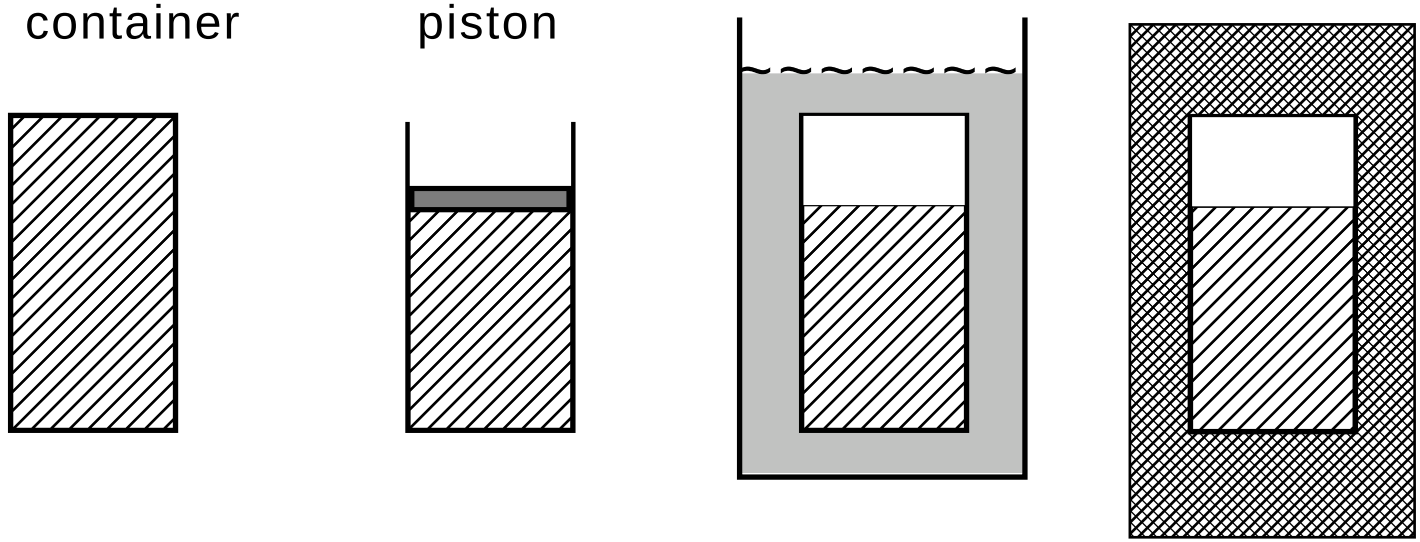
8.044 Statistical Physics I
Spring 2008

<u>Paths</u> Experimental conditions, not just math

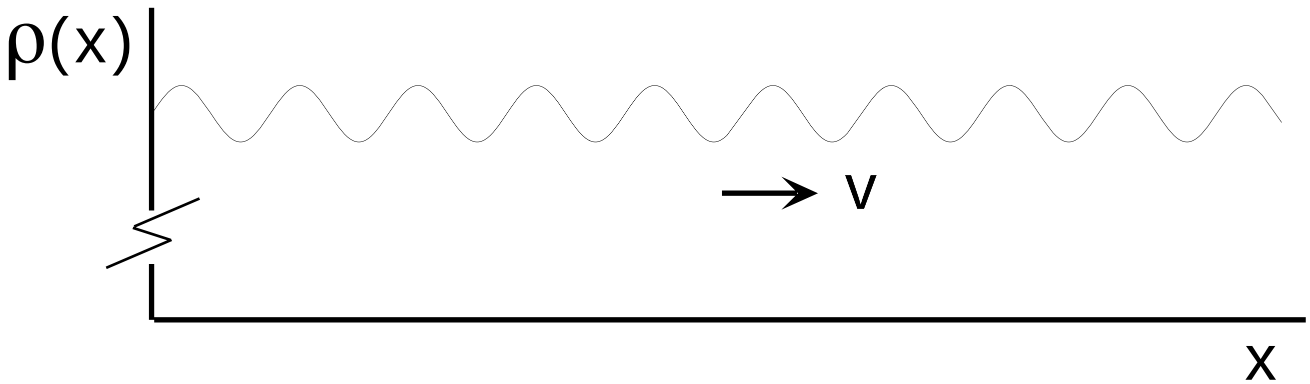
fills container	floating piston	bath	insulation



$\Delta V=0$	$\Delta P=0$	$\Delta T=0$	$\Delta Q=0$

$\Delta Q = 0$ could come from time considerations

<u>Example</u> Sound Wave



too fast for heat to flow out of compressed regions

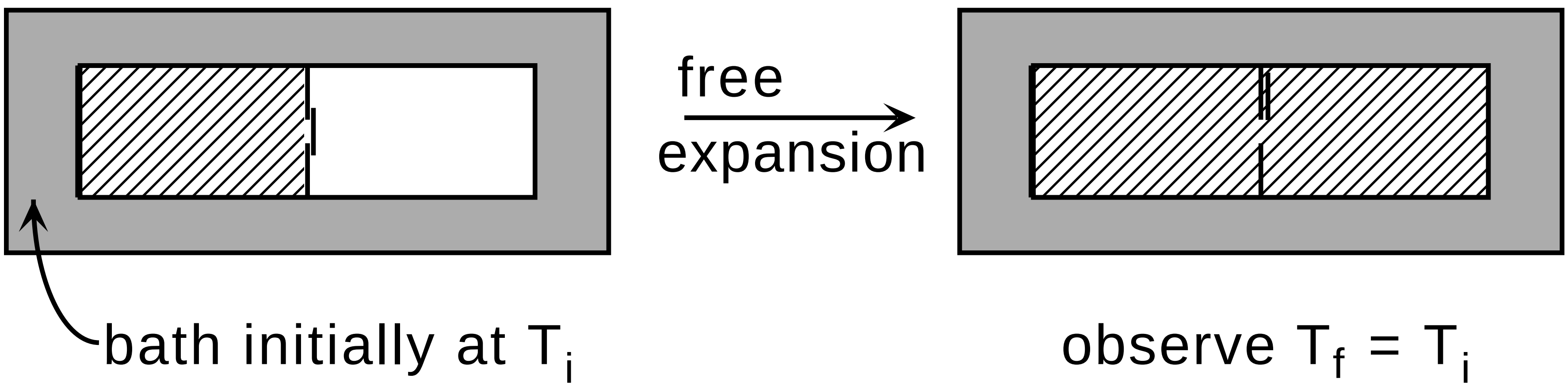
$$v = \sqrt{\frac{1}{\rho \kappa_S}}$$

Example Hydrostatic system: an ideal gas, PV=NkT

New information $$\left.\frac{\partial U}{\partial V}\right|_T = 0\,,$$

3 possible sources

- Experiment



No work done so $\Delta W = 0$

$T_f = T_i \Rightarrow \Delta Q = 0$

together $\quad \Rightarrow \quad \underbrace{\Delta U = 0}_{\text{here}} \quad \rightarrow \quad \underbrace{(\partial U/\partial V)_T = 0}_{\text{quasi-static changes}}$

- Physics: no interactions, single particle energies only $\Rightarrow (\partial U/\partial V)_T = 0$

- Thermo: 2^{nd} law $+ \; (PV = NkT) \Rightarrow (\partial U/\partial V)_T = 0$

Consequences

$$dU = \underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_{C_V} dT + \underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{0} dV$$

$$U = \int_0^T C_V(T')\,dT' + \underbrace{\text{constant}}_{\text{set}=0}$$

In a monatomic gas one observes $C_V = \frac{3}{2}Nk$.

Then the above result gives $U = C_V T = \frac{3}{2}NkT$.

$$
\begin{aligned}
C_P - C_V &= (\underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{0} + P)\ \underbrace{\left(\frac{\partial V}{\partial T}\right)_P}_{\frac{\partial}{\partial T}(NkT/P)_P = Nk/P} \\
&= Nk \qquad \text{for any ideal gas}
\end{aligned}
$$

Applying this to the monatomic gas one finds

$$
\begin{aligned}
C_P &= \frac{3}{2}Nk + Nk = \frac{5}{2}Nk \\
\gamma &\equiv C_P/C_V = \frac{5}{3}
\end{aligned}
$$

Adiabatic Changes $\quad đQ = 0$

Find the equation for the path.

Consider a hydrostatic example.

$$đQ = \underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_{C_V} dT + \underbrace{\left(\left(\frac{\partial U}{\partial V}\right)_T + P\right)}_{(C_P - C_V)/\alpha V} dV = 0$$

$$\left(\frac{\partial T}{\partial V}\right)_{\Delta Q=0} = -\left(\frac{C_P - C_V)}{C_V}\right)\frac{1}{\alpha V} = -\frac{(\gamma - 1)}{\alpha V}$$

This constraint defines the path.

Apply this relation to an ideal gas.

$$\alpha \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{V}\,\frac{\partial}{\partial T}\left(\frac{NkT}{P}\right)_P = \frac{1}{V}\left(\frac{Nk}{P}\right) = \frac{1}{V}\frac{V}{T} = \frac{1}{T}$$

Path

$$\frac{dT}{dV} \;=\; -(\gamma-1)\frac{T}{V}$$

$$\frac{dT}{T} \;=\; -(\gamma-1)\frac{dV}{V} \rightarrow \ln\left(\frac{T}{T_0}\right) = -(\gamma-1)\ln\frac{V}{V_0}$$

$$\left(\frac{T}{T_0}\right) \;=\; \left(\frac{V}{V_0}\right)^{-(\gamma-1)}$$

Adiabatic

Isothermal

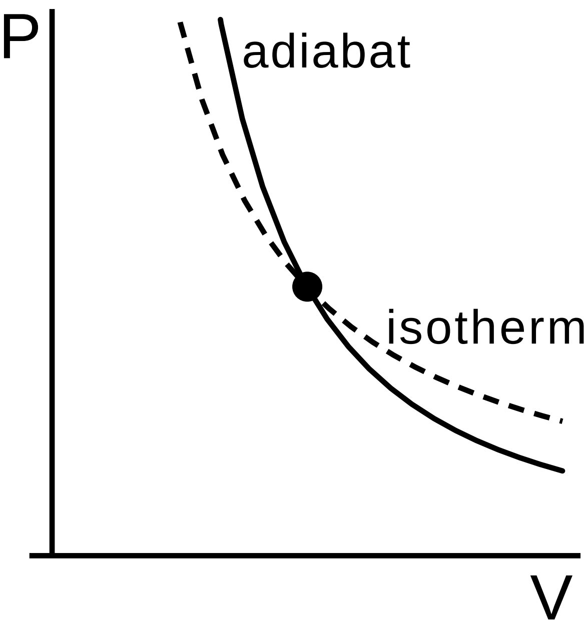
$$TV^{\gamma-1} = c$$

$$PV^{\gamma} = c'$$

$\gamma = 5/3$ (monatomic)

$$P \propto V^{-5/3}$$

$$\frac{dP}{dV} = -\frac{5}{3}\frac{P}{V}$$



$$PV = c''$$

$$P \propto V^{-1}$$

$$\frac{dP}{dV} = -\frac{P}{V}$$

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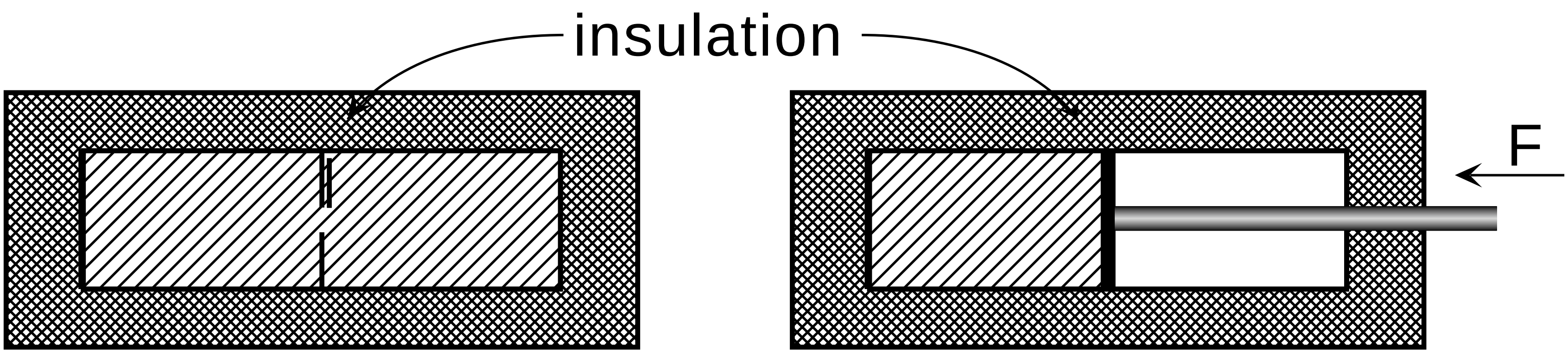
Expansion of an ideal gas



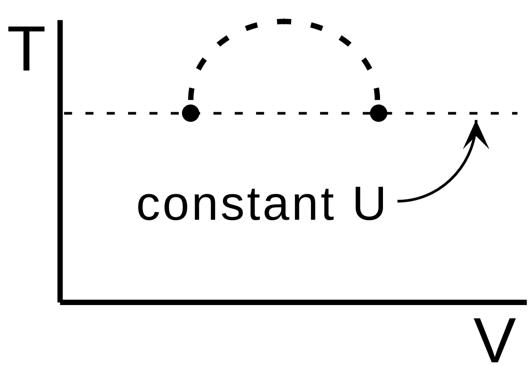
rupture diaphragm

adiabatic $\Delta Q = 0$

not quasistatic

$\Delta W = 0$

$\rightarrow \Delta U = 0$



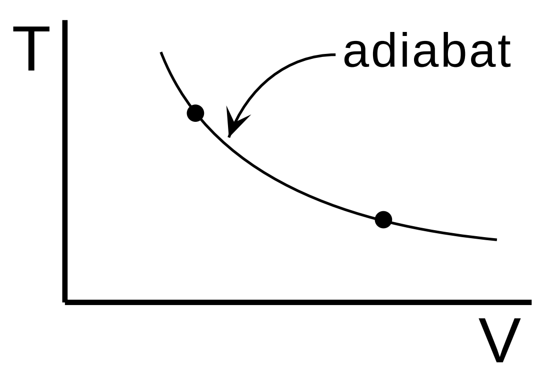
slowly move piston

adiabatic $\Delta Q = 0$

quasistatic

ΔW is negative

$\rightarrow \Delta U =$ is negative



Starting with a few known facts,

1^{st} law, $\not{d}W$, and state function math,

one can find

relations between some thermodynamic quantities,

a general expression for dU,

and the adiabatic constraint.

Adding models for the equation of state and the heat capacity allows one to find

the internal energy U

and the adiabatic <u>path</u>.