8.044 Statistical Physics I
Spring 2008

PARAMAGNETS

Spin 1/2 Paramagnet, Microcanonical Ensemble

Assume the moment is parallel to the spin. Take n to be the state variable. μ is the magnetic moment of the particle.

$$\vec{H} = H\widehat{z}$$

$\downarrow$ $\underline{\qquad n \qquad}$ μH

$\uparrow$ $\underline{\qquad N\text{-}n \qquad}$ -μH

$(\uparrow),\ N_\uparrow = N - n,\ \vec{\mu}_\uparrow = \mu\widehat{z},\ \epsilon_\uparrow = -\mu H$

$(\downarrow),\ N_\downarrow = n,\ \vec{\mu}_\downarrow = -\mu\widehat{z},\ \epsilon_\downarrow = \mu H$

$$M \;=\; \mu(N_\uparrow - N_\downarrow) = \mu(N - 2n) = M(n)$$

$$E \;\equiv\; N\langle\epsilon\rangle = N_\uparrow\epsilon_\uparrow + N_\downarrow\epsilon_\downarrow = -\mu H(N - 2n) = \underbrace{-HM(n)}_{H \,\&\, n}$$

$$S \;=\; k\ln\left(\frac{N!}{n!(N-n)!}\right) = S(n)$$

$$\approx\; k\left[\,N\ln N - n\ln n - (N - n)\ln(N - n)\,\right]$$

$$\frac{dS}{dn} = k\left[-1 - \ln n + 1 + \ln(N - n)\right] = -k\ln\left(\frac{n}{N-n}\right)$$
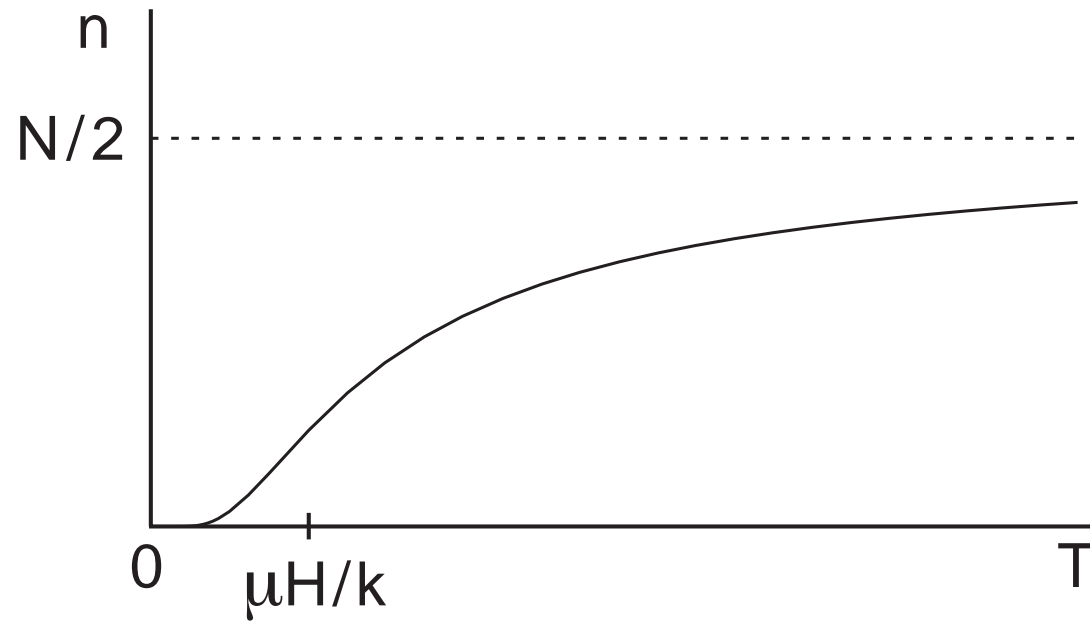
$$S = S(n) \text{ and } M = M(n) \Rightarrow S = S(M)$$

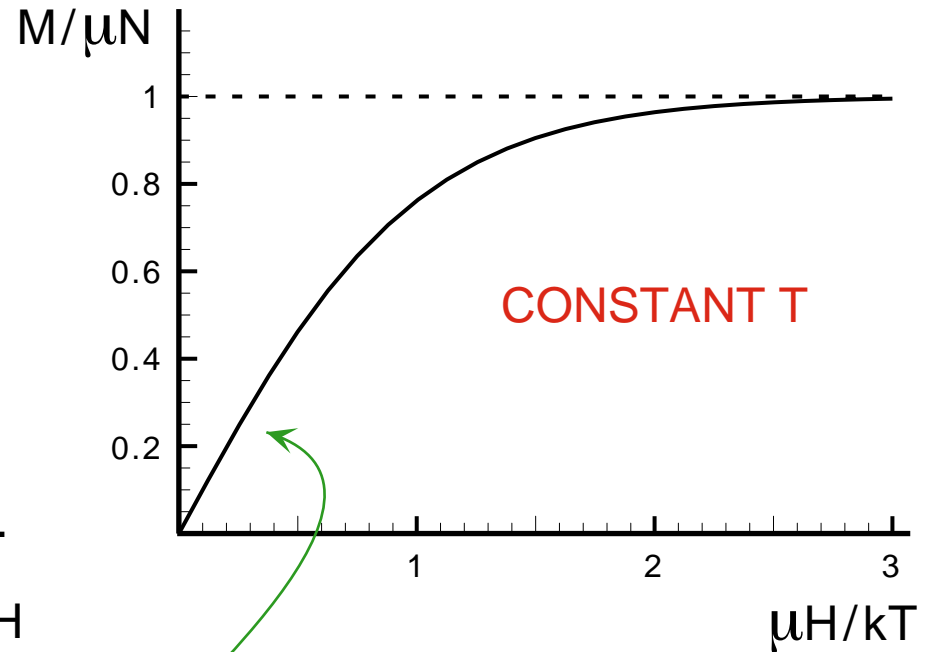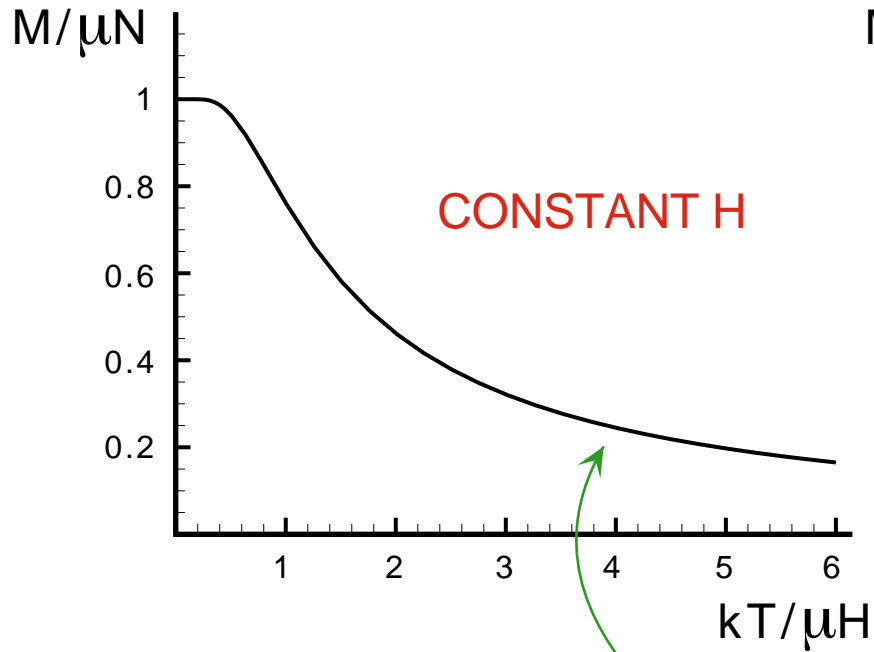$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_M \quad \text{does not exist!}$$

$$-\frac{H}{T} = \left(\frac{\partial S}{\partial M}\right)_E = \frac{dS}{dn}\left(\frac{\partial n}{\partial M}\right)_E = \left(-k\ln\frac{n}{N-n}\right)\left(\frac{-1}{2\mu}\right)$$

$$\frac{n}{N-n} = e^{-2\mu H/kT} \rightarrow n = N \frac{1}{e^{2\mu H/kT} + 1}$$

$$M = \mu(N - 2n) = \mu N \tanh(\mu H/kT)$$

MAGNETIZATION IN SPIN 1/2 PARAMAGNET

CURIE LAW BEHAVIOIR: $M = \dfrac{N\mu^2 H}{kT} \propto H/T$

Internal Energy

$$dU = \bar{d}Q + \bar{d}W = TdS + HdM$$

$$dU \equiv \text{adiabatic } (\bar{d}Q = 0) \text{ work}$$

$$\bar{d}Q = TdS, \quad \bar{d}Q = 0 \Rightarrow dS = 0 \Rightarrow dM = 0 \Rightarrow dU = 0$$

$dU = 0$ for any change: $\underline{U = 0 \text{ for this model}}$

But $E \equiv N < \epsilon > = -HM \neq 0$!!

Energy $=$ energy to create H field   ①

       $+$ energy to assemble M ②

       $+$ energy to move M into H ③

$\textcircled{1}$ does not appear when using $\not{d}W = H\,dM$. It is included in U when using $\not{d}W = -M\,dH$.
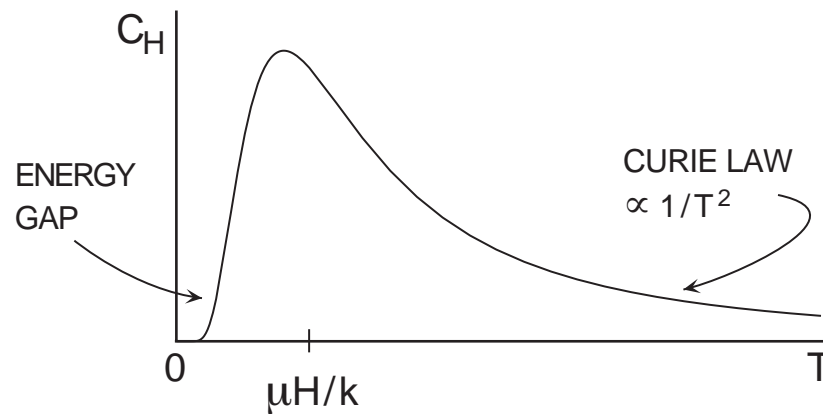
$\textcircled{2}$ We did not create the $\vec{\mu}$. They do not interact. $\Rightarrow U = 0$ in the current example.

$\textcircled{3}$ $-\vec{H} \cdot \vec{M}$, energy of macroscopic moment in $\vec{H}$ equals $N < \epsilon >$ in the current example.

$$C_M \equiv \left(\frac{dQ}{dT}\right)_M = T\left(\frac{\partial S}{\partial T}\right)_M = 0 \text{ since } S = S(M)$$

$$C_H \equiv \left(\frac{dQ}{dT}\right)_H = \frac{1}{dT}(\underbrace{dU}_{0} - HdM)_H = -H\left(\frac{\partial M}{\partial T}\right)_H$$

$$= Nk\left(\frac{2\mu H}{kT}\right)^2 \frac{e^{2\mu H/kT}}{(e^{2\mu H/kT}+1)^2}$$

C_H

ENERGY GAP

CURIE LAW $\propto 1/T^2$

0

$\mu H/k$

T

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Spin 1/2 Paramagnet, Canonical Ensemble

$$Z_1 = e^{+\mu H/kT} + e^{-\mu H/kT}$$

↓ ———— μH

↑ ———— -μH

$$p_\uparrow = \frac{e^{+\mu H/kT}}{Z_1} \qquad p_\downarrow = \frac{e^{-\mu H/kT}}{Z_1}$$

$$M = N\mu(p_\uparrow - p_\downarrow)$$

$$= \mu N \tanh(\mu H/kT) \quad \checkmark$$