ captures the variance of the distribution. It reflects the amount of variation within the distribution. A bigger θ implies less variability as the log of z has standard deviation $\pi/(\theta\sqrt{6})$. θ governs the heterogeneity across goods in the countries' relative efficiencies and as such it governs the comparative advantage over the set of goods.

For instance, consider the extreme case of an economy with two goods and two countries, where the North has an absolute advantage in both goods and there is no variance between the productivities of the two goods in either country. In this case, no country is relatively more efficient than the other country in producing one good relative to the other good. As such there is no comparative advantage in production. This simple example illustrates how the parameter θ will govern the comparative advantage. As the heterogeneity in the productivity accross the set of goods increase for each country (*low* θ), the comparative advantage will also play a stronger role for trade.

Overall, the gains from trade should be higher, the lower is θ. If there is a lot of variation in the productivity accross goods for each country, then there should be a lot of efficiency gains to be made from trading. Likewise, a change in the barriers to trade should have a higher impact the trade flow the higher is θ (the force of comparative advantage weakens). Hence there are fewer efficiency outliers that overcome differences in geographic barriers. Finally, the sensitivity of any economic outcome in the model to an exogeneous technology shock abroad (ie the 'interdependence of nations') will be higher the higher the scope for trade through comparative advantage, ie the lower is θ (assuming that the technology shock leaves the variance unchanged and only affects T). See Hseih and Ossa (2011) for estimates of such effects.

(b) Derive the distribution of prices in country n, $G_n(p)$ (equation (6) in EK2002). Comment on the attractions of the Frechet productivity distribution in this derivation—when did it simplify things? Would a simple expression like that in equation (6) be possible if countries each had their own paramter θ_i?

Solution: Let d_{ni} be the "iceberg" cost, i.e. delivering a unit from country i to country n requires producing d_{ni} units in country i. The price of a good in a given country n is given by:

$$p_n(j) = \min\{p_{ni}(j),\ i = 1, ...N\},\ \text{where } p_{ni}(j) = \frac{w_i}{z_i(j)} d_{ni}.$$

By definition, the distribution of the price in a country n is given by $G_n(p) = \Pr(p_n \leq p)$. Given the price is defined as above:

$$\begin{aligned} G_n(p) &= 1 - \prod_i (1 - G_{ni}(p)) \\ \text{where } G_{ni}(p) &= \Pr(p_{ni}(j) \leq p) \\ &= \Pr\left(\frac{w_i}{p} d_{ni} \leq z_i(j)\right) = 1 - e^{\left\{-T_i(w_i d_{ni})^{-\theta} p^{\theta}\right\}} \end{aligned}$$

Hence,

$$G_n(p) = 1 - e^{\left(-\varphi_n p^{\theta}\right)}$$

where $\varphi_n = \sum_i T_i (w_i d_{ni})^{-\theta}$. Hence, the prices in country n follow a Frechet distribution with parameter φ (capturing the country's effective state of technology with technology available from other countries, discounted by input costs and geographic barriers) and θ. This results follows directly from both the Frechet distribution because the extremum of a number of iid draws from an extreme value distribution (like the Frechet) is also an extreme value distribution; one can show that this is only true for an extreme value distribution. Note also that the uniformity of θ was important as it allowed us to take p^{θ} out of the sum in $\prod_i \exp\left\{-T_i (w_i d_{ni})^{-\theta} p^{\theta}\right\} = \exp\left\{-p^{\theta} \sum_i T_i (w_i d_{ni})^{-\theta}\right\}$ and hence neatly characterize the price distribution as Frechet with location parameter φ_n.

(c) Prove that the probability that country i provides a good at the lowest price in country n is simply country i's contribution to country n's price parameter φ_n, or that $\pi_{ni} = \frac{T_i(w_i\ d_{ni})^{\theta}}{\varphi_n}$. What sort of data could you use to test this prediction?

Solution: First, define π_{ni} as the probability that country i provides a good at the lowest price in country n. For $p_{ni}(j) = p$, this probability is given by $\pi_{ni} = \Pr(p_{ni}(j) \leq p_{ns}(j), s \neq i)$, ie the probability that the price at p is smaller than the price from any other country s. This is:

$$\Pr(p \leq p_{ns}(j), s \neq i) = e^{p^{\theta}\left\{-\sum_{s\neq i} T_s(w_s d_{si})^{-\theta}\right\}}$$

In order to find, the probability that the country i provides a good in country n at the lowest price, we need to integrate this expression over all possible p, from the distribution of prices p that country i can offer country n, to obtain:

$$\begin{aligned}
\pi_{ni} &= \int_0^{\infty} e^{p^{\theta}\left\{-\sum_{s\neq i} T_s(w_s d_{si})^{-\theta}\right\}} dG_{ni}(p) \\
&= \int_0^{\infty} e^{p^{\theta}\left\{-\sum_{s\neq i} T_s(w_s d_{si})^{-\theta}\right\}} \theta p^{\theta-1} T_i (w_i d_{ni})^{-\theta} \exp\left\{-T_i (w_i d_{ni})^{-\theta} p^{\theta}\right\} dp \\
&= \frac{T_i (w_i d_{ni})^{-\theta}}{\varphi} \int_0^{\infty} \varphi e^{p^{\theta}\left\{-\sum_{i} T_s(w_s d_{si})^{-\theta}\right\}} \theta p^{\theta-1} dp \\
&= \frac{T_i (w_i d_{ni})^{-\theta}}{\varphi}
\end{aligned}$$

From the first line to the second line, we used the definition of $G_{ni}(p)$ from the previous question and I simply differentiate the cdf of the price a good from country i to country n. From the third to the fourth line, I used the fact that:

$$\int_0^{\infty} \varphi e^{p^{\theta}\left\{-\sum_{i} T_s(w_s d_{si})^{-\theta}\right\}} \theta p^{\theta-1} dp = 1 \tag{8}$$

As $\varphi e^{p^{\theta}\left\{-\sum_{i} T_s(w_s d_{si})^{-\theta}\right\}} \theta p^{\theta-1}$ is $dG_n(p)$, i.e. the kernel of a Frechet distribution. By definition, it should integrate to one.

We have seen that the probability that country i is the least cost supplier of country n, is given by the relative contribution of the country i in the price distribution parameter of the price in country n, i.e. φ_n. As such the higher is country i's absolute advantage in producing all goods, (high T_i), the higher is the probability that country i will be the least cost supplier. Likewise, the lower is the labor cost or the trade barriers between country i and n, the more likely country i will be the least cost supplier.

To test this prediction, one possibility would be to use data on producer prices in country i (which should be equal to $T_i (w_i d_{ni})^{-\theta}$) and estimate a reduced-form empirical relationship between these producer prices in i and the total price distribution in country n (ie $G_n(p)$). One could then compare these reduced-form coefficients with data on $T_i (w_i d_{ni})^{-\theta}$, which could be estimated from trade data (recall that the exporter fixed effect for country i should be equal to $T_i (w_i)^{-\theta}$, and standard gravity tools can be used to back out or estimate determinants of $d_{ni}^{=\theta}$.

(d) Prove that the price of a good that country n actually buys from any country i also has the distribution $G_n(p)$—or that, for all goods consumed in country n, conditioning on the source of their production has no bearing on the good's price. Give the intuition for why this is true. What sort of data could you use to test this prediction?

Solution: If country n buys from country i, then counrty $i's$ price offered in country n must be the lowest cost supplier of that good to country n. If the price at which the country i sells the good is q, then, as we know from the previous exercise, the probability that country i is the lowest cost supplier of the good in n is given by $\exp\left(q^\theta\left\{-\sum_{s\neq i} T_s\left(w_s d_{si}\right)^{-\theta}\right\}\right)$. Integrating over all possible prices q such that $q \leq p$ this probability becomes:

$$
\begin{aligned}
& \int_0^p q^\theta\left\{-\sum_{s\neq i} T_s\left(w_s d_{si}\right)^{-\theta}\right\} \\
= & \int_0^q e^{q^\theta\left\{-\sum_{s\neq i} T_s\left(w_s d_{si}\right)^{-\theta}\right\}}\theta q^{\theta-1} T_i\left(w_i d_{ni}\right)^{-\theta} e^{\left\{-T_i\left(w_i d_{ni}\right)^{-\theta} q^\theta\right\}} dq \\
= & \frac{T_i\left(w_i d_{ni}\right)^{-\theta}}{\varphi}\int_0^p \varphi e^{q^\theta\left\{-\sum_i T_s\left(w_s d_{si}\right)^{-\theta}\right\}}\theta q^{\theta-1} dq \\
= & \pi_{ni} G_n(p)
\end{aligned}
$$

Given that π_{ni} is the probability that country i is the least cost supplier of a good in country n, then $G_n(p)$ is the conditional distribution of the price charged by country i in n for the goods that i sells in n:

$$G_n(p) = \frac{\int_0^p q^\theta\left\{-\sum_{s\neq i} T_s\left(w_s d_{si}\right)^{-\theta}\right\}}{\pi_{ni}}$$

Hence the price of a good that country n actually buys from any country i also has the distribution $G_n(p)$. The intuition behind this result is that all the adjustment is on the extensive margin. A country with a higher T, lower w, or lower d will exploit its advantage by selling a wider range of goods, exactly to the point at which the distribution of prices for what it sells in n is the same as n's overall price distribution. Another source of intuition for this result is that, in equilibrium, if country i were able to provide goods to n more cheaply than some other country j could provide goods to country n, then consumers in n would buy from i and not from j. This could not be an equilibrium and hence something has to adjust (country i in selling more goods will push up its wage and move into goods at which it is relatively less productive).

To test this prediction one simply needs import price data—that is, data on the price of imported goods as they enter the importing country. These data would have to be in so-called 'cif' terms, ie they are the prices after all trade costs have been paid. One could then simply compare the distribution of import prices by source and test the prediction of the model that these price distributions do not depend on the source. One could do this simply by regressing import prices on source country dummies, or more sophisticatedly by comparing the entire distribution of these prices across sources (for example, as one student pointed out, via a Kolmogarov-Smirnoff test for the equality of distributions).

(e) Explain why the (constant) elasticity of substition on consumer preferences, σ, does not enter the equation for trade flows (equation (10)). Was the assumption of CES preferences necessary for the derivation of equation (10)?

Solution: The CES parameter does not enter equation (10) because in this model all of the adjustment is on the extensive margin.

At first glance it appears that the assumption of CES preferences is not necessary for the derivation of the equation (10)—we appeared to use no assumption on tastes whatsoever. That should be a very surprising result—that we could presume to derive an expression for country n's imports from country i without any attention to the tastes of consumers in country n. In fact, imagine an extreme case where consumers in country n only want the 'first' good in the continuum; in this case country n will simply buy this good from the country that offers it the lowest price. Note that we were a bit sloppy in the answer to (d) above. We derived the distribution of prices of goods that country i can supply to country n when these are goods for which country i is the cheapest supplier in the world. We then equated this distribution to the distribution of prices of goods that country n will actually buy from country i but that last equation implicity required a statement about demand—that consumers want all goods and will buy them from whoever is cheapest.

So it should be clear that, for the logic in the derivation in part (d) above to go through, we at least require a restriction on tastes that is such that consumers demand at least some of every good. CES preferences deliver that restriction because the marginal utility of consumption of any good tends to infinity as the level of consumption of that good tends to zero. (And note that this is true no matter what the actual CES parameter is. Hence it is unsurprising that this parameter doesn't enter the expression for trade flows.) But would another demand system (that still has infinite marginal utility at zero demand) be sufficient for the derivation of equation (10) to go through?

(f) Derive equation (12). Interpret this result and give the intuition behind it.

Solution: Remember that $\pi_{ni} = \dfrac{T_i (w_i d_{ni})^{-\theta}}{\varphi_n}$ and $\pi_{ii} = \dfrac{T_i (w_i)^{-\theta}}{\varphi_i}$ (under the normalization/assumption that intra-national trade is free, or $d_{ii} = 1$). Hence,

$$\frac{X_{ni}}{X_n} / \frac{X_{ii}}{X_i} = \frac{\varphi_i}{\varphi_n} d_{ni}^{-\theta}$$

Using the fact that for a CES utility function, the price index is defined as (assuming that $\sigma < 1 + \theta$) :

$$P_n = \gamma \varphi_n^{-1/\theta}$$

where γ is a constant that depends on θ and σ, and hence is not country specific given that θ and σ are not country specific. Hence,

$$\frac{X_{ni}}{X_n} / \frac{X_{ii}}{X_i} = \frac{\varphi_i}{\varphi_n} d_{ni}^{-\theta} = \left(\frac{P_i}{P_n} d_{ni} \right)^{-\theta}$$

This expression characterizes how much country n will buy from country i relative to how much country i will buy from country i. The RHS says this should be related, in a loose sense, to arbitrage opportunities. If goods are cheap in country i, and they are expensive in country n, and the cost of getting them from i to n is not too high, then country n will buy a lot from country i. However, the parameter θ moderates these arbitrage opportunities because if θ is high then the scope for comparative advantage is low (loosely, countries' goods are more substitutable in terms of productivities), so it takes huge price gaps to induce much trade.

(g) Write the welfare of country i in the model as a function of just one intuitive endogenous variable. Can you explain the intuition behind this result?

Solution: The welfare of (a representative agent in) country i is simply its real wage, $\frac{w_i}{P_i}$. This can be written simply as:

$$\frac{w_i}{P_i} = \gamma \left(\frac{T_i}{\pi_{ii}} \right)^{1/\theta}$$

which is derived from $\pi_{ii} = \frac{T_i (w_i)^{-\theta}}{\varphi_i}$ and the definition of the price index. Hence, the welfare of country i is a function of a single endogenous variable, π_{ii}, which is the share of total expenditure that this country spends on its own goods. Recall that this model has a fixed set of varieties, and that a given country's productivity on any given variety is a constant (ie it is not affected by openness in any way). It should therefore not be surprising that the only way trade can raise welfare here is by allowing country i to source some of those varieties from other countries that are relatively more productive at making those varieties. The structure of this model means that the productivity with which these foreign-sourced varieties are made, relative to how productive the home country is making them, is a function of π_{ii} alone.

(h) Some authors like to think of the welfare of country i in models like this as the product of two (endogenous) terms or variables. The first term is often termed 'consumer market access' (CMA), which is meant to summarize how well consumers in country i are positioned for accessing markets that sell the goods they want. The second term is often termed 'firm market access' (FMA), and

this is meant to summarize how well firms in country i are positioned to access markets at which they can sell the goods they produce. Obviously this is vague. But try to interpret these notions in a way that you think is sensible in the context of the EK2002 model and come up with an expression in which welfare can be written simply as the product of CMA and FMA (and some exogenous variables/parameters).

Solution: This question was inspired by Redding and Sturm (AER, 2008) who use CMA and FMA (their terms) to study how welfare fell in West German cities that were close to the East-West Germany border after partition. Welfare is simply the nominal wage times one over the consumer price index. Start with the price index, which is equal to $\varphi_n = [\sum_i T_i (w_i d_{ni})^{-\theta}]^{-1/\theta}$. This is a natural measure of 'consumer market access' (CMA) because it tells you how close consumers in n are to all markets i (and where market distances are weighted according to how 'attractive' each market is, which depends on $T_i (w_i)^{-\theta}$.

Turning now to the nominal wage of country w_i, one can think of this as a measure of FMA because, with perfectly competitive product and labor markets and one factor, labor, firms will pass on to workers any benefits of beneficial market access to workers. Now, to make the connection explicit, one can use the trade balance condition to write:

$$\begin{aligned} w_i L_i &= \sum_i X_{ni} = \sum_i T_i (w_i d_{ni})^{-\theta} X_n p_n^{\theta} \\ \iff \quad w_i &= \left(\frac{T_i}{L_i} \sum_i (d_{ni})^{-\theta} X_n p_n^{\theta} \right)^{1+\theta} \end{aligned}$$

So this expresses the nominal wage in country i as a sum, over all markets, of terms that involve the total expenditure in those markets (X_n), the price level in those markets (ie how competitive they are, p_n), and these terms are weighted by the cost of accessing these markets from country i. Written in this way one can see how the nominal wage can be though of as 'firm market access'.

(i) Imagine that trade is free between all countries. Derive a closed-form expression for the welfare level in country i as a function of exogenous variables only. Interpret this expression.

Solution: If trade is free, then there is no trade cost (ie $d_{ni} = 1$). As such, the law of one price must hold and from the equilibrium in the labor market:

$$\frac{w_i}{w_n} = \left(\frac{T_i / L_i}{T_n / L_n} \right)^{1/(1+\theta)}$$

Hence, combining this expression in the definition of welfare, one can show that:

$$\frac{w_i}{P_i} = \gamma^{-1} L_i w_i \varphi_i^{1/\theta} = \gamma^{-1} T_i^{(1/(1+\theta))} \left[\sum_{k=1}^{N} T_k^{1/(1+\theta)} (L_k / L_i)^{\theta/(1+\theta)} \right]^{1/\theta}$$

where we use (i) $\varphi_i = \sum_s T_s (w_s)^{-\theta}$ and (ii) $w_s = w_i \left(\frac{T_s/L_s}{T_i/Li}\right)^{1/(1+\theta)}$.

Absent any trade barriers, one can notice from the expression above that welfare is increasing in the technology everywhere (because all countries sell some goods to the country in question). However, home technology is especially important to the welfare of home residents because they own this technology. The home technology term enters divided by the size of the home labor force—this is because as a country grows it will produce more goods and have its terms of trade move against it. (Note that this terms-of-trade effect is not present under autarky. Under autarky there are pure constant returns to scale, but trade leads to effective decreasing returns to scale through terms of trade effects, as in Acemoglu and Ventura (2002).) The foreign technology terms enter multiplied by these country's labor force sizes—this is because the home country benefits if it can trade with large and highly productive countries. (When a foreign country grows in size the home country sees a terms of trade improvement.)

(j) Imagine now that there are just two identical countries in the world (but trade is not free and instead incurs the standard iceberg cost d). Derive the simplest possible expression for the level of welfare in either of these countries as a function of d (feel free to make all the normalizations you want in order to focus on the role of just d). Interpret this expression and give the intuition for it. Try evaluating this expression numerically at different values of d and θ that you think are plausible and discuss your answers. Are the gains from trade in this simple model 'large' or 'small'?

Solution: Normalize such that $T_k = 1$ for all countries k. Further, set the wage in country 1 to be the numeraire. By symmetry this will also be the wage in country 2. Then we know that:

$$\pi_{21} = \frac{1}{1 + (d)^{-\theta}}$$

Hence, if we also normalize $\gamma = 1$ we have:

$$\frac{w_i}{p_i} = (1 + d^{-\theta})^{1/\theta}$$

so that welfare in this 2-country symmetric world is a simply function of just d and θ. As expected given our normalizations, for autarky ($d \to \infty$) and at any finite θ, $\frac{w_i}{p_i} = 1$. On the other hand, with $d = 1$ (*no* trade cost) $\frac{w_i}{p_i} = 2^{1/\theta}$. For the value of θ estimated by EK (2002) of approximately 8, we can notice that $\frac{w_i}{p_i} = 2^{1/8} = 1.09$.So the gains from free trade here (free trade welfare divided by autarky welfare) are just 9 percent of real GDP. With $\theta = 4$ this rises to 19 percent. I am not sure whether these numbers are 'large' or 'small'.

Staring at this welfare formula one can see that as d rises from 1 (ie trade gets costly) the gains from trade disappear quickly because d enters to the power of minus θ. For example, with $d = 1.25$ (a 25 percent ad valorem trade cost), $d^{-\theta} = 0.16$ for $\theta = 8$, and the cost of going to autarky is just a fall in welfare from 1.02 to 1. (For $\theta = 4$ these numbers become $d^{-\theta} = 0.41$ and welfare falls from 1.09 to 1.)

(k) EK2002's preferred estimate of θ is 8.28. Explain how this estimate was arrived at. How well does this method fit with the EK2002 modeling approach? Offer your criticisms of this method more generally.

Solution: The equation in exercise (6) suggests that a way to estimate θ if one had data on trade shares, aggregate prices, and trade costs. However, trade costs are not observable. In order to disentangle trade costs from θ, EK (2002) propose to approximate trade costs by using disaggregate price information (on the prices of identical goods) across countries and the maximum price difference (across all sampled goods) between two countries as a bound for the trade cost. They then estimate θ via both a method of moment and an ols estimation.

One criticism of this method is that the max difference between two country's prices (of an identical good) could still be smaller than the true trade cost as one observes only the subset of all the prices (any finite sample will be a subset of the continuum of goods in the model). This criticism is developed in Simonovska and Waugh (2010), and a correction is proposed (which raises estimates of trade costs, and hence lowers estimates of θ.). A second criticism is that it is extremely hard to find price data for identical goods around the world, so the estimates will be sensitive to unobserved quality differences. A third criticism is that if real-world producers/distributors/retailers charge variable mark-ups then real world price observations will not differe by just pure trade costs. A fourth criticism is that if trade costs differ across goods (unlike in the model) then the max price gap need not be identifying any good's trade cost.

(l) Is the EK2002 preferred estimate of $\theta = 8.28$ 'large' or 'small'? Defend your answer (there is obviously no right answer!) with reference to your discussion of part (a).

Solution: Obviously there is no right answer to this. The question is just whether the auxiliary observable implications of $\theta = 8.28$ make sense. Perhaps one could argue it seems small. For starters, this value of θ implies that the standard deviation of log productivity of producers under autarky should be just 0.15, which seems low (and it would be even lower once the country starts trading and bad draws don't get used in production.) Second, the implied gains from trade are very small—lower than introspection might suggest (perhaps try to think of the chunk of your consumption basket that you import from abroad and how much more you might have to pay for those goods if they were produced domestically?). Third, the implied magnitude of international transmission channels (remember, everything forein enters mutliplied by $d^{-\theta} = 0.16$ for $\theta = 8$) seems smaller than introspection would suggest, or than the discussion of these forces merits in, eg, the popular business press.

6. *(15 marks) This question asks you to discuss some recent empirical work on estimating the (reduced-form) gains from trade, and how this work relates to theoretical work on the gains from trade.*

(a) *State how large the estimated 'gains from openness' are in Frankel and Romer (1999), and Feyrer (2009) Paper 1 and Feyrer (2009) Paper 2. Discuss whether you think the estimates in the two Feyrer (2009) papers are smaller or larger than the true average treatment effect of openness to international trade.*

Solution: Frankel and Romer (1999) and Feyrer (2009) use different methods to instrument for the effect of trade on GDP, but both find important effects of trade on GDP. Frankel and Romer (1999) instrument for a country's openness to trade by using a measure of its distance from its relatively important trade partners. The first stage consists of two steps. The first step involves estimating a gravity-like equation:

$$\ln\left(\frac{X_{ij}+M_{ij}}{GDP_i}\right) = a_0 + a_1 lnD_{ij} + a_2 N_i + a_3 N_j + a_4 B_{ij} + e_{ij} \qquad (9)$$

where $\ln(X_{ij}+M_{ij})$ is the sum of import plus export of a country i with country j, D_{ij} is the distance, B_{ij} is a shared border dummy (see the paper for full detail). The second step (of the first stage) involves summing the estimated gravity equation over all of country i's imports from all of its bilateral partners, j, and use this as the IV. Frankel and Romer (1999) then regress log GDP per capita on the trade share (instrumented). They find (in IV regressions) that a 1 *percentage point* increase in the trade openness variable is associated with a percentage increase in GDP per capita of between 2-3 percent. If the trade openness variable has a mean of somewhere around 50 percent (see table in Appendix), then this implies an elasticity (evaluated at the mean) of 1-1.5.

Feyrer (2009) Paper 1 uses a *panel* of country-level GDP and trade data from 1960-1995 and exploits the fact that marginal cost of shipping via air fell faster over this period than the marginal cost of shipping via sea; this technological change in the transportation industry affects country pairs differently because country pairs have differing relative sea-to-air distances. Feyrer (2009) Paper 2 uses the closing and the re-opening of the Suez canal (in an era in which sea shipping dominated), which changed the bilateral sea distance separating some country pairs from one another. Feyrer Paper 1 finds that a 10% increase in the trade share increases real GDP per capita between 4% to 6% (ie the estimated elasticity is around 0.4-0.6). Feyrer Paper 2 finds that a 10% increase in the trade share increases real GDP per capita by 1.5% to 2.5% (elasticity 0.15-0.25).

The results in Feyrer Paper 1 are larger then those in Feyrer Paper 2 because the former paper was unable to control for omitted variables of international interactions (such as FDI, migration or knowledge flows) between the trading partners which could positively affect GDP and be encouraged by a shortening of effective (ie transportation cost-weighted) distance separating countries. Put more simply, these are papers about the effects of 'more openness' and not just 'lower trade costs' on GDP. Feyrer Paper 2, on the other hand, is working with something that is probably closer to a pure trade cost shock and is therefore likely to better identify the true 'treatment effect' of openness to international trade.

(b) Assuming that the estimates in Feyrer (2009) Paper 2 are unbiased estimates of the average treatment effect of an extra unit of international trade on a country's real income, do these estimates make quantitative sense in the context of the models we have seen so far in 14.581?

Solution: While we have not seen quantitative versions of neoclassical trade models so far in 14.581, one should expect much smaller gains from lower trade costs in these models than the effects found by Feyrer (2009). In all of these theories, distortions to trade (ie trade costs) induce dead-weight losses that are well approximated by Harberger triangles—proportional to the square of the size of the distortion.

(c) Do the estimates in Feyrer (2009) Paper 2 line up with the predictions for the size of the gains from trade in the Eaton and Kortum (2002) model?

Solution: As we saw above, the expected gains from trade in an EK2002 model (when a value of θ is used that makes the model best match trade flows) are very small, far smaller than the estimates in Feyrer (2009). As Arkolakis, Costinot and Rodriguez-Clare (2011) point out, this is true for a wide class of models.

(d) Discuss an amendment to Feyrer (2009) Paper 2 that would explore the extent to which the theoretical predictions about the size of the gains from trade in Eaton and Kortum (2002) fit the data. Be clear about what regression you're proposing, why you're proposing it, and what the estimates would tell us.

Solution: The theory in EK2002 could potentially be used to inform Feyrer's (2009) papers (or, put differently, Feyrer's papers could be used to directly test the EK2002 model) in at least two ways.

First, Feyrer (2009), like FR (1999), estimates (in the first stage) an equation that looks a lot like a gravity equation (and is referred to as such) in a manner that is not totally consistent with the theory of gravity models (of which EK2002 is a part). Note that, as highlighted by Anderson and van Wincoop (2003), bilateral trade depends not only on bilateral trade costs, but also on the importing and exporting countries' 'multilateral resistance' terms. These could be easily controlled for using importer-times-year and exporter-times-year fixed effects.

Second, as we saw above, EK2002 predicts that trade can only affect welfare through the sufficient statistic π_{ii}—how much of country i's expenditure it buys, in equilibrium, from itself. This is not equal to the 'trade share' that FR (1999) and Feyrer (2009) use, so according to EK2002 the FR and Feyrer second stage is also mis-specified. Note that a particularly attractive feature of using $\ln \pi_{ii}$

rather than $\ln(\frac{X_i+M_i}{GDP_i})$ as the explanatory variable is that the EK2002 theory tells us that the coefficient on $\ln \pi_{ii}$ should be $\frac{-1}{\theta}$. Hence in EK2002 there is a cross-equation restriction between the first stage and second stage equations. Or put another way, a straightforward comparison could be made between the reduced-form estimated gains from openness in a Feyrer-type study (a 'large' elasticity between GDP per capita and 'openness', π_{ii}) and those that one would expect in the EK2002 model (a 'small' predicted elasticity equal to $\frac{1}{\theta}$).

(e) Have a quick look at Woodland (1980, ReStud) on "Direct and Indirect Trade Utility Functions". Can you suggest an empirical application of these tools that would put restrictions on a cross-country empirical approach to estimating the gains from trade?

Solution: The following is a highly speculative 'solution'. Woodland (1980) highlights both the direct and indirect trade utility functions. Consider first the direct trade utility function which is defined as:

$$U(e, v) \equiv \max_{x} \{u(x - e) | x \in Y(v)\} \tag{10}$$

This says that the utility in a country that exports an amount e and is endowed with factors v will be $U(e, v)$. Dixit and Norman (1980) refer to this as the 'Meade utility function' and discuss it extensively. The function $U(e, v)$ is implicitly the function that FR (1999) and Feyrer (2009) are trying to estimate: well-being as a function of trade flows (ie something related to e). I think it would be worth exploring whether one could make flexible (in the Diewert (1976) sense) functional form assumptions about $u(.)$ and $Y(.)$, and pass these through into the function $U(e, v)$ so that it inherits the functional forms implied by the functional form assumptions on $u(.)$ and $Y(.)$. This would then be an considerably general (at least as general as the functional form assumptions, but if these are chosen 'flexibly' then these are weak assumptions) way of estimating the magnitude of the gains from trade in a neoclassical environment. This would be a sort of bridge between the entirely reduced-form estimation in FR (1999) and Feyrer (2009), and the more structural approach in ACRC (2010).

Another approach inspired by Woodland (1980) would be to use the indirect trade utility function. This would be easier but less connected to the FR/Feyrer approach. The indirect trade utility function is $H(p, b, v)$, or the max utility that can be obtained by a country with endowments v when it faces prices p and trade imbalances b. Woodland shows how this function can be easily estimated if one assumes functional forms on $u(.)$ and $Y(.)$. Taking this to the data could be done in the manner of Harrigan (1997) (which, recall, did something similar on estimating the GNP function): assume free trade so that all countries face the same p and hence all of the p terms drop out into year fixed effects. Harrigan (1997) further shows how one can allow a decent amount of cross-country heterogeneity, which might have analogous applications here (to weaken the assumption of 'identical tastes and technologies' across countries.)

7. (10 marks) This question asks you to comment on the work of Costinot, Donaldson and Komunjer (2010).

(a) Describe what you see to be the most serious criticism of this paper.

Solution: Here are two serious criticisms of this paper. First, this paper is concerned with estimating the Ricardian model but is unable to cleanly distinguish Ricardian forces from H-O forces that may be correlated with Ricardian forces. Second, estimating the partial correlation between relative exports (the dependent variable) and relative productivity levels (the key independent variable of interest), or equivalently the parameter θ, or equivalently testing the model by examining how relative productivity affects relative exports, all hinge on an orthogonality condition between relative productivity levels and the error term in this regression. The model used by CDK (2010) suggests that the error term contains all elements of the trade cost term τ_{ij}^{k} that cannot be decomposed into the form $\tau_{ij}^{k} = \tau_{ij}\tau_{j}^{k}$; the error term of course also contains specification error (ie this equation is unlikely to describe the world completely) and measurement error in trade flows. There are various potential explanations for why relative productivity levels might be correlated with the error term (see the paper for details). For this reason CDK propose an IV for relative productivity, which is relative log R&D expenditure. The concern then becomes one of whether trade costs (other than those that take the $\tau_{ij}^{k} = \tau_{ij}\tau_{j}^{k}$ form) or other elements of the error term are correlated with relative R&D expenditure. This is definitely possible.

(b) Can you suggest a better (or at least alternative) instrument for producer prices than the one used by CDK (2010)?

Solution: An alternative IV could be based on Nunn (QJE 2007), which models country-industry productivity as the product of a country-specific term ('contract enforcement') and an industry-specific term ('relationship-specific input intensity'). Nunn (2007) assumes that the latter (measured in the United States) is exogenous from the perspective of the non-US countries in his sample. However, he worries about potential endogeneity of the country-specific term and instruments for it using measures of legal origin. A similar approach could be applied here.

(c) Are there additional theoretical restrictions in the CDK (2010) model that are not being tested in the paper?

Solution: Some possibilities: First, the coefficients on z_i^k and trade costs should be the same, which is testable. Second, the main prediction is that within an pair of exporters and industries, the relationship between relative productivities and relative ('corrected') exports should be the same to any export destination (j); this is testable. Third, perhaps one could look at actual firm size distributions and estimate the truncation (ie selection effect) directly and compare this to the prediction in the model.

8. (10 marks) Consider the section of Costinot (Ecta 2009) that deals with the Ricardian model. Describe the best possible empirical paper you can imagine writing that would test this model's predictions.

Solution: Solutions to this question are entirely open-ended. Students who are interested in discussing this as a direction for research (and it is probably a promising one) should talk to Arnaud or Dave directly.

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