8.044 Statistical Physics I
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

8.044: Statistical Physics I — Spring Term 2008

Problem Set #4

Problem 1: Moving Impurities along a Wire

In an effort to clear impurities from a fabricated nanowire, a laser beam is swept repeatedly along the wire in the presence of a parallel electric field. After one sweep, an impurity initially at $x = 0$ has the following probability density of being found at a new position x:

$$\begin{aligned} p(x) &= \frac{1}{3}\delta(x) + \frac{2}{3a}\exp[-x/a] \qquad && x \geq 0 \\ &= 0 && \text{elsewhere} \end{aligned}$$

where a is some characteristic length.

a) Find the cumulative function $P(x)$. Make a sketch of the result which displays all of the important features.

b) What is the probability that x will be displaced by at least an amount a by a single sweep of the laser beam.

c) Find the mean and the variance of x in terms of a.

d) Give an approximate probability density for the total distance d the impurity has moved along the wire after 36 sweeps of the laser beam.

Problem 2: Thermal Equilibrium and the Concept of Temperature

Systems A, B, and C are gases with coordinates P, V; P', V'; and P'', V''. When A and C are in thermal equilibrium, the equation

$$PV - nbP - P''V'' = 0$$

is found to be satisfied. When B and C are in equilibrium, the relation

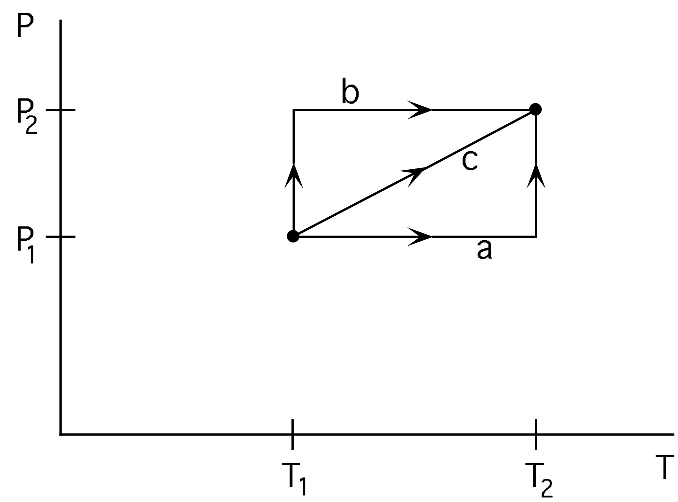
$$P'V' - P''V'' + \frac{nB'P''V''}{V'} = 0$$

holds. The symbols n, b, and B' represent constants.

a) What are the three functions which are equal to one another at thermal equilibrium and each of which is equal to t where t is the empirical temperature?

b) What is the relation expressing thermal equilibrium between A and B?

Problem 3: Work in a Simple Solid



In the simplest model of an elastic solid

$$dV = -V\mathcal{K}_T dP + V\alpha dT$$

where $\mathcal{K}_T$ is the isothermal compressibility and α is the thermal expansion coefficient. Find the work done on the solid as it is taken between state (P_1, T_1) and (P_2, T_2) by each of the three paths indicated in the sketch. Assume that the fractional volume change is small enough that the function $V(P, T)$ which enters the expression for dV can be taken to be constant at $V = V_1 = V(P_1, T_1)$ during the process.

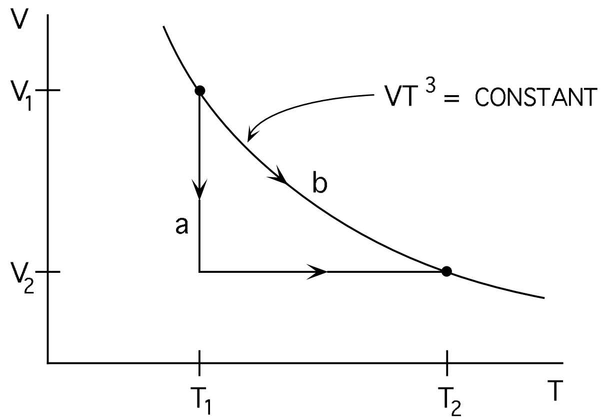
Problem 4: Work in a Non-Ideal Gas

An approximation to the equation of state for a real gas is

$$(P + a/V^2)(V - b) = NkT$$

where a, b, and k are constants. Calculate the work necessary to compress the gas isothermally from V_1 to $V_2 < V_1$.

Problem 5: Work and the Radiation Field



The pressure P due to the thermal equilibrium radiation field inside a cavity depends only on the temperature T of the cavity and not on its volume V,

$$P = \frac{1}{3}\sigma T^4.$$

In this expression σ is a constant. Find the work done on the radiation field as the cavity is taken between states (V_1, T_1) and (V_2, T_2) along the two paths shown in the diagram.