

MIT OpenCourseWare
<http://ocw.mit.edu>

8.044 Statistical Physics I
Spring 2008

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

Thermodynamic Approach

$$u(T) \equiv \int_0^\infty u(\nu, T) d\nu$$

Then

$$E(T, V) = u(T)V$$

$$P(T, V) = \frac{1}{3}u(T)$$

This is enough to allow us to find $u(T)$.

$$dE = TdS - PdV$$

$$\begin{aligned}\left(\frac{\partial E}{\partial V}\right)_T &= T\left(\frac{\partial S}{\partial V}\right)_T - P = T\left(\frac{\partial P}{\partial T}\right)_V - P \\ &= \frac{1}{3}Tu'(T) - \frac{1}{3}u(T)\end{aligned}$$

$$\text{also} = u(T)$$

$$\Rightarrow u'(T) = \frac{4}{T}u(T)$$

$$u(T) = AT^4$$

Emissive Power of a Black ($\alpha = 1$) Body

$$e(\nu, T) = \frac{1}{4}c u(\nu, T) \Rightarrow e(T) = \frac{1}{4}c u(T) = \frac{1}{4}AcT^4$$

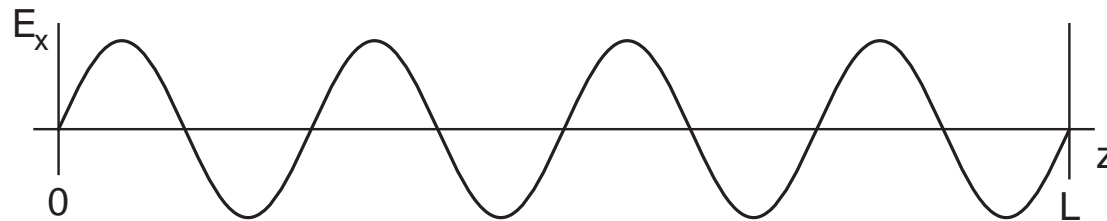
$$e(T) \equiv \sigma T^4$$

This is known as the STEFAN-BOLTZMANN LAW.

$$\sigma = 56.7 \times 10^{-9} \text{ watts}/m^2K^4$$

Statistical Mechanical Approach

\mathcal{H} ? Single normal mode (plane standing wave) in a rectangular conducting cavity.



$$\vec{E}_{0,0,n,\vec{1}_x}(\vec{r}, t) = E(t) \sin(n\pi z/L) \vec{1}_x$$

$$\vec{B}_{0,0,n,\vec{1}_y}(\vec{r}, t) = (n\pi c^2/L)^{-1} \dot{E}(t) \cos(n\pi z/L) \vec{1}_y$$

$$\text{Energy density} = \frac{1}{2}\epsilon_0 \vec{E} \cdot \vec{E} + \frac{1}{2}\frac{1}{\mu_0} \vec{B} \cdot \vec{B} \quad [\text{no } \vec{r} \text{ or } t \text{ average}]$$

$$\begin{aligned} \mathcal{H} &= \frac{V}{2} \left[\frac{1}{2}\epsilon_0 E^2(t) + \frac{1}{2}\frac{1}{\mu_0} (n\pi c^2/L)^{-2} \dot{E}^2(t) \right] \\ &= \frac{V}{2} \frac{\epsilon_0}{2} \left[E^2(t) + (n\pi c/L)^{-2} \dot{E}^2(t) \right] \end{aligned}$$

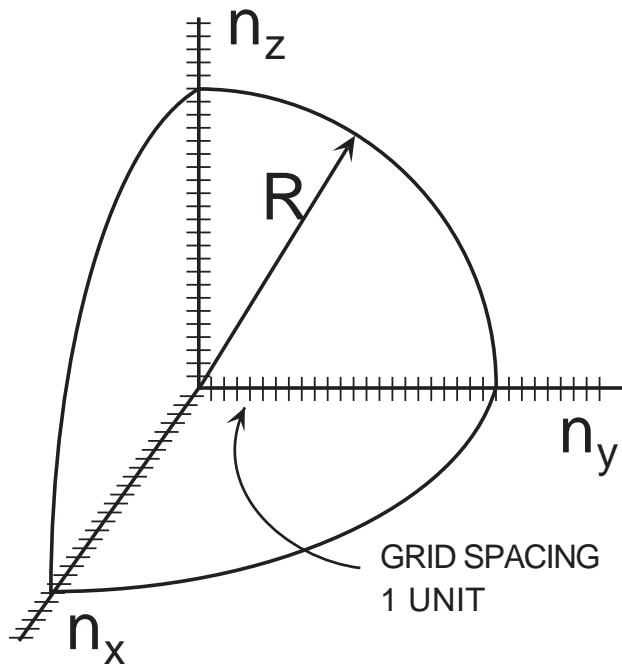
⇒ Each mode corresponds to a harmonic oscillator.

Count the modes.

$$\vec{E}_{n_x, n_y, n_z} = |E| \vec{e}_j \sin(n_x \pi x / L) \sin(n_y \pi y / L) \sin(n_z \pi z / L) e^{i\omega t}$$

The unit polarization vector \vec{e}_j has 2 possible orthogonal directions and $n_i = 1, 2, 3 \dots$.

$$\frac{\partial^2 \vec{E}}{\partial t^2} - c^2 \vec{\nabla}^2 E = 0 \quad \Rightarrow \quad \omega^2 = \left(\frac{\pi c}{L} \right)^2 (n_x^2 + n_y^2 + n_z^2)$$



If the radian frequency $< \omega$

$$R = \sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{L}{\pi c} \omega$$

modes (freq. $< \omega$)

$$= 2 \times \frac{1}{8} \times \frac{4}{3} \pi R^3$$

$$= \frac{\pi}{3} \left(\frac{L}{\pi c} \right)^3 \omega^3$$

$$D(\omega) = \frac{d\#}{d\omega} = \pi \left(\frac{L}{\pi c} \right)^3 \omega^2 = \frac{V}{\pi^2 c^3} \omega^2$$

