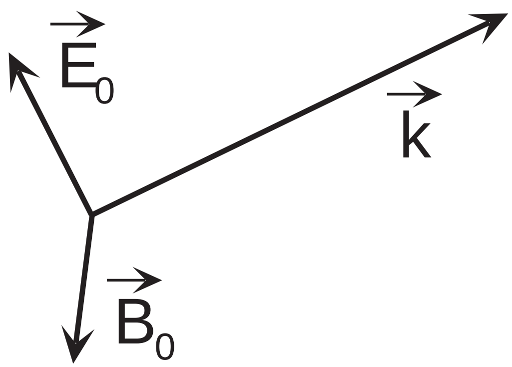
8.044 Statistical Physics I
Spring 2008

Thermal Radiation Radiation in thermal equilibrium with its surroundings

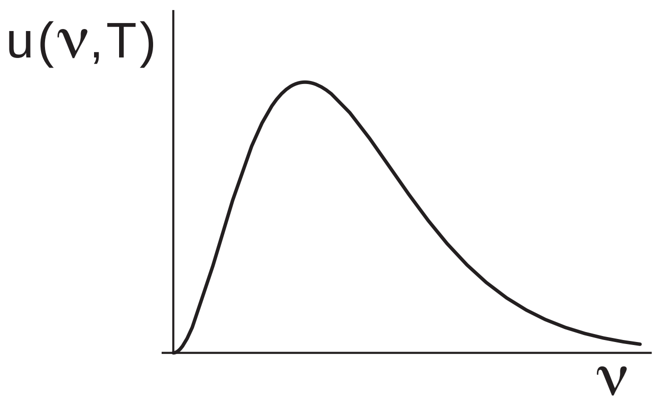


$$\vec{E} = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} \qquad \omega = c|\vec{k}|$$

$$\vec{B} = \vec{B}_0\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} \qquad \vec{B}_0 = \vec{1}_k \times \vec{E}_0\,/c$$

Time average energy density	$\overline{u} = \frac{1}{2}\epsilon_0 \|\vec{E}_0\|^2$
Time average energy flux	$\vec{j}_{\mathrm{E}} = (c\overline{u})\,\vec{1}_k$
Time average pressure ($\perp$ to $\vec{k}$)	$P = \overline{u}$

Thermal radiation has a continuous distribution of frequencies.

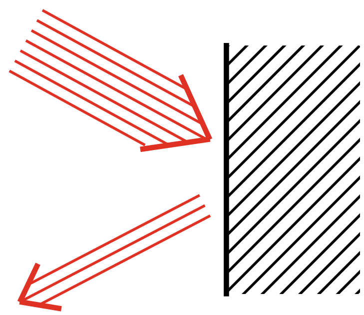


Peaks near $h\nu = 3k_BT$

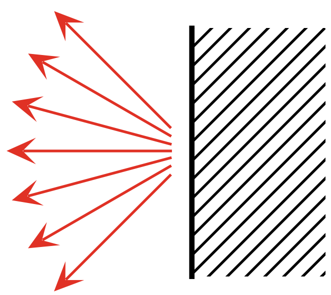
$(h/k_B \sim 5 \times 10^{-11}$ K-sec$)$

Spectral Region	ν (Hz)	T (K)	Thermal Rad.
Radio	10^6	5×10^{-5}	
Microwave	10^{10}	0.5	cosmic background
Infrared	10^{13}	5×10^2	room temp.
Visible	$\frac{1}{2} \times 10^{15}$	2×10^4	sun's surface
Ultraviolet	10^{16}	5×10^5	
X ray	10^{18}	5×10^7	black holes
γ ray	10^{21}	5×10^{10}	

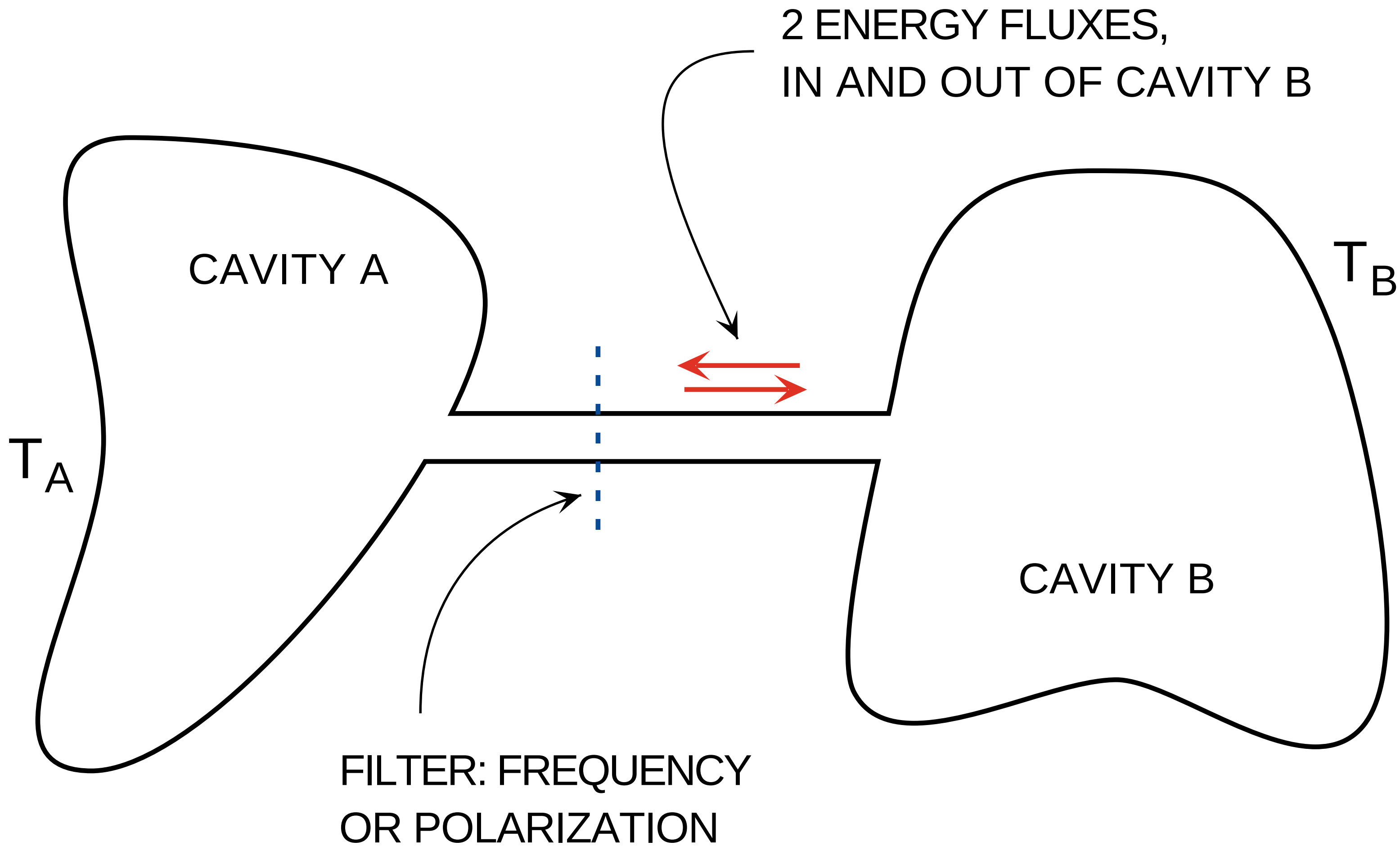
$$\text{ABSORPTIVITY } \alpha(\nu,T) \equiv \left\langle \frac{\text{ENERGY ABSORBED}}{\text{ENERGY INCIDENT}} \right\rangle_{\text{ISOTROPIC}}$$



$$\text{EMISSIVE POWER } e(\nu,T) \equiv \left\langle \frac{\text{ENERGY EMITTED}}{\text{AREA}} \right\rangle_{\text{ISOTROPIC}}$$



THERMAL RADIATION: PROPERTIES



ASSUME $T_A = T_B$ AND THERMAL EQUILIBRIUM

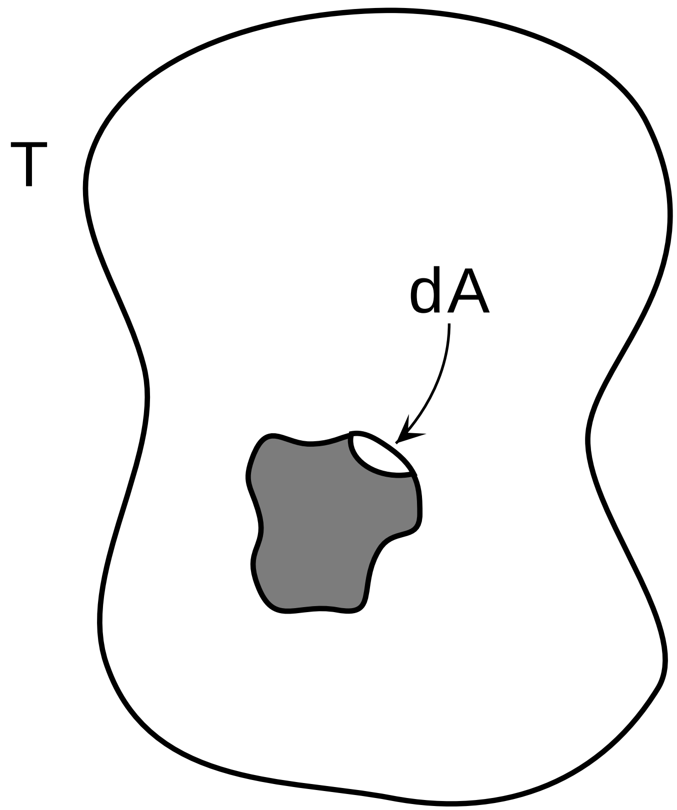
CONCLUSIONS:

- $u(\nu, T)$ is independent of shape and wall material

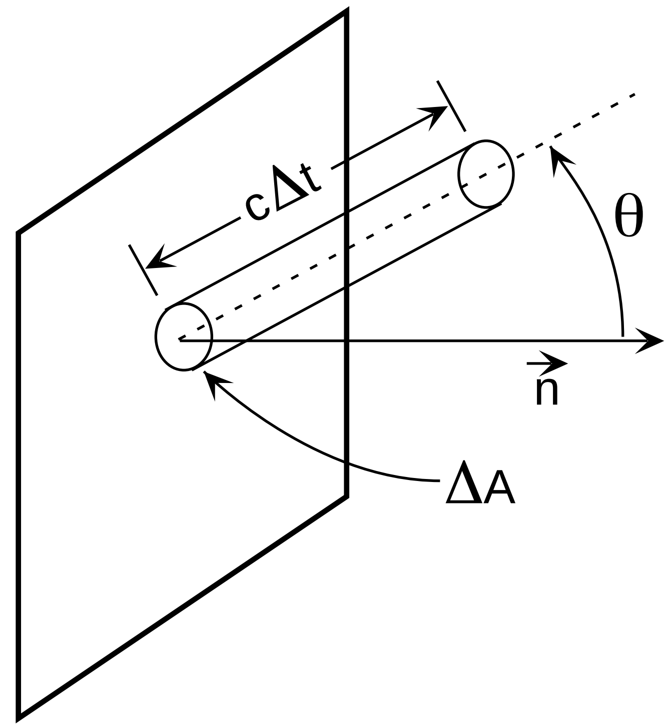
- $u(\nu, T)$ is isotropic

- $u(\nu, T)$ is unpolarized

CONSIDER AN OBJECT IN THE CAVITY, IN THERMAL EQUILIBRIUM



COMPUTE THE ENERGY FLUX



$$
\begin{aligned}
\Delta E &= \int (E \text{ in cylinder})\, p(\theta,\phi)\, d\theta\, d\phi \\
&= \int (u \;\; \Delta A \;\; \cos\theta \;\; c\Delta t) \left(\frac{\sin\theta}{2}\, \frac{1}{2\pi}\right)\, d\theta\, d\phi \\
&= c\, u\, \Delta A\, \Delta t\, \underbrace{\int_0^{\pi/2} \frac{\cos\theta \sin\theta}{2}\, d\theta}_{1/4}\, \underbrace{\int_0^{2\pi} \frac{1}{2\pi}\, d\phi}_{1}
\end{aligned}
$$

$\Rightarrow$ energy flux onto $dA = \underline{\frac{1}{4} c\, u(\nu, T)}$

Momentum Flux

Plane wave momentum density $\vec{p} = \frac{u}{c}\,\vec{1}_k$

$|\Delta p| = 2|p_\perp|$ since $\vec{p}_{\perp\,\text{in}} = -\vec{p}_{\perp\,\text{out}}$

$$
\begin{aligned}
|\Delta p|_\nu &= \int \left(\frac{2\cos\theta}{c}\right)(E \text{ in cylinder})\, p(\theta,\phi)\, d\theta\, d\phi \\
&= u(\nu,T)\,\Delta A\,\Delta t \underbrace{\int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta}_{1/3} \underbrace{\int_0^{2\pi} \frac{1}{2\pi}\, d\phi}_{1} \\
&= \tfrac{1}{3} u(\nu,T)\,\Delta A\,\Delta t
\end{aligned}
$$

$$
\Rightarrow P(T) = \underline{\tfrac{1}{3}\int_0^\infty u(\nu,T)\, d\nu}
$$

Apply detailed balance to the object in the cavity.

$$E_{\text{out}} = E_{\text{in}}$$

$$e\,dA = \alpha\left(\tfrac{1}{4}c\,u(\nu,T)\right)dA$$

$$\Rightarrow \quad \frac{e(\nu,T)}{\alpha(\nu,T)} = \tfrac{1}{4}c\,u(\nu,T)$$

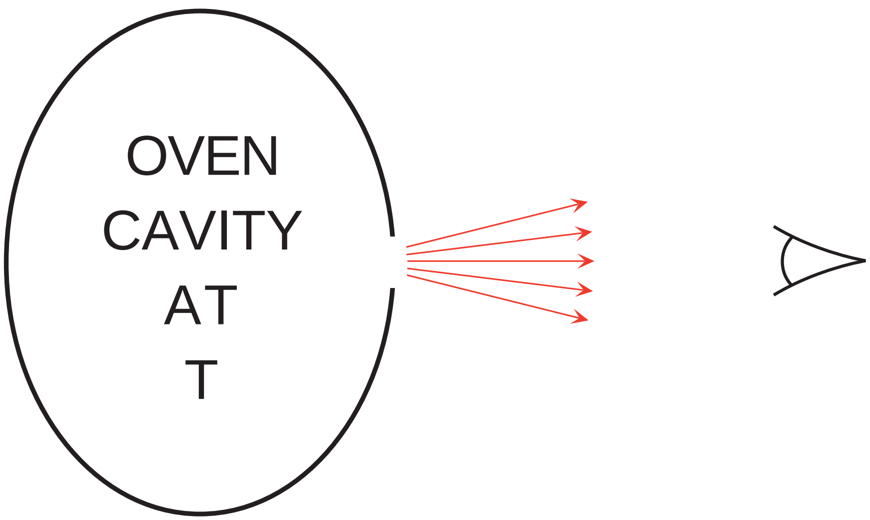
This ratio has a universal form for <u>all</u> materials.

The result is known as KIRCHOFF'S LAW.

Black Body Radiation

If $\alpha \equiv 1 \equiv$ "Black"

Then $e(\nu, T) = \frac{1}{4} c\, u(\nu, T)$



Measure $e(\nu, T)$

and obtain $u(\nu, T)$