8.044 Statistical Physics I
Spring 2008

Review Paramagnet: a collection of non-interacting magnetic moments

Quantum Mechanics

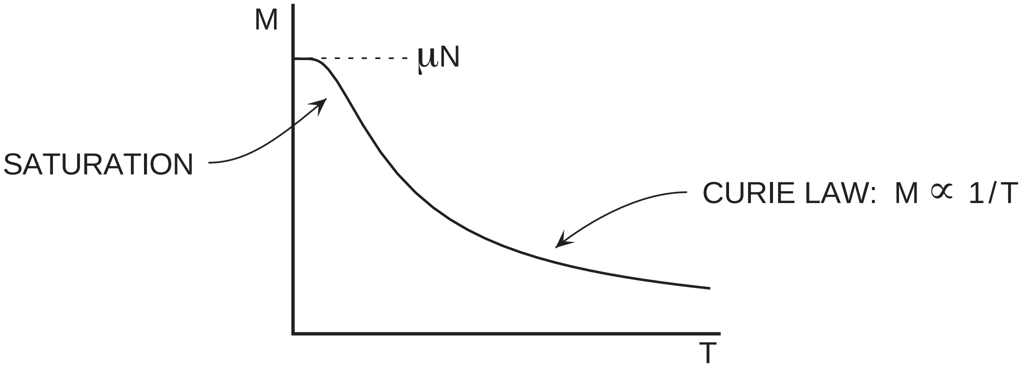
$\vec{\mu} = g_J \mu_B \vec{J}$

$\Rightarrow 2J + 1$ levels

The topic was saved until the last because it is subtle.

First $J = 1/2$, Microcanonical: $S(M) \rightarrow M(T, H)$

Second $J = 1/2$, Canonical: $p(m_J) \rightarrow <\mu> \rightarrow M(T,H)$



Third $J > 1/2$, Canonical: $<\mu> \rightarrow M(T,H)$

$$Z = Z(\eta = g\mu_B H/kT)$$

So what's the problem?

$$< \mathcal{H} >^{\text{S. M.}} \neq \; U^{\text{Thermo}}$$

$$= U^{\text{assembly}} + (-\vec{H} \cdot \vec{M})^{\text{position}} \quad (\text{for } \not{d}W = HdM)$$

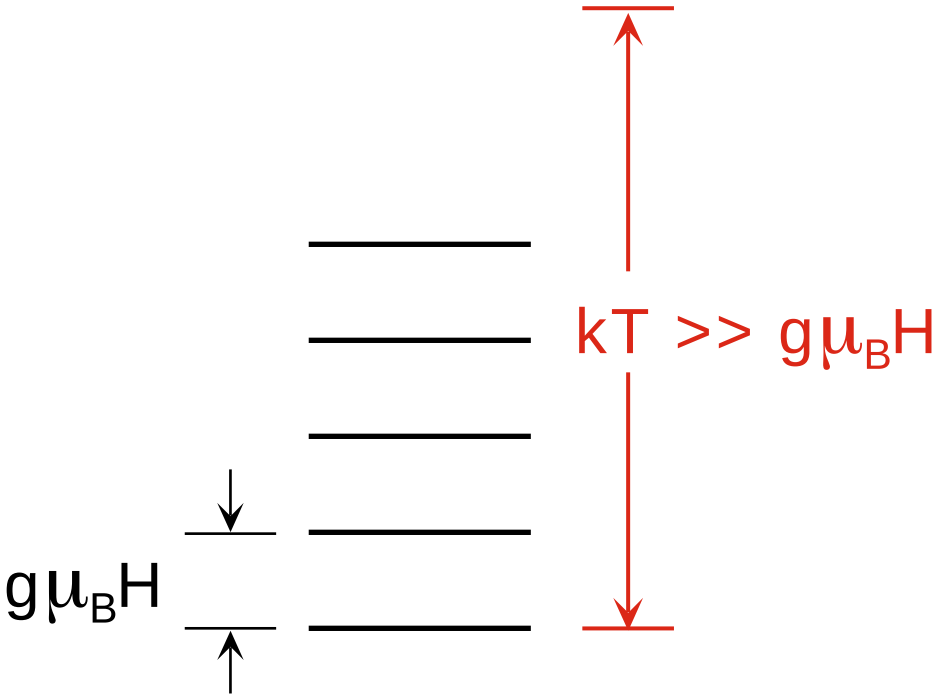
$$< \mathcal{H} > = -\frac{1}{Z}\left(\frac{\partial Z}{\partial \beta}\right)_H = - \; \underbrace{\frac{1}{Z}\frac{dZ}{d\eta}}_{M/g\mu_B} \; \underbrace{\left(\frac{\partial \eta}{\partial \beta}\right)_H}_{g\mu_B H} = -HM \quad \surd$$

Entropy of a Quantum Paramagnet

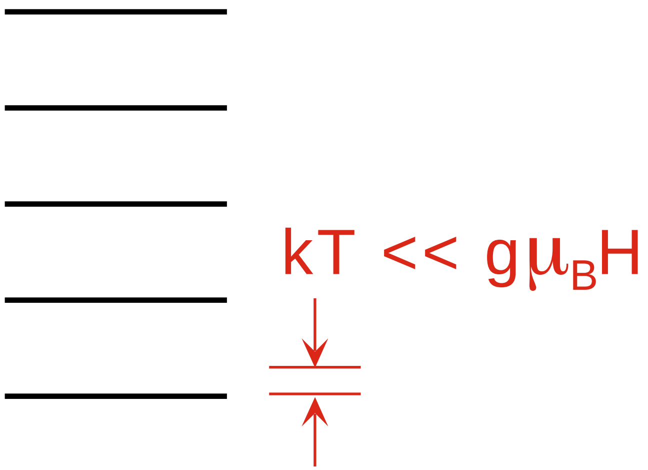
- When is $-kT \ln Z \neq F$?

- How is a paramagnet like a sponge?

HIGH AND LOW TEMPERATURE BEHAVIOR OF A QUANTUM PARAMAGNET



ENERGY LEVELS ALMOST
EQUALLY POPULATED,
CURIE LAW BEHAVIOR



MOMENT ALMOST SATURATED,
ENERGY GAP BEHAVIOR

$$S(kT \ll g\mu_B H) \rightarrow Nk\,\mathsf{ln}(1) = 0$$

$$S(kT \gg g\mu_B H) \rightarrow Nk\,\mathsf{ln}(2J+1)$$

$$Z_1 = \sum_{m=-J}^{J} (e^{\eta})^m \qquad \eta \equiv g\mu_B H/kT$$

Try

$$-kT\,\mathsf{ln}\,Z \;=\; F = \underbrace{U}_{0} - TS \Rightarrow S = k\,\mathsf{ln}\,Z = Nk\,\mathsf{ln}\,Z_1$$

$$S(kT \ll g\mu_B H) \rightarrow Nk \ln(1) = 0$$

$$S(kT \gg g\mu_B H) \rightarrow Nk \ln(2J+1) \quad \underline{Nk \ln(2J+1) \text{ O.K.}}$$

$$Z_1 = \sum_{m=-J}^{J} (e^{\eta})^m \qquad \eta \equiv g\mu_B H/kT$$

Try

$$-kT \ln Z = F = \underbrace{U}_{0} - TS \Rightarrow S = k \ln Z = Nk \ln Z_1$$

$$S(kT \ll g\mu_{B}H) \rightarrow Nk\ln(1) = 0 \quad \underline{NkJ(g\mu_{B}H/kT)\ \text{ wrong!}}$$

$$S(kT \gg g\mu_{B}H) \rightarrow Nk\ln(2J+1) \quad \underline{Nk\ln(2J+1)\ \text{ O.K.}}$$

$$Z_1 = \sum_{m=-J}^{J} (e^{\eta})^m \qquad \eta \equiv g\mu_{B}H/kT$$

Try

$$-kT\ln Z \underbrace{=}\, F = \underbrace{U}_{0} - TS \Rightarrow S = k\ln Z = \underbrace{Nk\ln Z_1}$$

$$\text{wrong}\ \underline{\hspace{4cm}} \Leftarrow \underline{\hspace{4cm}}\text{wrong}$$

In the derivation of the canonical ensemble we found

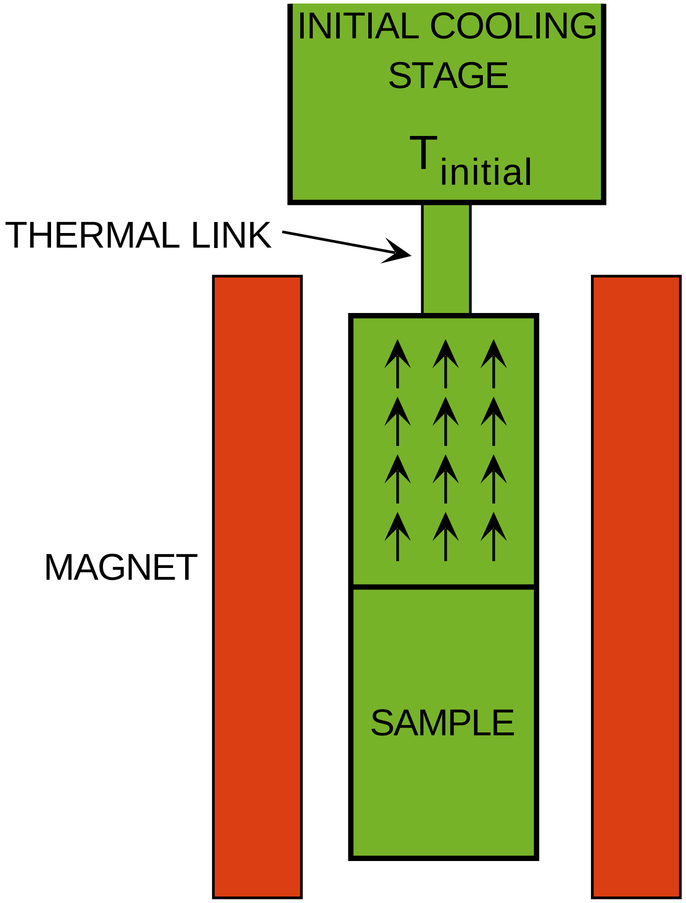
$-kT \ln Z = < E_1 > -TS_1$ where $< E_1 > = < \mathcal{H}(\{p,q\}) >$

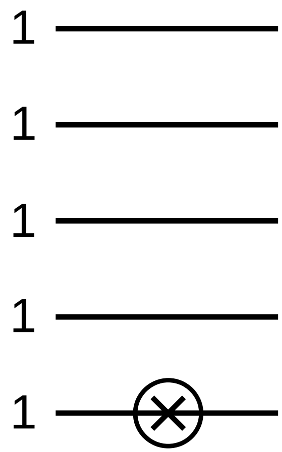
Then we set $< E_1 > = U$. But in the paramagnet $< E_1 > = U - HM$, thus

$-kT \ln Z = U - HM - TS = G(T, H)$ for our model.

$\Rightarrow S = k \ln Z - HM/T = \underline{Nk \ln Z_1(\eta) - NkJ\,\eta B_J(\eta)}$

ADIABATIC DEMAGNETIZATION
(MAGNETIC COOLING)





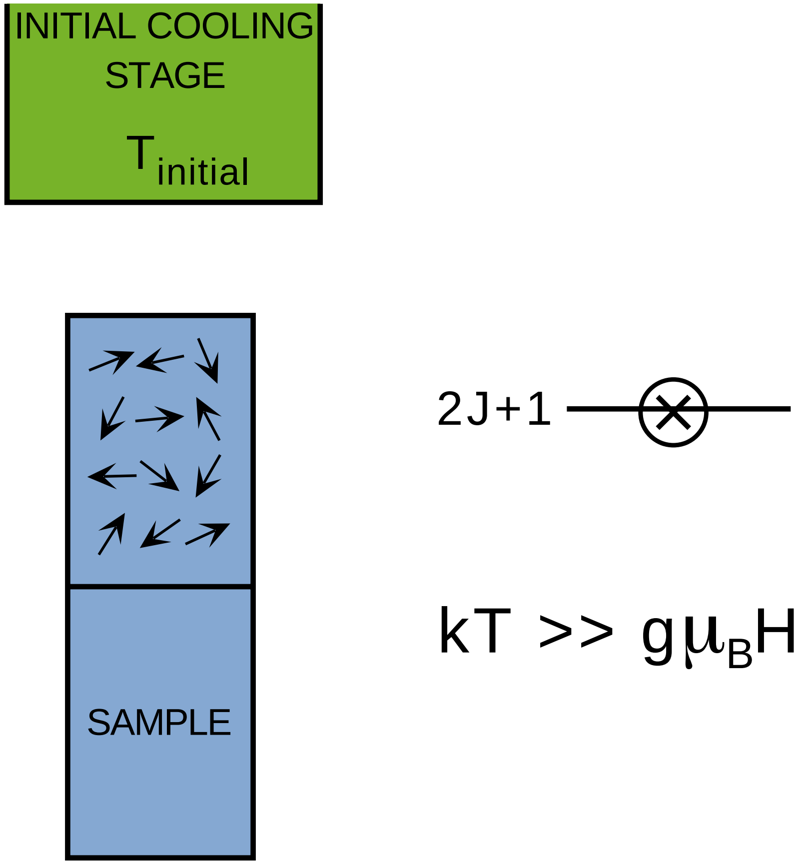
$$kT << g\mu_B H$$

$$H = H_{initial}$$

$$\left.\begin{array}{l} S_M \sim 0 \\ S_S \text{ high} \end{array}\right\} S_{total}$$

$$T_S = T_{initial}$$

ADIABATIC DEMAGNETIZATION
(MAGNETIC COOLING)



$kT >> g\mu_B H$

$H \sim 0$

$$\left.\begin{array}{l} S_M \sim Nk\ln(2J+1) \\ \\ S_S \text{ low} \end{array}\right\} S_{total}$$

$T_S << T_{initial}$

Electronic Example, CMN

Cerium Mangesium Nitrate

$2Ce(NO_3)_3 \cdot 3Mg(NO_3)_2 \cdot 24H_2O$

Ce^{+++} $J=5/2$ $T_{\text{ordering}} \sim 1.9$ mK

Cools ^{3}He and samples therein to $\sim$ 2 mK.

Nuclear Example, Cu

Metallic Copper

Cu I=3/2 $T_{\text{ordering}} \sim 1\ \mu$K

Cools Cu electrons and lattice to $\sim 10\ \mu$K.