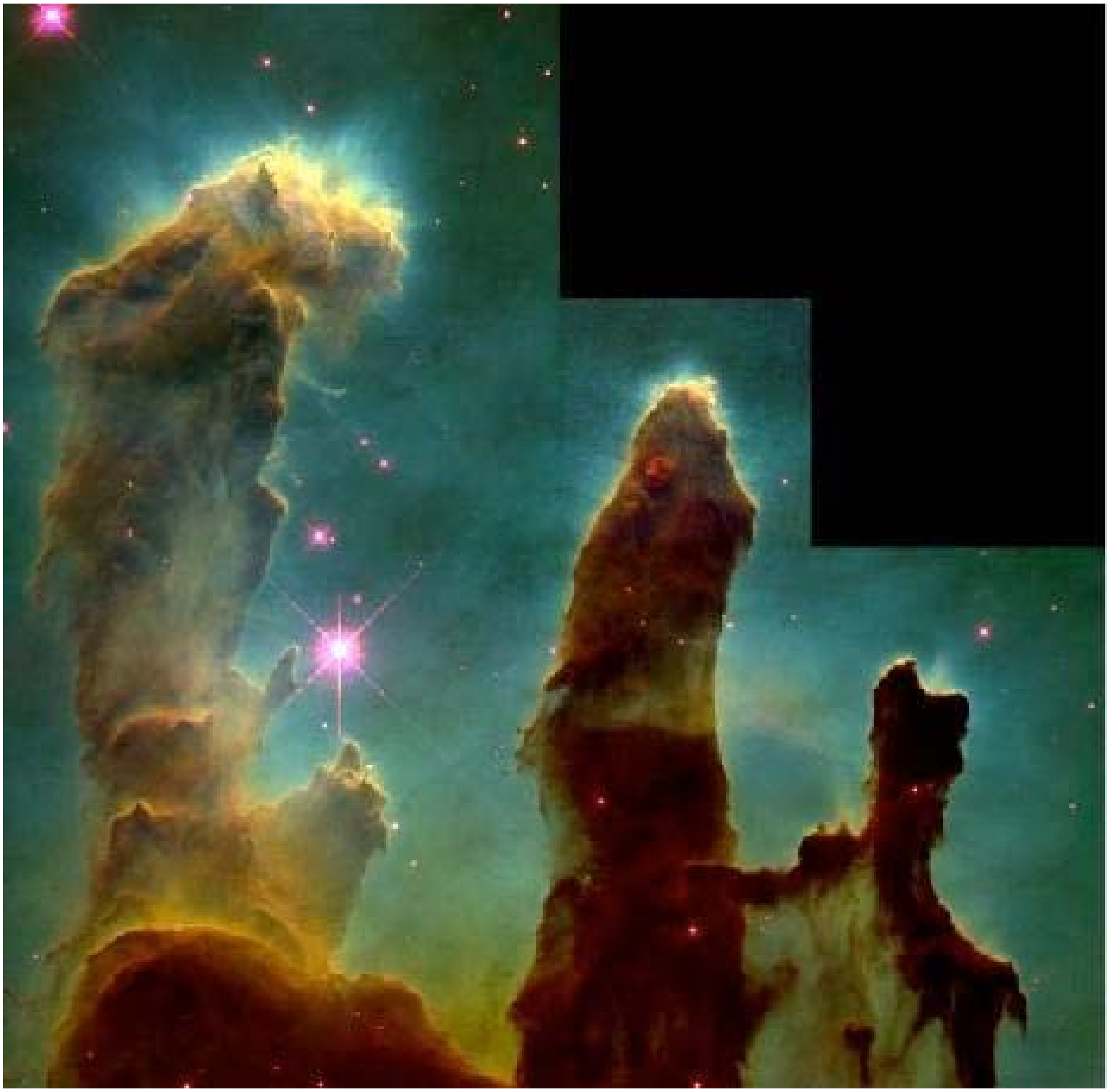
8.044 Statistical Physics I
Spring 2008



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Stellar Configurations

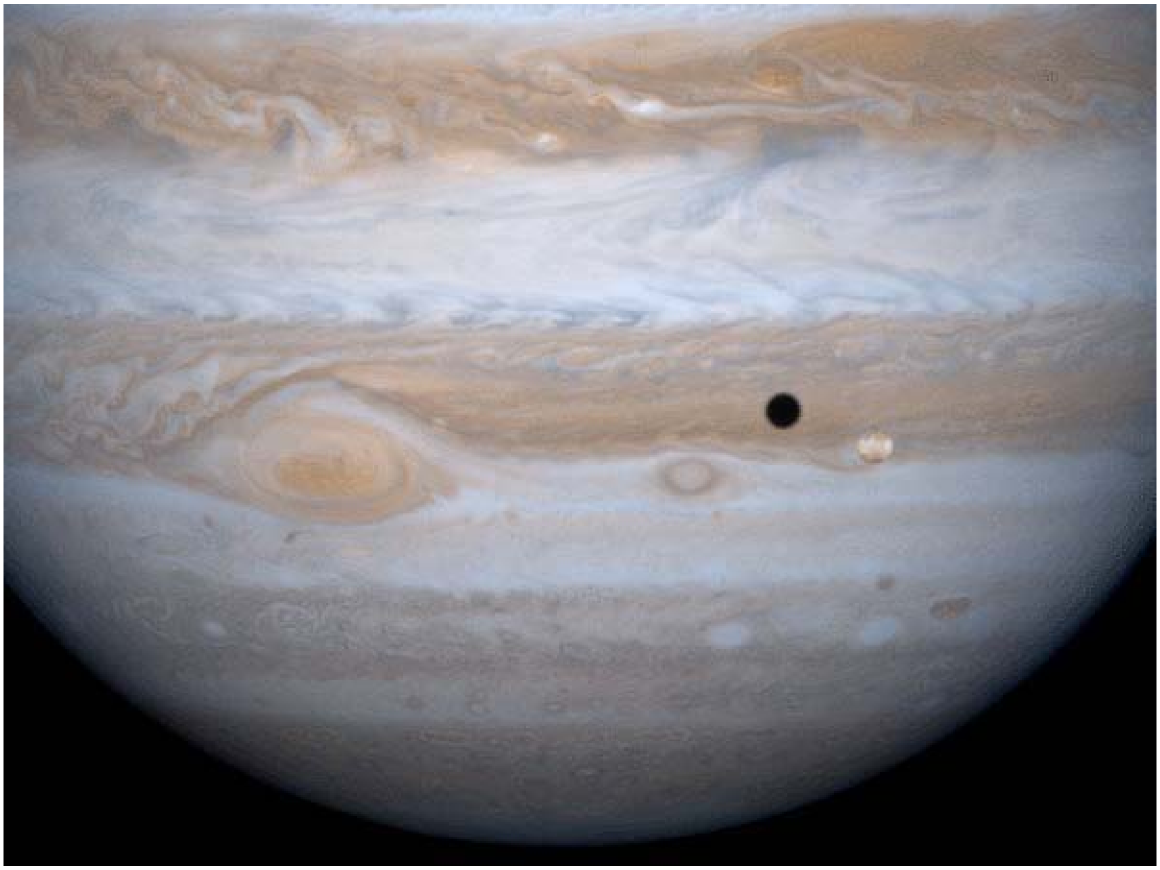
- Self gravitating

- Self-consistent solution needed

- Different processes resist collapse

Planets

- Gravity weak because of small M
- Atomic forces provide balancing pressure



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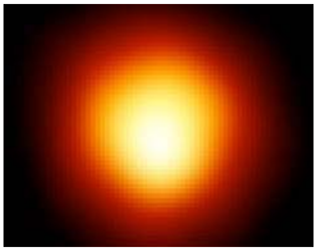
Normal Stars

- Gravitational energy starts process
- Fusion then supplies energy
- Plasma of electrons and nuclei
- Kinetic pressure, $P = nkT$
- Radiation pressure, $P = \frac{1}{3}u(T)$, helps and dominates above about $10M_{\odot}$



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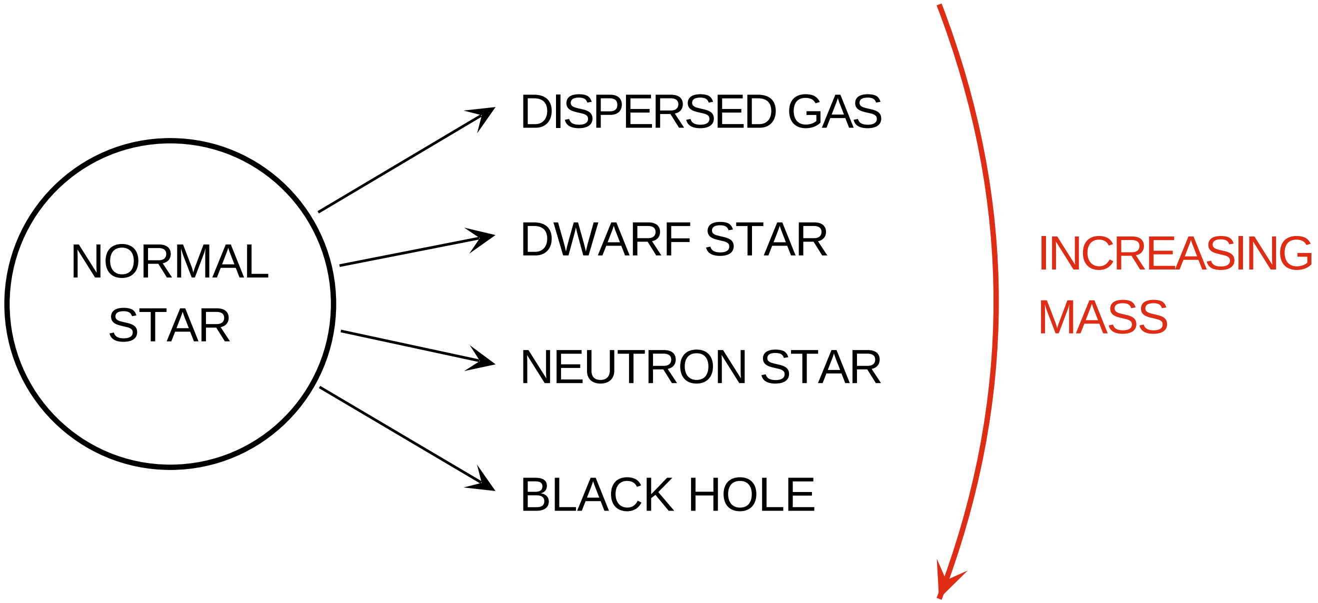
FOUR POSSIBLE END STATES OF STARS



NORMAL STAR

DISPERSED GAS

DWARF STAR

NEUTRON STAR

BLACK HOLE

INCREASING MASS

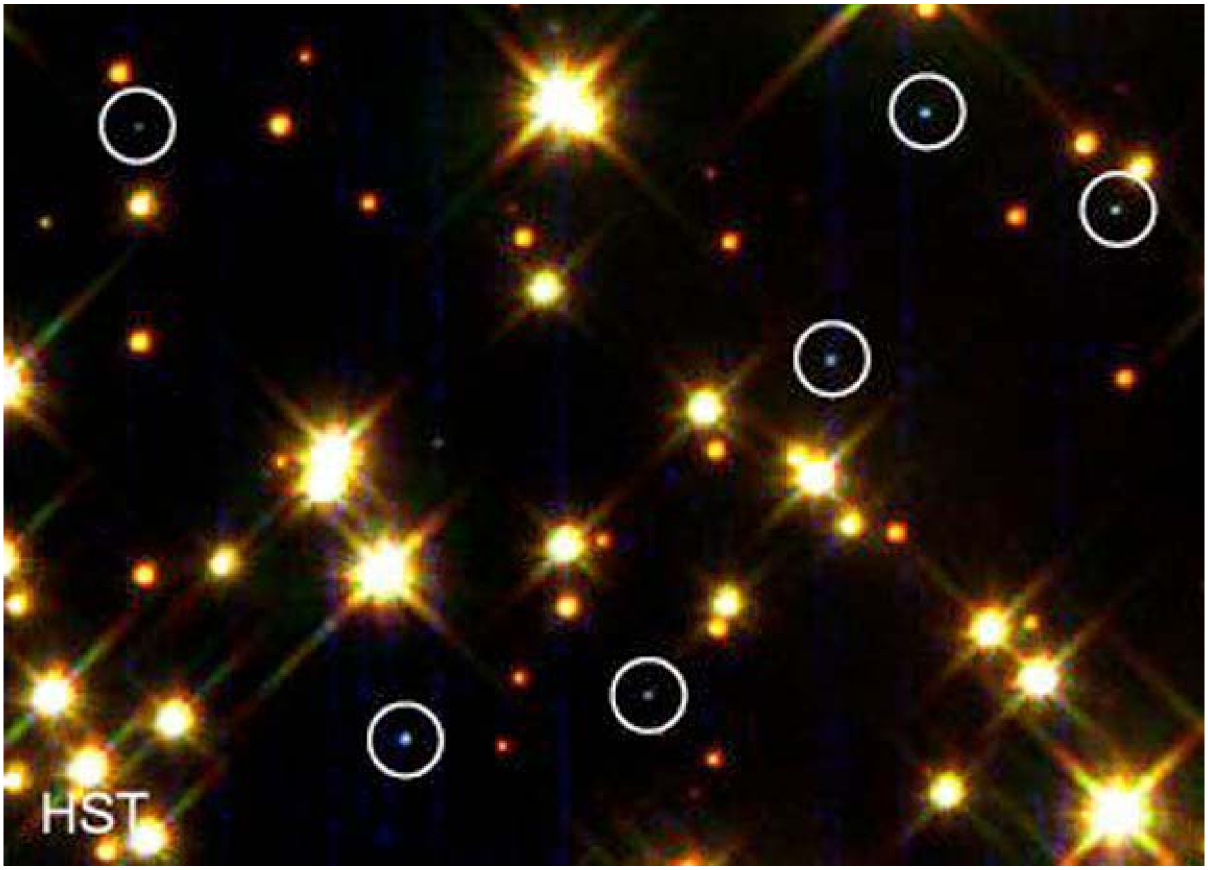
White Dwarf

- Fusion has stopped

- Collapses to a small size, nuclear spacing $\sim 1/100$ that of a solid

- Electron degeneracy pressure supports it, $P \propto \frac{1}{m_e}\, n^{5/3}$

- White → gray → brown (dead, cold)



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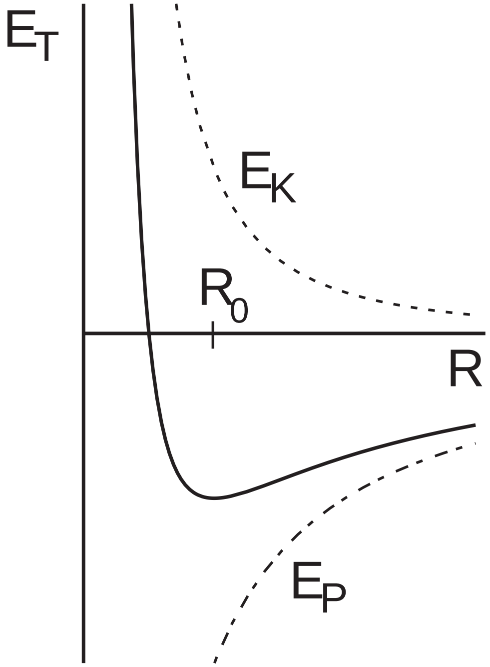
Assume uniform density of α^{++} and e^{-}

$$E_K = \underbrace{E_K^{(\alpha)}}_{\text{small}} + E_K^{(e)} = \frac{3}{5} N_e \epsilon_F = \frac{3}{5} N_e \frac{\hbar^2}{2m_e} \left(3\pi^2 (N_e/V)\right)^{2/3}$$

$$V = \frac{4}{3}\pi R^3 \quad M \approx N_\alpha m_\alpha = (N_e/2) m_\alpha \Rightarrow N_e = 2M/m_\alpha$$

$$E_K = \frac{3}{5}\left(\frac{9\pi}{2}\right)^{2/3} \frac{\hbar^2}{m_e} \left(\frac{M}{m_\alpha}\right)^{2/3} \frac{1}{R^2} \qquad E_P = -\frac{3}{5} G \frac{M^2}{R}$$

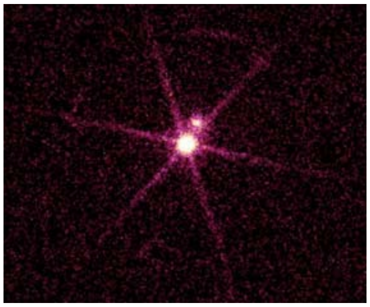


$$E_T = E_K + E_P$$

$$MR_0^3 = 2(9\pi)^2 \frac{\hbar^6}{G^3 m_e^3 m_\alpha^5}$$

$$= 0.74 \times 10^{51} \text{ kg-m}^3$$

$R_0 \propto 1/M^{1/3}$ <u>Stable for any M</u>.



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Sirius B: $M = 2.1 \times 10^{30}$ kg

R

observed	5.6×10^{6} m	
our model	7.1×10^{6} m	(good)
better model	8.6×10^{6} m	($\Rightarrow$ a problem)

Our model of Sirius B implies

$n_e = 8.6 \times 10^{29}\ \text{cm}^{-3}$

$\epsilon_F = 4.7 \times 10^{-7}\ \text{ergs} \rightarrow 3.4 \times 10^{9}\text{K}$

$(T_{\text{surface}} \sim 2 \times 10^{7}\ \text{K})$

<u>But</u> $m_e c^2 = 8.2 \times 10^{-7}$ ergs $\Rightarrow$ relativity needed

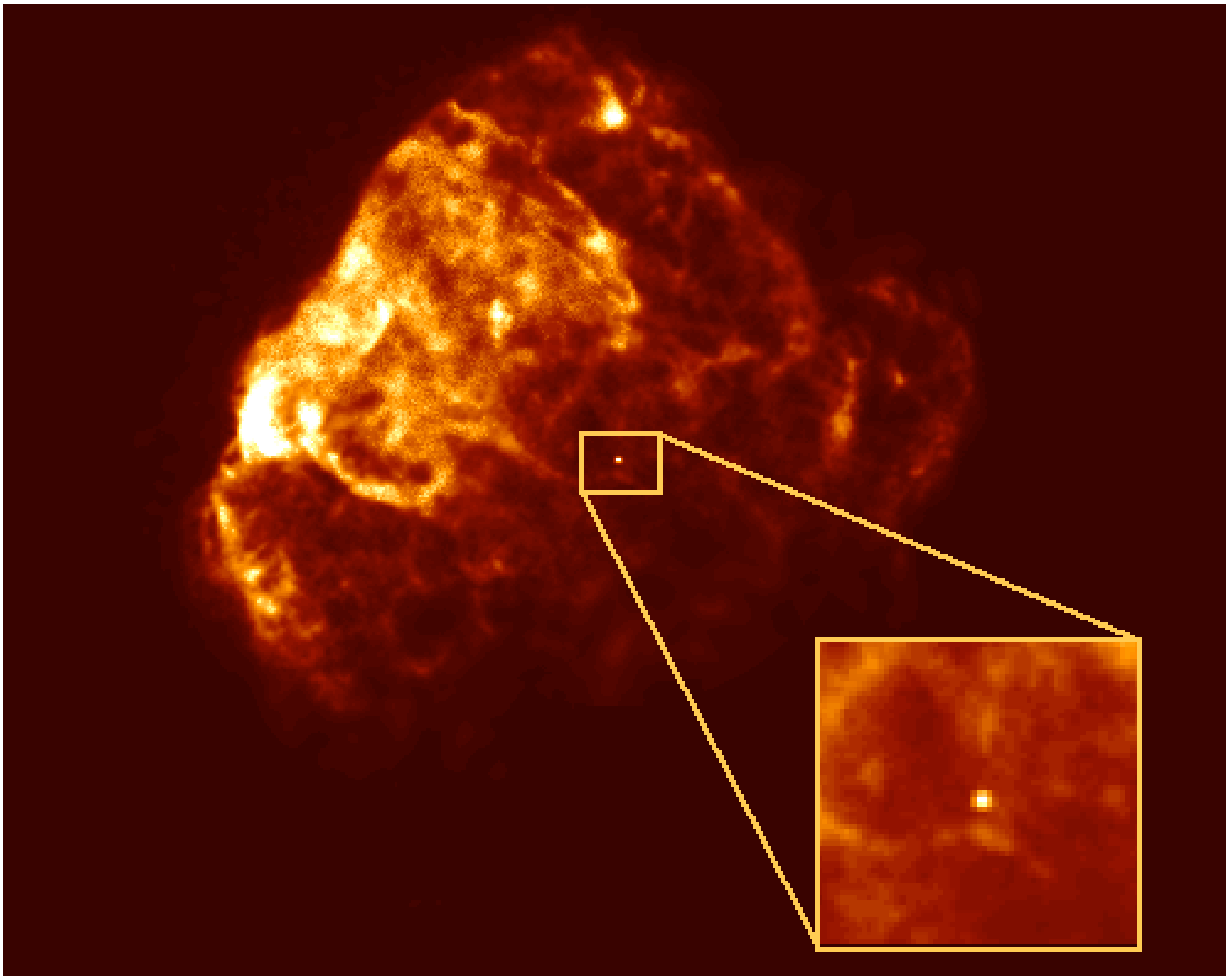
Homework problem examines extreme relativistic gas and finds

- Softer equation of state, $P \propto \frac{1}{m_e} n^{4/3}$
- Potential for collapse if M is too large

Chandrasekhar limit ($\sim$1935) $M_{Ch} = 1.4 M_{\odot}$

Neutron Star

- $p^+ + e^- \rightarrow n$ to lower coulomb energy
- Degeneracy pressure of neutrons
  $MR_0^3 \propto \hbar^6/G^3 m_n^8 \;\Rightarrow\; R_0 \sim 15$ km if $M = 1.4M_\odot$
- Nuclear forces also contribute to P
- Rotating neutron stars seen as pulsars
- Also subject to stability limit, $M \sim 2M_\odot$



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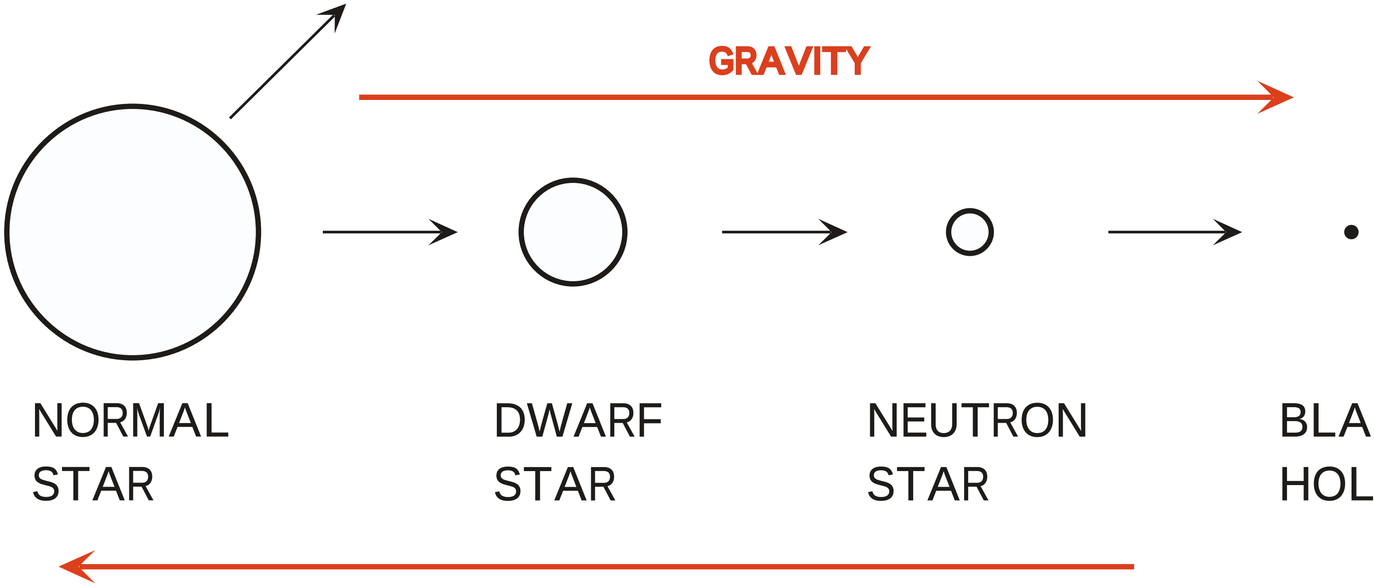
STELLAR CONFIGURATIONS



DISPERSED GAS

GRAVITY

NORMAL
STAR

DWARF
STAR

NEUTRON
STAR

BLACK
HOLE

**GAS PRESSURE
(+ RADIATION)**

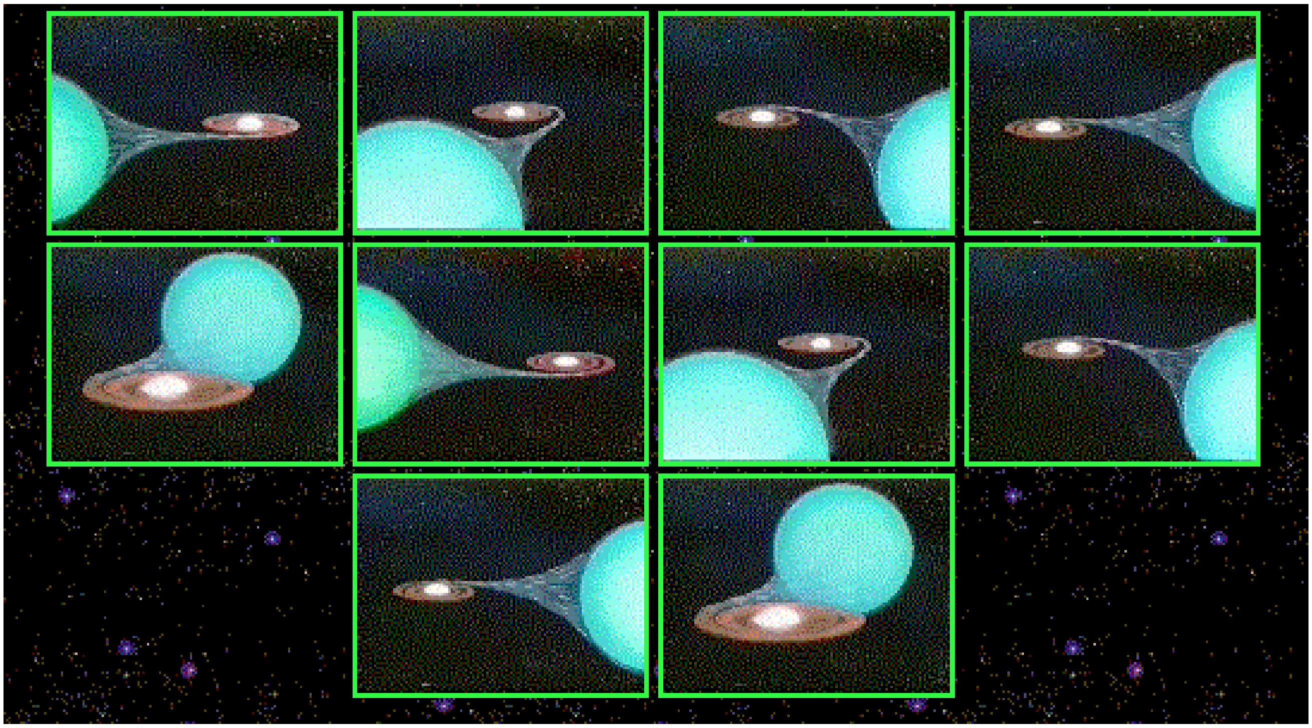
**ELECTRON
DEGENERACY**

**NEUTRON
DEGENERACY
+ NUCLEAR
FORCES**

$1\ g/cm^3$

$10^6\ g/cm^3$

$10^{14}\ g/cm^3$



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