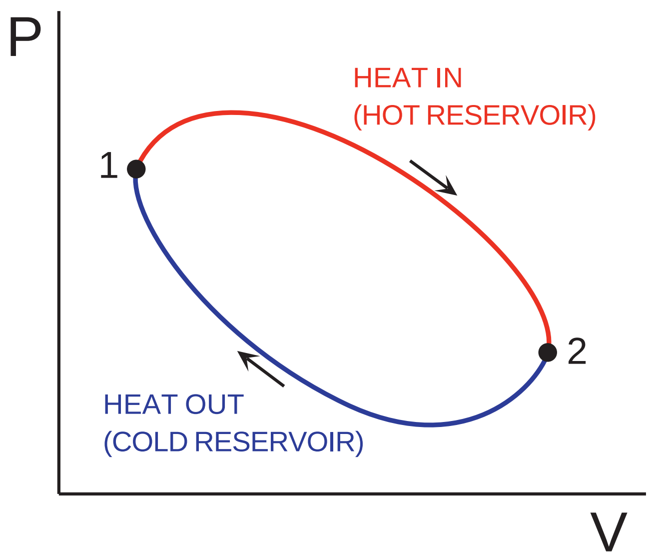
8.044 Statistical Physics I
Spring 2008

Heat Engine

- Takes a substance around a closed cycle
- Heat is put into the substance and taken out
- Work is taken out
- Efficiency, $\eta \equiv$ (work out) / (heat in)

Closed cycle $\Rightarrow \Delta U = \Delta Q + \Delta W = 0 \Rightarrow \Delta Q = -\Delta W$



$$\Delta Q \equiv \oint đQ$$

$$= \underbrace{\int_1^2 đQ}_{\equiv |Q_H|} + \underbrace{\int_2^1 đQ}_{\equiv -|Q_C|}$$

Most General Case

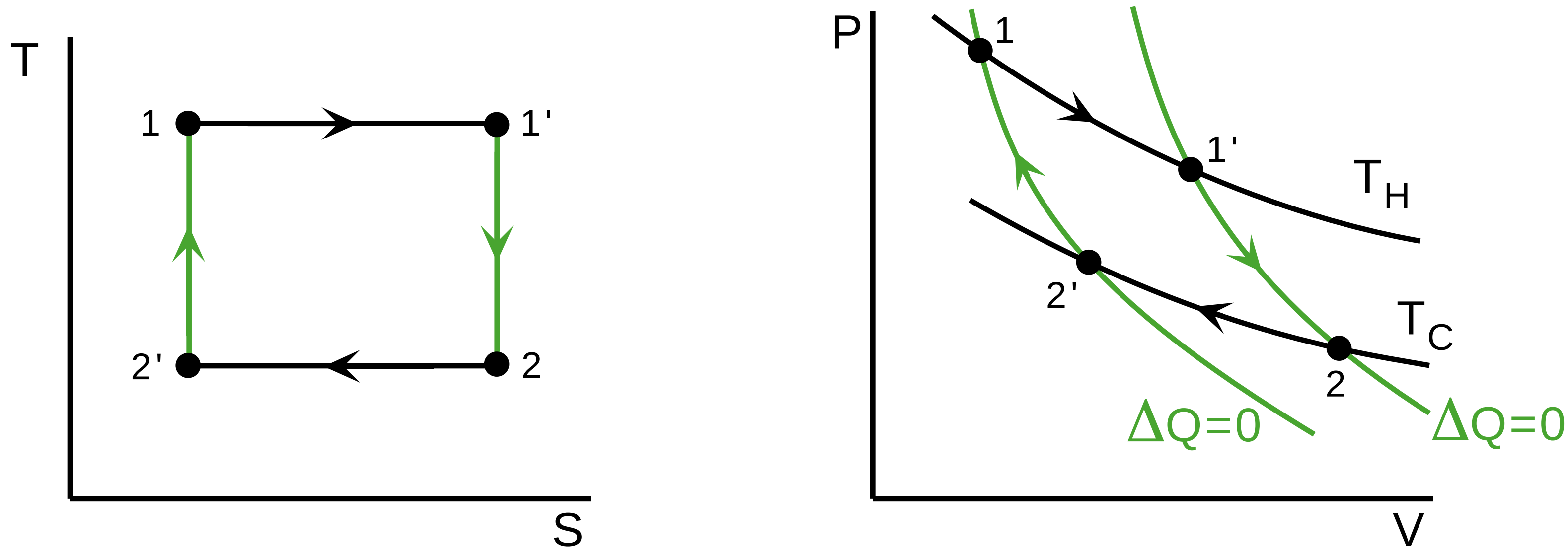
$$W_{\text{out}} = -\Delta W = \Delta Q = |Q_H| - |Q_C|$$

$$\eta \equiv \frac{W_{\text{out}}}{|Q_H|} = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \frac{|Q_C|}{|Q_H|}$$

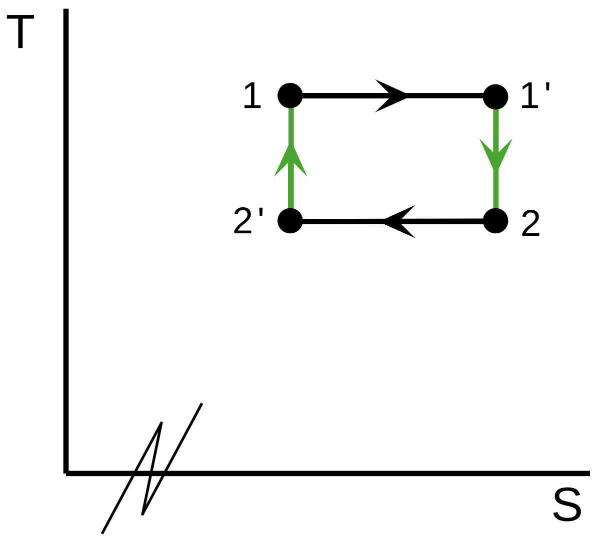
Very Special Case   Example: Carnot Cycle

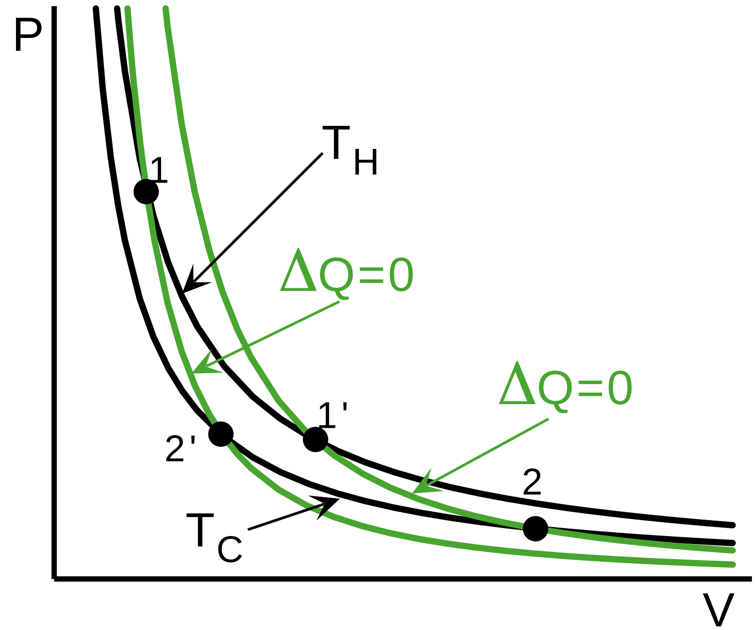
- Any substance

- Isothermal and adiabatic changes



Use the second law: $đQ \leq TdS$





DRAWN TO SCALE FOR AN IDEAL GAS: PV=NkT

$T_H = 1.5\ T_C \quad S_{HIGH} - S_{LOW} = (3/2)\ Nk\ \ln 2$

$$|Q_H| \leq T_H \int_1^{1'} dS$$

$$-|Q_C| \leq T_C \int_2^{2'} dS, \quad \text{use} \quad \int_2^{2'} dS = -\int_1^{1'} dS$$

$$\leq -T_C \int_1^{1'} dS \Rightarrow |Q_C| \geq T_C \int_1^{1'} dS \quad \text{and} \quad \frac{|Q_C|}{|Q_H|} \geq \frac{T_C}{T_H}$$

$$\eta = 1 - \frac{|Q_C|}{|Q_H|} \; \underline{\leq 1 - \frac{T_C}{T_H}}$$

Arbitrary Engine Cycle

$\not{d}Q \leq TdS$ for each element along the path.

$$\underbrace{\int_1^2 \not{d}Q}_{|Q_H|} \;\leq\; \int_1^2 TdS \;\leq\; T_{\text{max}} \underbrace{\int_1^2 dS}_{\text{positive}}$$

$$\int_2^1 đQ \;\leq\; \int_2^1 T dS, \quad \text{both sides are negative}$$

$$|Q_C| \;\geq\; |\int_2^1 T dS| \geq T_{\text{min}} \, |\int_2^1 dS|$$

$$\geq\; T_{\text{min}} \, |\int_1^2 dS| \;\; \text{since} \oint dS = 0$$

$$\frac{|Q_C|}{|Q_H|} \;\geq\; \frac{T_{\text{min}}}{T_{\text{max}}}$$

$$\eta = 1 - \frac{|Q_C|}{|Q_H|} \;\underline{\leq 1 - \frac{T_{\text{min}}}{T_{\text{max}}}}$$

Carnot cycle in a pure thermodynamic approach

- Used to define temp. $\eta = 1 - \frac{|Q_C|}{|Q_H|} \equiv 1 - \frac{T_C}{T_H}$

- Used to define the entropy

$$\oint \frac{đQ}{T} \leq 0 \Rightarrow \frac{đQ}{T} \text{ is an exact differential}$$