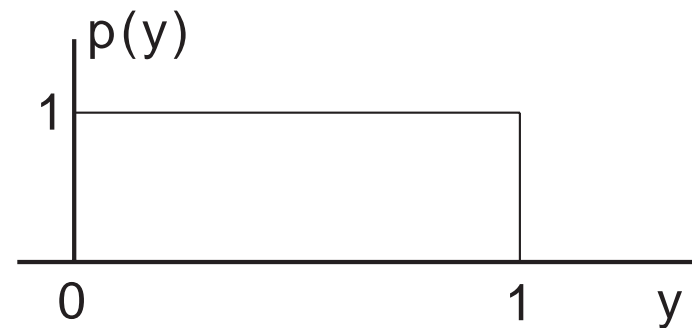
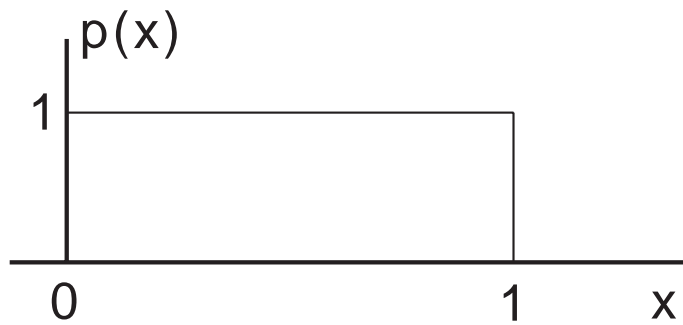


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8.044 Statistical Physics I
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Example Random number generator for programmers



x and y are statistically independent

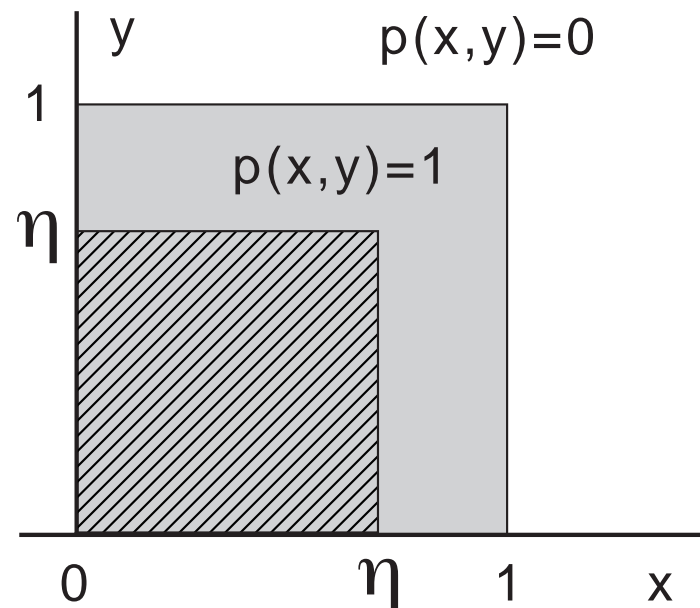
$z \equiv \text{MAX}(x, y)$ Find $p(z)$

$$p(x, y) = p(x) p(y)$$

Where is $\text{MAX}(x, y) = \eta$?

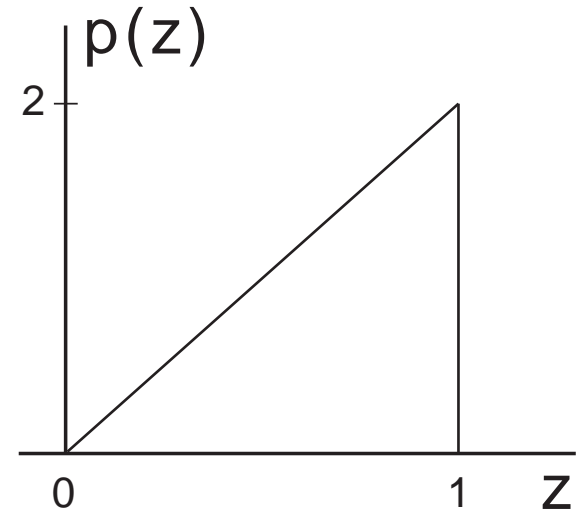
Where is $\text{MAX}(x, y) < \eta$?

A



B $P_z(\eta) = \eta^2$

C $p_z(\eta) = 2\eta \quad 0 \leq \eta \leq 1$



$$\langle z \rangle = \int_0^1 2\eta^2 d\eta = (2/3) \left[\eta^3 \right]_0^1 = 2/3$$
$$\langle z^2 \rangle = \int_0^1 2\eta^3 d\eta = (2/4) \left[\eta^4 \right]_0^1 = 1/2$$

$$\text{Var}(z) = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}, \quad \text{S.D.} = \frac{1}{\sqrt{18}} = 0.24$$

Example Desorbing atom

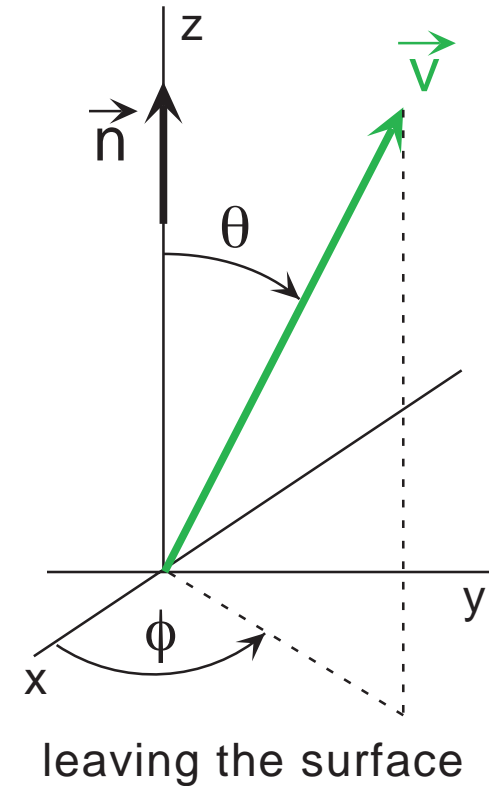
$$p(v, \theta, \phi) = p(v) p(\theta) p(\phi)$$

$$p(v) = (1/2\sigma^4) v^3 \exp[-v^2/2\sigma^2]$$

$$p(\theta) = 2 \sin \theta \cos \theta$$

$$p(\phi) = 1/2\pi$$

Find $p(v_z)$

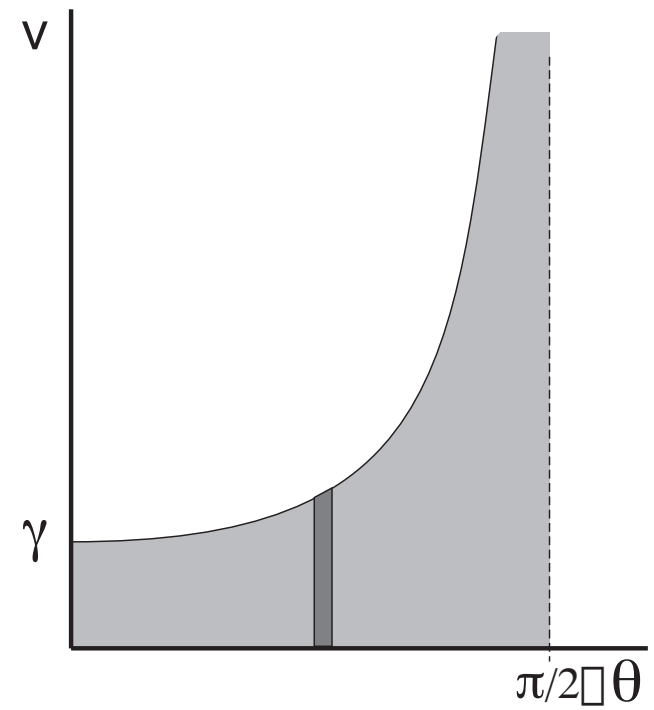


$$v_z = v \cos \theta$$

$$v \cos \theta < \gamma$$

$$\Rightarrow v < \gamma / \cos \theta$$

A



8.044 L6B5

B

$$\begin{aligned} P_{v_z}(\gamma) &= \int_0^{\pi/2} \int_0^{\gamma/\cos\eta} p_v(\zeta) p_\theta(\eta) d\zeta d\eta \\ &= \int_0^{\pi/2} p_\theta(\eta) \left[\int_0^{\gamma/\cos\eta} p_v(\zeta) d\zeta \right] d\eta \end{aligned}$$

C

$$p_{v_z}(\gamma) = \frac{dP_{v_z}(\gamma)}{d\gamma} = \int_0^{\pi/2} p_\theta(\eta) \left[\frac{1}{\cos\eta} p_v\left(\frac{\gamma}{\cos\eta}\right) \right] d\eta$$

$$p_{v_z}(\gamma) =$$

$$\int_0^{\pi/2} (2 \sin \eta \cos \eta) \left[\frac{1}{\cos \eta} \frac{1}{2\sigma^4} \left(\frac{\gamma}{\cos \eta} \right)^3 \exp\left[-\frac{1}{2\sigma^2} \frac{\gamma^2}{\cos^2 \eta}\right] \right] d\eta$$

$$\text{Let } \frac{1}{2\sigma^2} \frac{\gamma^2}{\cos^2 \eta} \equiv X$$

$$dX = -\frac{1}{\sigma^2} \frac{\gamma^2}{\cos^3 \eta} (-\sin \eta) d\eta$$

$$\eta = 0 \quad \Rightarrow \quad X = \gamma^2 / 2\sigma^2; \quad \eta = \pi/2 \quad \Rightarrow \quad X = \infty$$

$$\begin{aligned} p_{v_z}(\gamma) &= \frac{\gamma}{\sigma^2} \int_{\gamma^2/2\sigma^2}^{\infty} e^{-X} dX = -\frac{\gamma}{\sigma^2} \left[\gamma^2/2\sigma^2 \ e^{-X} \right. \\ &= \frac{\gamma}{\sigma^2} \exp[-\gamma^2/2\sigma^2] \quad \gamma > 0 \end{aligned}$$

