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8.044 Statistical Physics I
Spring 2008

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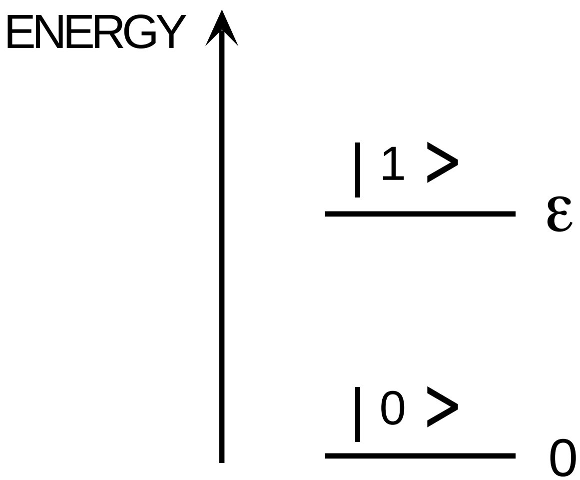
# EXAMPLE 2 Level System

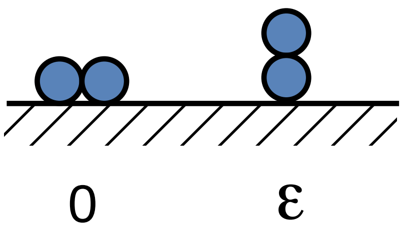
Ensemble of N "independent" systems



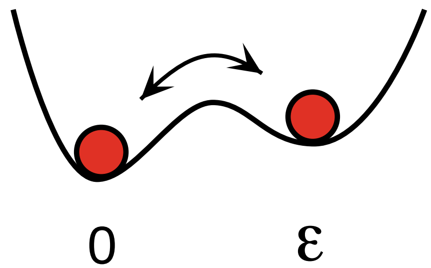

$$N = N_0 + N_1$$

$$E = \varepsilon N_1$$

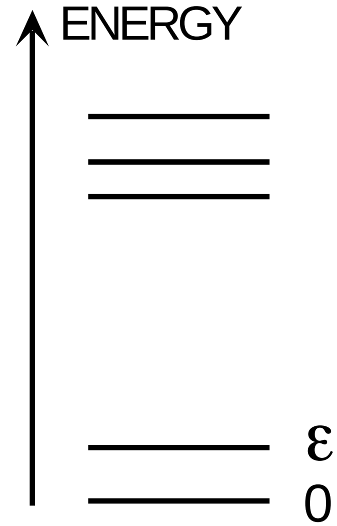
SURFACE MOLECULES




IONS IN A CRYSTAL




LOWEST LYING STATES



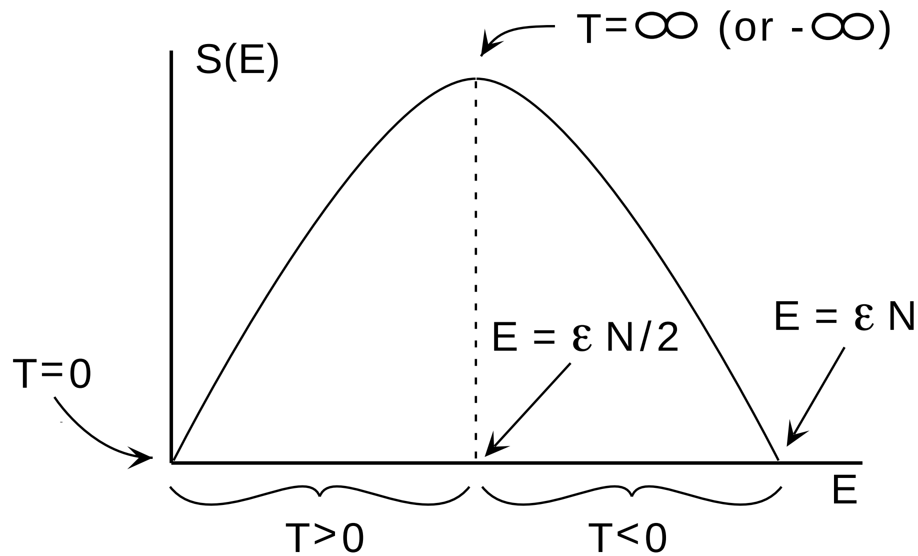

- $E \leftrightarrow N_1$
- NO WORK POSSIBLE (JUST HEAT FLOW)

$$\Omega(E) = \frac{N!}{N_1!(N-N_1)!}$$

1 when $N_1 = 0$ or $N$

Maximum when $N_1 = N/2$

$$S(E) = k \ln \Omega(E)$$



$$\ln N! \approx N \ln N - N$$

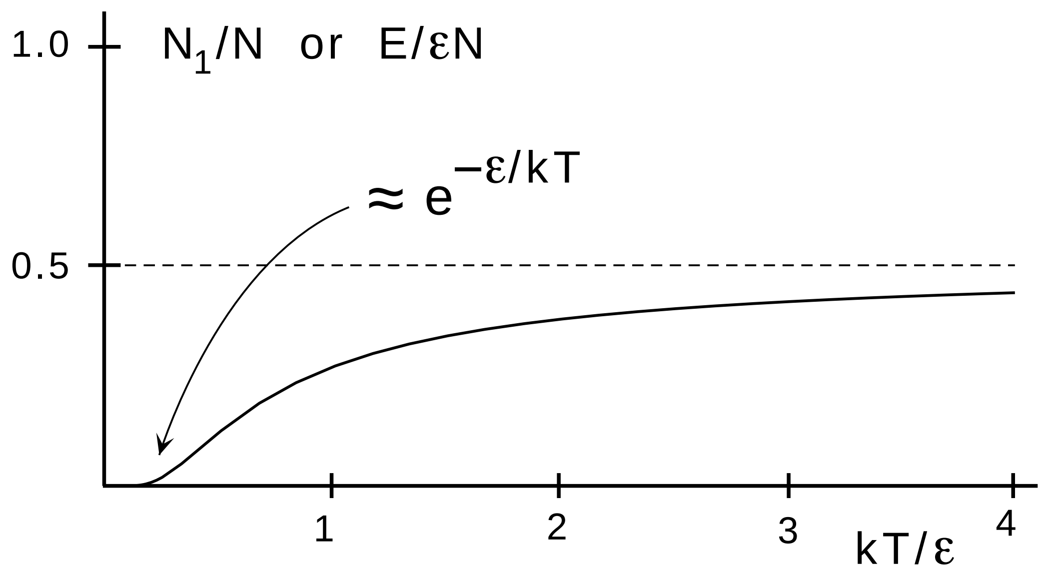
$$S(E) = k[N \ln N - N_1 \ln N_1 - (N - N_1) \ln(N - N_1) \\ - N + N_1 + N - N_1]$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N = \frac{\partial S}{\partial N_1} \underbrace{\frac{\partial N_1}{\partial E}}_{1/\epsilon} = \frac{k}{\epsilon}[-1 - \ln N_1 + 1 + \ln(N - N_1)]$$

$$= \frac{k}{\epsilon} \ln\left(\frac{N - N_1}{N_1}\right) = \frac{k}{\epsilon} \ln\left(\frac{N}{N_1} - 1\right)$$

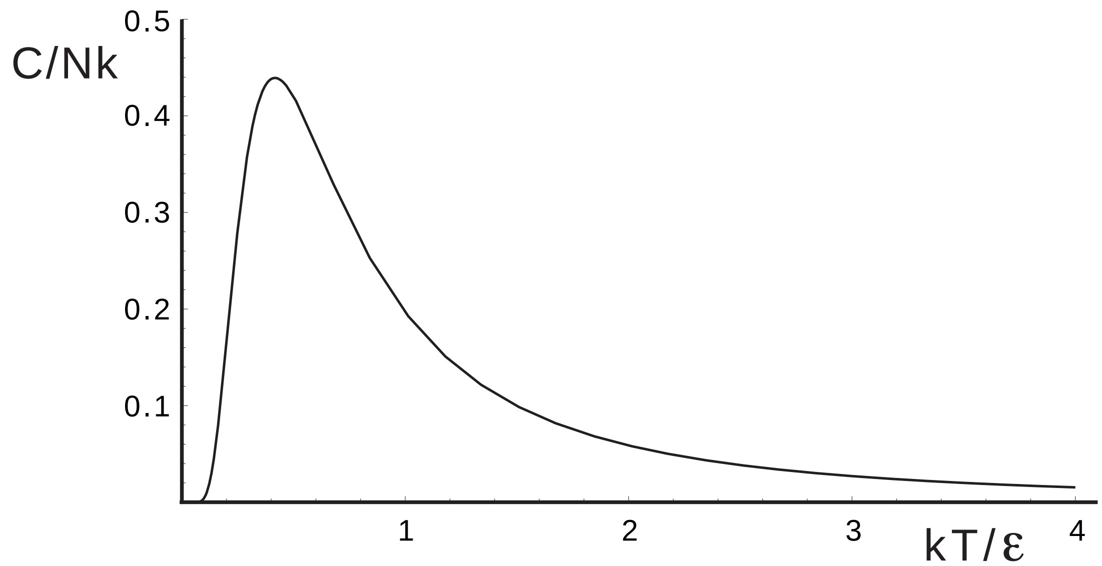
$$\frac{N}{N_1} - 1 = e^{\epsilon/kT} \quad \rightarrow \quad N_1 = \frac{N}{e^{\epsilon/kT} + 1}$$

$$E = \epsilon N_1 = \frac{\epsilon N}{e^{\epsilon/kT} + 1}$$



$$C \equiv \frac{\partial E}{\partial T} = Nk\left(\frac{\epsilon}{kT}\right)^2 \frac{e^{\epsilon/kT}}{(e^{\epsilon/kT}+1)^2}$$

$$\rightarrow Nk\left(\frac{\epsilon}{kT}\right)^2 e^{-\epsilon/kT} \quad \text{low } T, \qquad \rightarrow \frac{Nk}{4}\left(\frac{\epsilon}{kT}\right)^2 \quad \text{high } T$$



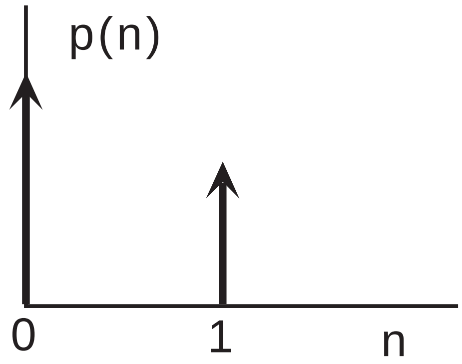
$p(n) = ? \quad n = 0, 1$ $$p(n) = \frac{\Omega'}{\Omega}$$

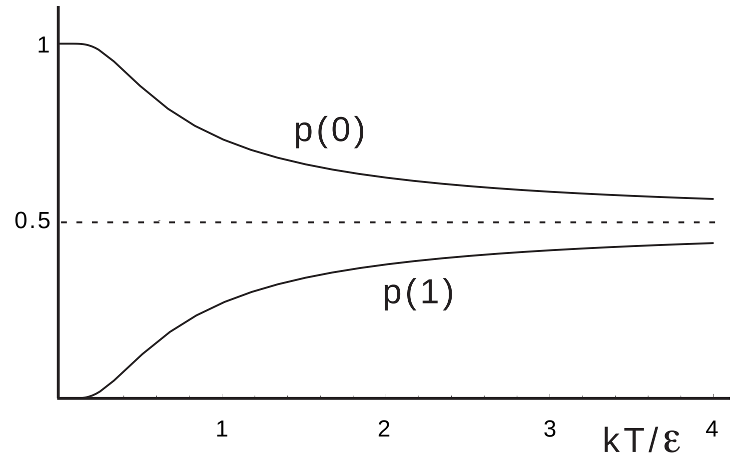
In $\Omega'$ $\quad N \rightarrow N - 1 \quad$ and $\quad N_1 \rightarrow N_1 - n$

$$p(n) = \frac{\frac{(N-1)!}{(N_1-n)!(N-1-N_1+n)!}}{\frac{N!}{N_1!(N-N_1)!}}$$

$$p(n) = \underbrace{\frac{(N-1)!}{N!}}_{1/N} \; \underbrace{\frac{N_1!}{(N_1-n)!}}_{\substack{1 \;\; n=0 \\ N_1 \;\; n=1}} \; \underbrace{\frac{(N-N_1)!}{(N-N_1-1+n)!}}_{\substack{N-N_1 \;\; n=0 \\ 1 \;\; n=1}}$$

$$\left.\begin{aligned} p(0) \; &= \tfrac{N-N_1}{N} = 1 - \tfrac{N_1}{N} \\ p(1) \; &= \tfrac{N_1}{N} = [e^{\epsilon/kT}+1]^{-1} \end{aligned}\right\} \quad p(0) + p(1) = 1$$




$$E = (0) N\, p(0) + (\epsilon) N\, p(1) = \frac{\epsilon N}{e^{\epsilon/kT} + 1}$$

But we knew $E$, so we could have worked backwards to find $p(1)$.

# MICROCANONICAL ENSEMBLE

MODEL THE SYSTEM

↓

FIND $\Omega(E,N,V \ldots.)$

THERMODYNAMIC RESULTS

FIND $S(E,N,V \ldots.)$

↓

$$\left.\frac{\partial S}{\partial E}\right|_{N,V} = \frac{1}{T}$$

$$\left.\frac{\partial S}{\partial V}\right|_{E,N} = \frac{P}{T}$$

etc.

MICROSCOPIC INFORMATION

$P(\sim\sim\sim) = \Omega' / \Omega$

The microcanonical ensemble is the starting point for Statistical Mechanics.

- We will no longer use it to solve problems.

- We will develop our understanding of the $2^{ND}$ law.

- We will derive the canonical ensemble, the real workhorse of S.M.