8.044 Statistical Physics I
Spring 2008

Facts about sums of RVs

- Exact expressions for $< s >$ and Var(s) if S.I.

- $p(s) = p(x) \otimes p(y)$ if S.I.

- $p(s)$ slightly more complicated if not S.I.

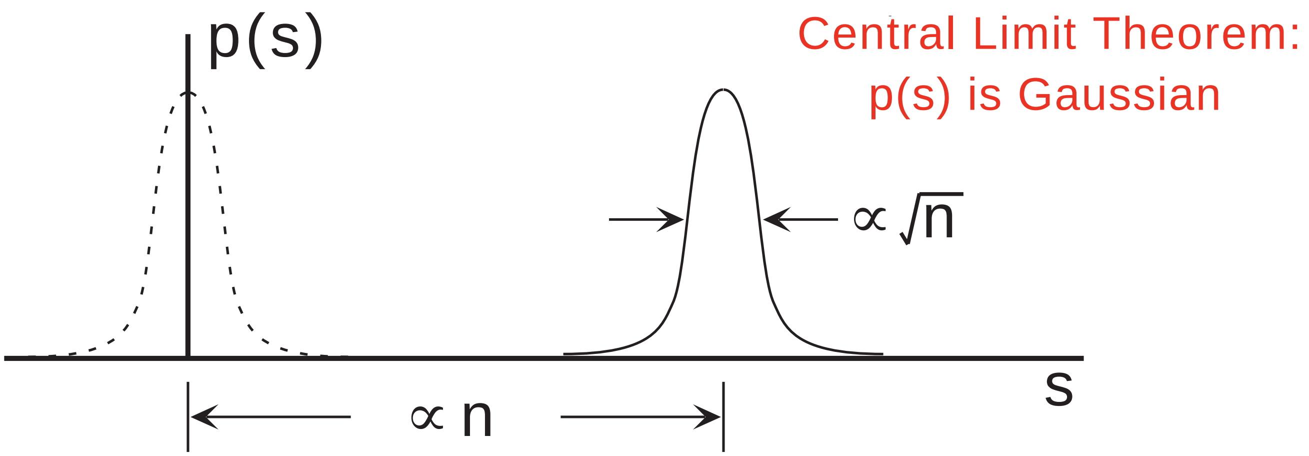
- $\otimes$ usually changes functional form

- But not always

- Fourier techniques are very useful

Very important special case: Central Limit Theorem

- RVs are S.I.
- All have identical densities $p(x_i)$
- Var(x) is finite but $< x >$ could be zero
- n is large



If x is continuous

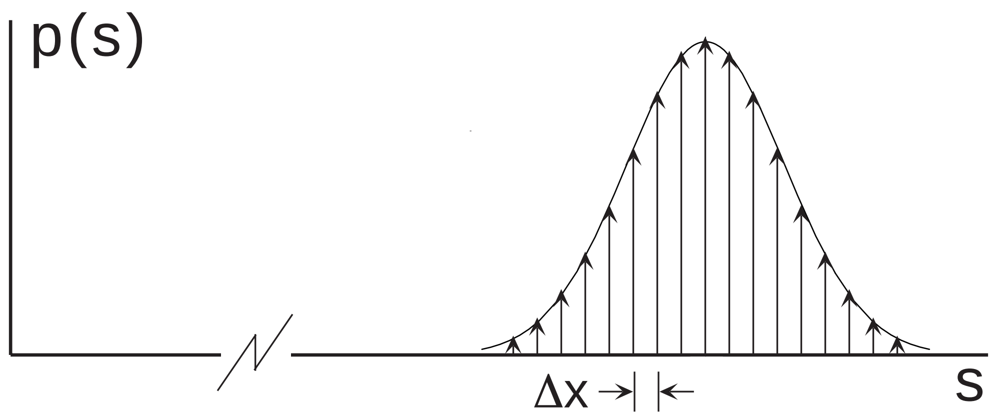
$$p(s) = \frac{1}{\sqrt{2\pi\sigma^2}}\, \mathrm{e}^{-(s-<s>)^2/2\sigma^2}$$

$$< s >= n < x >$$

$$\sigma^2 = n\, \sigma_x^2$$

If x is discrete in equal steps of Δx

$$p(s) = \sum_i \underbrace{\frac{\Delta x}{\sqrt{2\pi\sigma^2}}\, e^{-(s-<s>)^2/2\sigma^2}}_{\text{envelope}}\ \underbrace{\delta(s - i\,\Delta x)}_{\text{comb}}$$



Example K.E. of an ideal gas

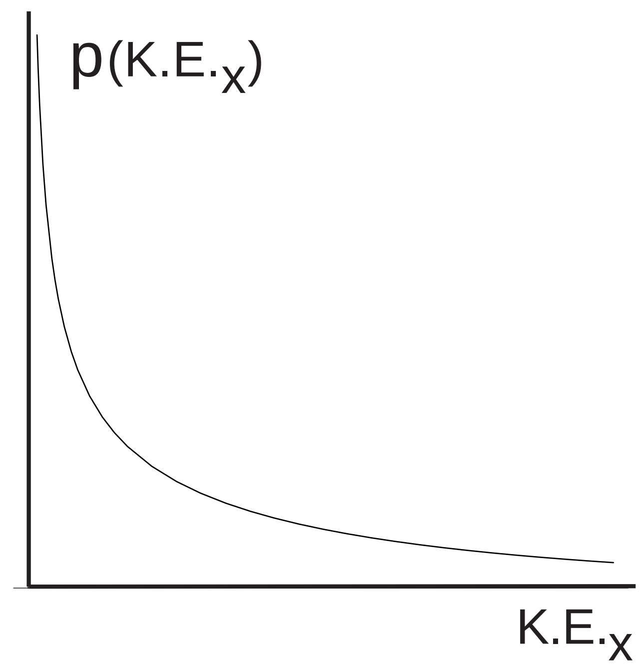
$$p(v_x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-v_x^2/2\sigma^2} \qquad \sigma = \sqrt{kT/m}$$

$$\text{K.E.}_x = \tfrac{1}{2} m v_x^2$$

$$\text{mean} = \tfrac{1}{2} m \underbrace{\langle v_x^2 \rangle}_{\sigma^2} = \tfrac{1}{2} kT$$

$$\text{mean sq.} = (\tfrac{1}{2} m)^2 \underbrace{\langle v_x^4 \rangle}_{3\sigma^4}$$

$$= \tfrac{3}{4} (kT)^2$$



$$\text{variance} = \tfrac{1}{2} (kT)^2$$

3 directions, N atoms

$E =$ total K.E.

$U \equiv \langle E \rangle = 3N \langle K.E._x \rangle = \frac{3}{2}NkT$

Variance$(E) = 3N$ Variance$(K.E._x) = \frac{3}{2}N(kT)^2$

$$p(E) = \frac{1}{\sqrt{2\pi\{(3/2)N(kT)^2\}}} \exp[-\frac{(E-(3/2)NkT)^2}{2\{(3/2)N(kT)^2\}}]$$

$$\frac{\text{s.d.}}{\text{mean}} = \frac{\sqrt{\frac{3}{2}N}\ kT}{\frac{3}{2}N\ kT} = \frac{1}{\sqrt{\frac{3}{2}N}}$$