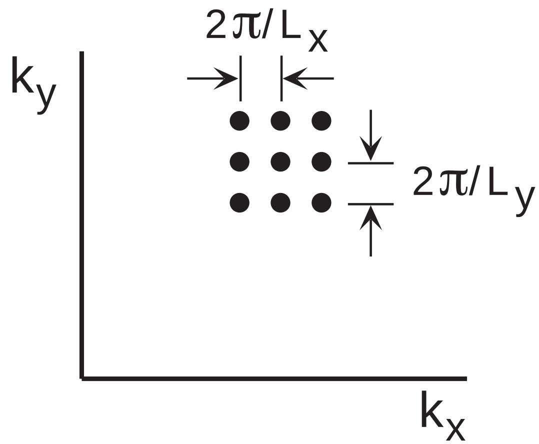
8.044 Statistical Physics I
Spring 2008

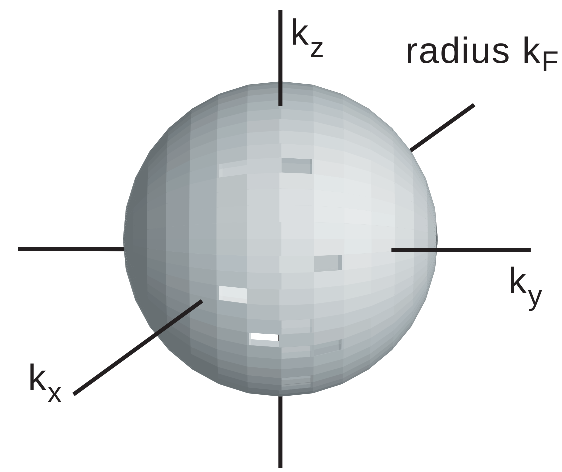


$$D(k) = \left(\frac{2\pi}{L_x}\right)^{-1} \left(\frac{2\pi}{L_y}\right)^{-1} \left(\frac{2\pi}{L_z}\right)^{-1}$$
$$= \frac{V}{(2\pi)^3}$$

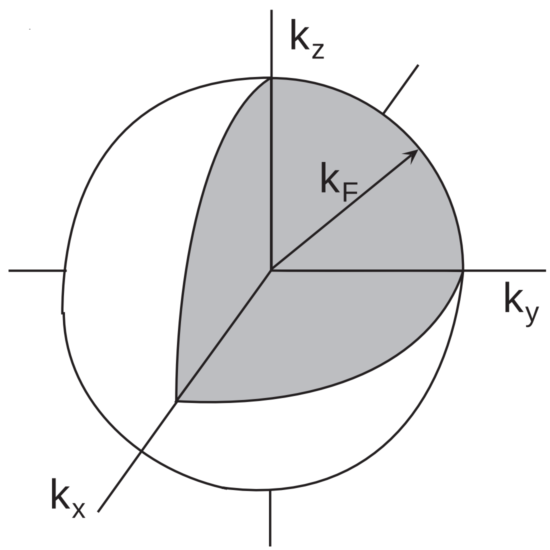
wave vectors only!

$$N = 2 \times D(k) \times \frac{4}{3}\pi k_F^3$$

$$k_F = \left(3\pi^2(N/V)\right)^{1/3}$$



Fermions: Non-interacting, free, spin 1/2, $T = 0$



Fermi sea

Fermi surface

Fermi wave vector

$$k_F = \left(3\pi^2(N/V)\right)^{1/3}$$

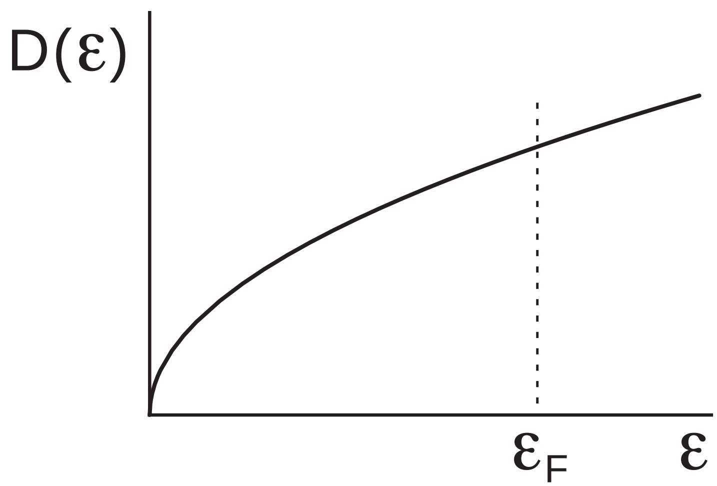
Fermi energy

$$\epsilon_F = \hbar^2 k_F^2/2m$$

$$\#(\epsilon) = 2 \times D(k) \times \frac{4}{3}\pi k^3(\epsilon) \qquad k(\epsilon) = \left(\frac{2m\epsilon}{\hbar^2}\right)^{1/2}$$

$$= \frac{8\pi}{3}\frac{V}{(2\pi)^3}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon^{3/2}$$

$$D(\epsilon) = \underline{\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon^{1/2}}$$



$$\epsilon_F = \frac{\hbar^2 k_F^2}{2m} \propto (N/V)^{2/3}$$

$$D(\epsilon) = a\,\epsilon^{1/2}$$

$$N = \int_0^{\epsilon_F} D(\epsilon)\,d\epsilon = \tfrac{2}{3} a\,\epsilon_F^{3/2}$$

$$E = \int_0^{\epsilon_F} \epsilon D(\epsilon)\,d\epsilon = \tfrac{2}{5} a\,\epsilon_F^{5/2} = \underline{\tfrac{3}{5} N \epsilon_F} \quad \propto N(N/V)^{2/3}$$

Consequences: Motion at $T = 0$

$$\vec{p} = \hbar\vec{k} \qquad \vec{p}_F = \hbar\vec{k}_F \qquad v_F = \frac{\hbar}{m}k_F$$

Copper, one valence electron beyond a filled d shell

$$N/V = 8.45 \times 10^{22} \text{ atoms-cm}^{-3}$$

$$k_F = 1.36 \times 10^{8} \text{ cm}^{-1} \qquad v_F = 1.57 \times 10^{8} \text{ cm-s}^{-1}$$

$$p_F = 1.43 \times 10^{-19} \text{ g-cm-s}^{-1} \qquad \epsilon_F/k_B = 81,000K$$

Consequences: $P(T=0)$

$E = \frac{3}{5} N \epsilon_F \qquad \epsilon_F \propto (N/V)2/3$

$$P = -\left(\frac{\partial E}{\partial V}\right)_{N,S} = -\frac{3}{5} N \underbrace{\left(\frac{\partial \epsilon_F}{\partial V}\right)_N}_{-\frac{2}{3}\frac{\epsilon_F}{V}} = \underline{\frac{2}{5}(N/V)\epsilon_F} \propto (N/V)^{5/3}$$

$$\left(\frac{\partial P}{\partial V}\right)_{N,T} = -\frac{5}{3}(P/V) \text{ at } T = 0$$

$$\mathcal{K}_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{N,T} = \frac{3}{5}\frac{1}{P} = \frac{3}{2}\frac{1}{(N/V)\epsilon_F}$$

For potassium

$$\epsilon_F = 2.46 \times 10^4 K = 3.39 \times 10^{-12} \text{ ergs}$$

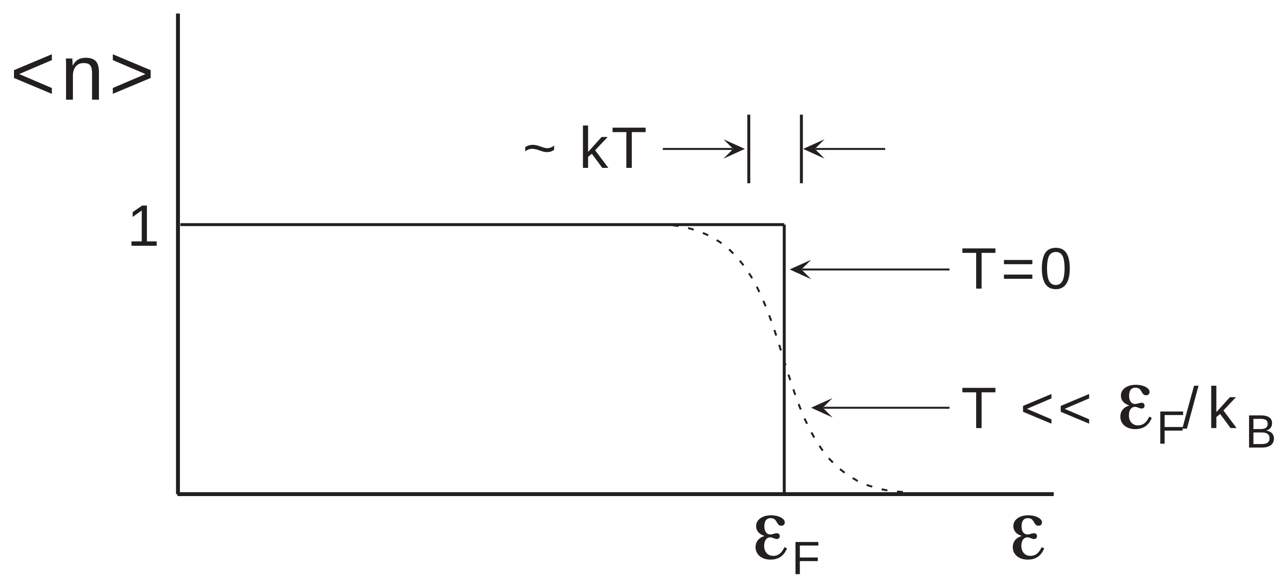
$$(N/V)_{\text{conduction}} = 1.40 \times 10^{22} \text{ cm}^{-3}$$

$$\mathcal{K}_T = \frac{1.5}{1.40\times10^{22}\times3.39\times10^{-12}} = 31.6\times10^{-12} \text{ cm}^3\text{-ergs}^{-1}$$

The measued value is 31×10^{-12} !

Temperature Dependence of $< n_{\vec{k},m_s} >$

$< n_{\vec{k},m_s} >= f(\epsilon)$ only, as in the Canonical Ensemble



Estimate C_V

	Classical	Quantum
# electrons	N	N
# electrons influenced	N	$\sim N \times \frac{kT}{\epsilon_F}$
$\Delta\epsilon$	$\frac{3}{2}kT$	$\sim kT$
ΔE	$\frac{3}{2}NkT$	$\sim N\,\frac{(kT)^2}{\epsilon_F}$
C_V	$\frac{3}{2}Nk$	$\sim 2Nk\,\frac{kT}{\epsilon_F}$

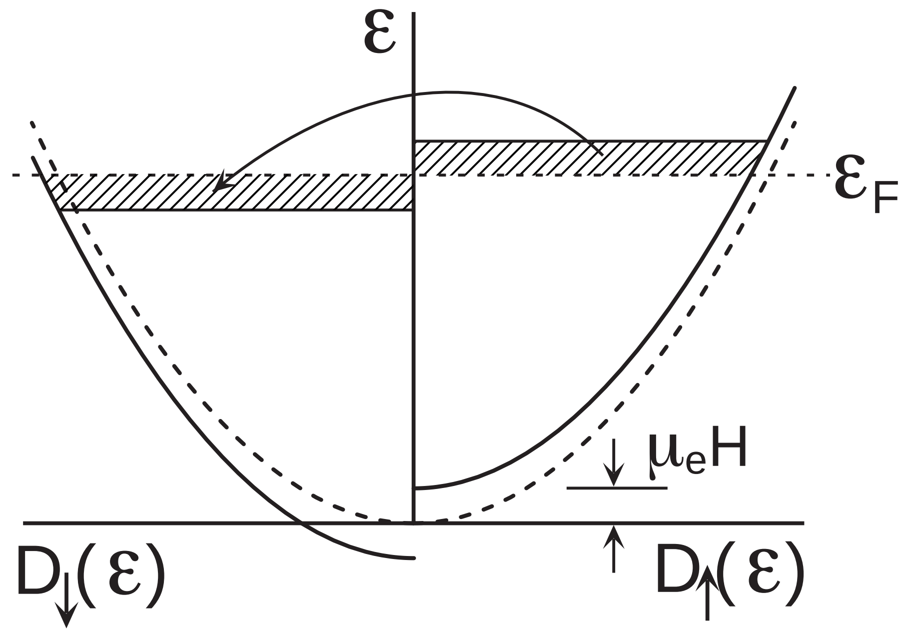
Exact result: $C_V = \frac{\pi^2}{2}\,Nk\,\frac{kT}{\epsilon_F}$

Magnetic Susceptibility

$\vec{H} = H\hat{z}$

$\epsilon_{k,\uparrow} = \epsilon(k) + \mu_e H$

$\epsilon_{k,\downarrow} = \epsilon(k) - \mu_e H$



$$N_\downarrow - N_\uparrow = \left(\frac{N}{2} + \mu_e H \, \frac{D(\epsilon_F)}{2}\right) - \left(\frac{N}{2} - \mu_e H \, \frac{D(\epsilon_F)}{2}\right)$$

$$= \mu_e H D(\epsilon_F) \quad \Rightarrow \quad M = \mu_e^2 H D(\epsilon_F) \equiv \chi H$$

$$\underline{\chi = \mu_e^2 D(\epsilon_F)}$$

This expression holds as long as $kT \ll \epsilon_F$, so χ is temperature independent in this region. This is <u>not</u> Curie law behavior. It is called Pauli paramagnetism.