

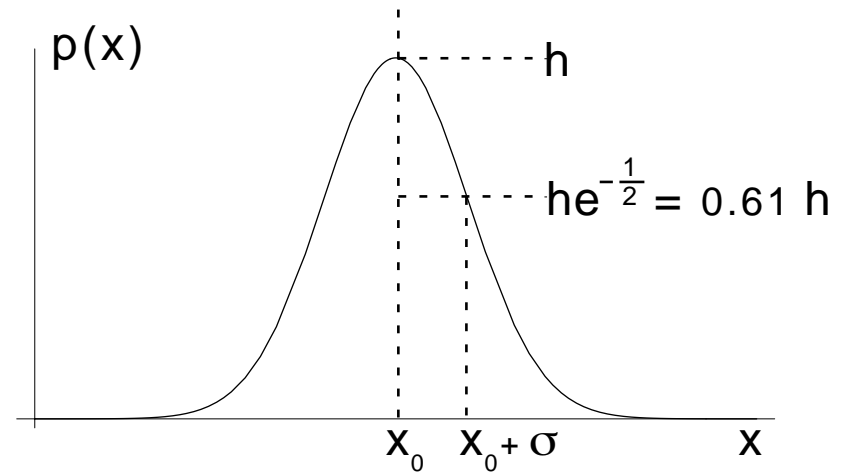
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8.044 Statistical Physics I
Spring 2008

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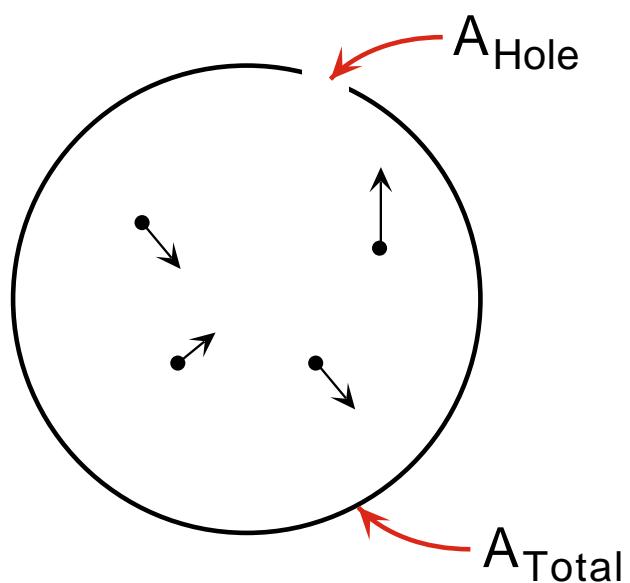
Gaussian density (memorize)

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$



2 parameters

Example Atom escaping from a cavity



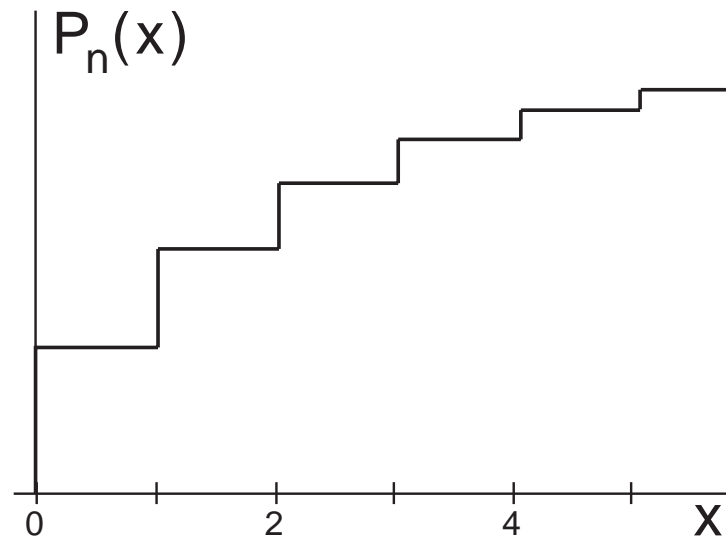
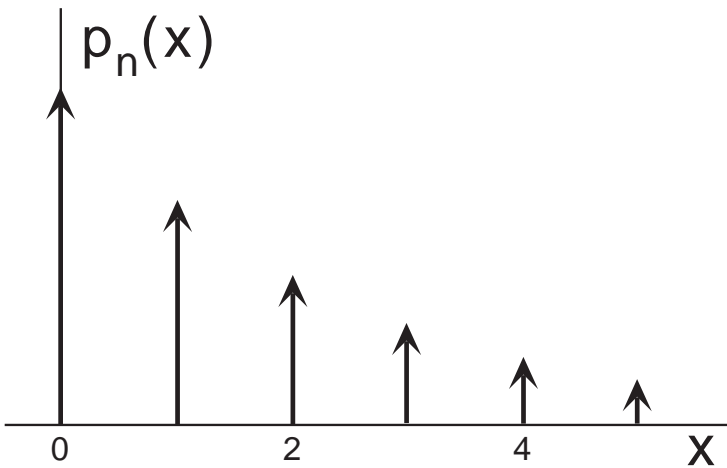
Atom escapes after n^{th} wall encounter

$$p(n) = \left(\frac{A_H}{A_T}\right) \left(1 - \frac{A_H}{A_T}\right)^n$$

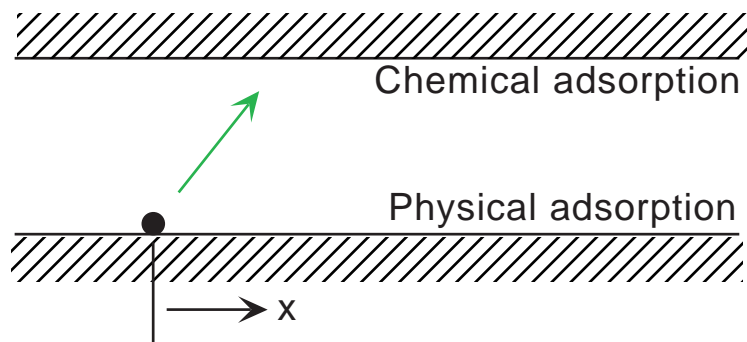
$$n = 0, 1, 2, \dots$$

$$p_n(x) = \sum_{n=0}^{\infty} \left(\frac{A_H}{A_T}\right) \left(1 - \frac{A_H}{A_T}\right)^n \delta(x - n)$$

Called a geometric or a Bose-Einstein density

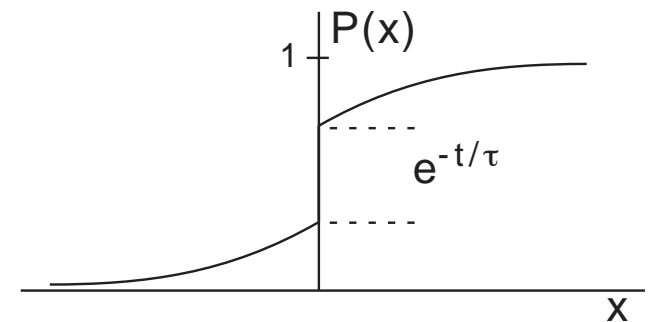
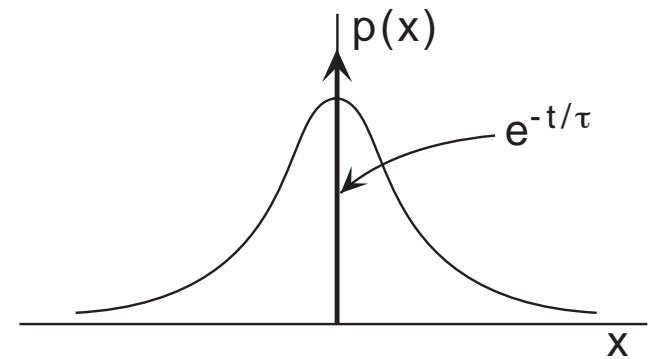


Example Mixed, t dependent RV



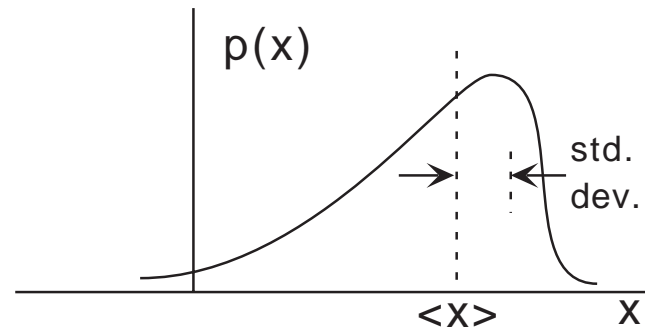
Given: atom on bottom at $t = 0$

$$p(x) = e^{-t/\tau} \delta(x) + (1 - e^{-t/\tau}) f(x)$$



Averages

$$\langle f(x) \rangle \equiv \int_{-\infty}^{\infty} f(x)p(x) dx$$



$\langle x \rangle$ is the **mean**

$\langle x^2 \rangle$ is the **mean square**

$$\begin{aligned} \langle (x - \langle x \rangle)^2 \rangle &= \langle (x^2 - 2x \langle x \rangle + \langle x \rangle^2) \rangle \\ &= \langle x^2 \rangle - 2 \langle x \rangle^2 + \langle x \rangle^2 \\ &= \langle x^2 \rangle - \langle x \rangle^2 \text{ is the } \mathbf{variance} \\ &\equiv \mathbf{(standard deviation)^2} \end{aligned}$$

Gaussian

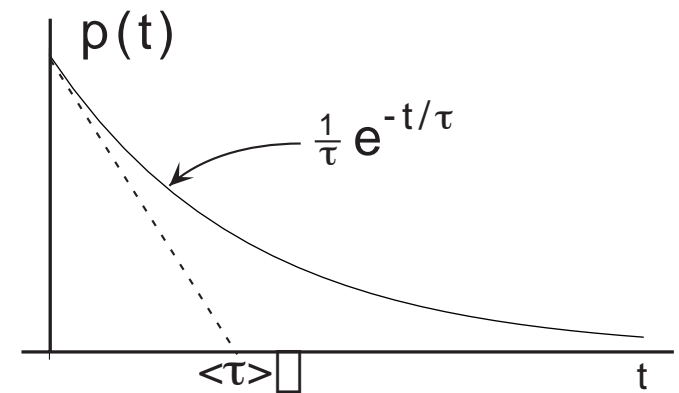
$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-x_0)^2/2\sigma^2}$$

$$\langle x \rangle = x_0$$

$$\text{Var}(x) = \sigma^2$$

Controlled separately

Exponential

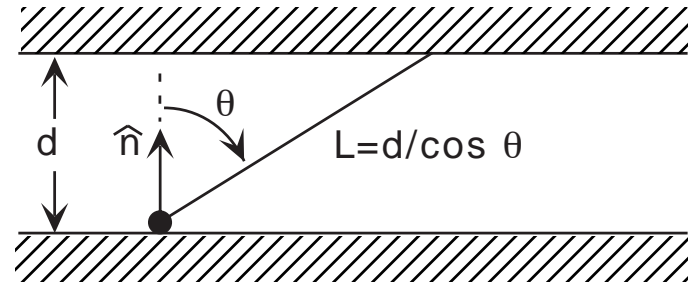


$$\langle t \rangle = \tau$$

$$\text{Var}(t) = \tau^2$$

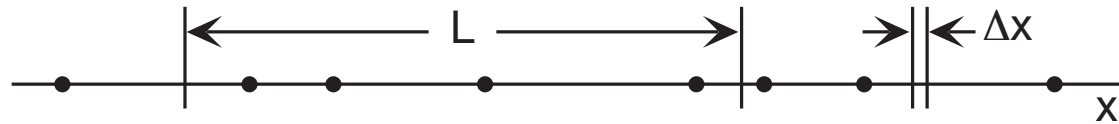
Determined by same
parameter

Example Mean free path



$$\begin{aligned}\langle L \rangle &= \int_0^{\pi/2} (d / \cos \theta) p(\theta) d\theta \\ &= \int_0^{\pi/2} (d / \cos \theta) 2 \sin \theta \cos \theta d\theta = 2d \int_0^{\pi/2} \sin \theta d\theta \\ &= 2d \left[-\cos \theta \right]_0^{\pi/2} = 2d\end{aligned}$$

Poisson density Events occur randomly along a line
at a rate r per unit length



$$p(1) \rightarrow r\Delta x \text{ as } \Delta x \rightarrow 0$$

Events are statistically independent

$$p(n) = \frac{1}{n!} (rL)^n e^{-rL} = \frac{1}{n!} \langle n \rangle^n e^{-\langle n \rangle}$$

Examples of Poisson probability densities

